Interaction theory – Photons

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Introduction

- Photons
- Charged particles
- Neutrons

Ionizing radiation

Atoms

Matter

Interaction theory

Dosimetry

Radiation source

- Photons
- Charged particles
- Neutrons

Ionizing radiation

Molecules

Cells

Humans

Effects

H₂O

DNA

Objectives

- To understand primary and secondary effects of ionizing radiation
- How radiation doses are calculated and measured
- To understand the principles of radiation protection, their origin and applications
Contents FYSKJM4710

- Interactions between ionizing radiation and matter
- Radioactive and non-radioactive sources
- Calculations and measurement of absorbed doses (dosimetry)
- Radiation chemistry
- Biological effects of ionizing radiation
- Principles of radiation protection

Relevant issues

- X-ray and CT investigations
- Radiotherapy
- Positron emission tomography
- Radiation protection
- Radiation Biology

Cross section 1

- Cross section s: "target area", effective target covering a certain area
- Proportional to the interaction strength between an incoming particle and the target particle
- Consider two discs, one target and one incoming:

\[
\pi (r_1^2 + r_2^2)
\]

- s is the total area: \( \pi (r_1^2 + r_2^2) \)
Cross section 2

- N particles move towards an area S with n atoms
- Probability of interaction: \( p = \frac{n\sigma}{S} \)
- Number of interacting particles: \( N_p = \frac{Nn\sigma}{S} \)

Cross section 3

- Separate between electronic and atomic cross section
- The cross section depends on:
  - Type of target (nucleus, electron, ...)
  - Type of and energy of incoming particle (photon, electron, ...)
- Cross section calculated with quantum mechanics - here visualized in a classical window

Cross section 4

- Differential cross section with respect to scattering angle

\[
\frac{d\sigma}{d\Omega} = \frac{\text{number of particles scattered into } d\Omega}{\text{number of particles per unit area } d\Omega}
\]

Photon interactions

- Photon represented by a plane wave \( \vec{A}_{\text{in}}(r, t) \) in quantum mechanical calculations
- In principle, two different processes:
  - Absorption \( \vec{A}_{\text{in}}(r, t) \) \( \rightarrow \) \( \vec{A}_{\text{out}}(r, t) \)
  - Scattering \( \vec{A}_{\text{in}}(r, t) \) \( \rightarrow \) \( \vec{A}_{\text{out}}(r, t) \)
- Scattering: coherent (elastic) og incoherent (inelastic)
Coherent (Rayleigh) scattering

- Scattering without loss of energy: $h\nu = h\nu'$
- Photon is absorbed by atom, thereby emitted at a small deflection angle
- Depends on atomic structure and photon energy
- Atomic cross section:
  $$\sigma_R \propto \left( \frac{2}{\hbar \nu'} \right)^2$$

Incoherent (Compton) scattering

- Scattering with loss of energy: $h\nu' < h\nu$
- Photon-electron scattering; electron may be assumed free (i.e. unbound)
- Thomson scattering: low energy limit, $h\nu \to 0$

Compton scattering – kinematics

- Conservation of energy and momentum:
  $$h\nu = h\nu' + T$$
  $$\frac{h\nu}{c} = \frac{h\nu'}{c} \cos \theta + p \cos \phi$$
  $$\frac{h\nu}{c} \sin \theta = p \sin \phi$$
  $$(pc)^2 = T^2 + 2TMc^2$$

$$\Rightarrow$$

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{mec^2}(1 - \cos \theta)}$$

$$\cot \phi = \left( 1 + \frac{h\nu}{mec^2} \right) \tan \left( \frac{\theta}{2} \right)$$
Compton scattering – example

- An X-ray unit is to be installed, with the beam direction towards the ground. Employees in the floor above the unit are worried. Maximum X-ray energy is 250 keV. What is the maximum energy of the backscattered photons?

\[
\theta = 180^\circ \Rightarrow h' = \frac{h}{1 + \frac{m_e c}{h}(1 - \cos \theta)} = \frac{h}{1 + \frac{2m_e c}{h}}
\]

\[
h' = 250 \text{ keV} \Rightarrow h' = \frac{250}{1 + \frac{2 \times 250}{511}} = 126 \text{ keV}
\]

Compton scattering – cross section 1

- Klein and Nishina derived the cross section for Compton scattering, assuming free electron

- Differential cross section:

\[
\frac{d\sigma}{d\Omega} = \frac{r_c^2}{2} \left( \frac{V'}{V} \right)^2 \left( \frac{V'}{V} + \frac{V}{V'} - \sin^2 \theta \right)
\]

\[
d\Omega = \sin \theta d\theta d\phi
\]

\[
r_c \text{ classical electron radius}
\]

\[
\text{incoming photon along } z\text{-axis}
\]

Compton scattering – cross section 2

- Cylinder symmetry results in:

\[
\frac{d\sigma}{d\theta} = \pi r_c^2 \left( \frac{V'}{V} \right)^2 \left( \frac{V'}{V} + \frac{V}{V'} - \sin^2 \theta \right) \sin \theta
\]

- \sim probability of finding a scattered photon in the interval \([\theta, \theta + d\theta]\)

- Total electronic cross section:

\[
\sigma = \int_0^{\pi} \pi r_c^2 \left( \frac{V'}{V} \right)^2 \left( \frac{V'}{V} + \frac{V}{V'} - \sin^2 \theta \right) \sin \theta d\theta
\]

- Atomic cross section: \(\sigma = Z_e \sigma\)

Compton scattering – cross section 3

- Scattered photons are more forwardly directed with increasing photon energy:
Compton scattering – cross section 3

- Cross section may be modified with respect to energy:

\[
\frac{d\sigma}{d(\nu')} = \frac{d\sigma}{d\Omega} \frac{d\Omega}{d(\nu')} = \frac{d\sigma}{d\Omega} \frac{2\pi \sin \theta}{d(\nu')}
\]

\[
\nu' = \frac{\nu}{1 + \frac{\nu}{m_ec^2}(1 - \cos \theta)}
\]

\[
\Rightarrow \frac{d\sigma}{d(\nu')} = \frac{\pi e^2 m_ec^2}{(\nu')} \left( \frac{\nu + \nu'}{\nu'} - 1 + \left( \frac{\nu}{\nu'} - 1 \right) \frac{m_ec^2}{\nu'} \right)^2
\]

Compton scattering – cross section 4

- Correct atomic cross section:

\[
\sigma_{\nu} = \frac{\sigma_{\nu'}}{\Omega \nu} + \Omega \nu \nu
\]

- Effect of electron binding energy

- Klein-Nishina
- Carbon (Z=6)
- Copper (Z=29)
- Lead (Z=82)

Compton scattering – transferred energy 1

- The energy transferred to an electron in a Compton process:

\[
T = \nu - \nu'
\]

- The cross section for energy transfer:

\[
\frac{d\sigma_{\nu}}{d\Omega} = \frac{d\sigma_{\nu - \nu'}}{d\Omega} = \frac{d\sigma}{d\Omega} \frac{\nu - \nu'}{\nu}
\]

- Mean energy transferred:

\[
\bar{T} = \frac{\int d\sigma_{\nu} d\Omega}{\sigma} = \frac{\int d\sigma_{\nu - \nu'} d\Omega}{\sigma} = \frac{d\sigma_{\nu}}{\sigma} \times \nu
\]
Compton scattering – transferred energy

- The fraction of incident energy transferred:

\[
\frac{\text{fraction of incident energy transferred}}{\text{fraction of incident energy absorbed by the photon}} \approx \frac{\theta}{\sin \theta}
\]

Photoelectric effect

- Photon is absorbed by atom/molecule; the result is an excitation or ionization:

\[\gamma + X \rightarrow X' + e^-\]

- Atom may deexcite and emit characteristic radiation:

\[\text{deexcitation} \rightarrow \text{emission of characteristic radiation}\]

Photoelectric effect 2

- In the kinematics, the binding energy of the ejected electron should be taken into account:

\[T = h\nu - E_b - T_a = h\nu - E_a\]

- Assuming \(E_b = 0\), the atomic cross section is:

\[
\frac{d\sigma}{d\Omega} = 2\sqrt{2}r^*\alpha'Z^2 \left( \frac{m_e c^2}{\hbar \nu} \right)^{3/2} \sin^2 \theta \left( 1 + 4 \left( \frac{2\hbar \nu}{m_e c^2} \cos \theta \right) \right)
\]

- \(\alpha\): The fine-structure constant
- Solid angle \(\Omega\) gives the direction of the ejected electron
Characteristic radiation

- Energy of characteristic radiation depends on
  electronic structure and transition probabilities
- "K- and L-shell" vacancies ↔ \( h\nu_K \) and \( h\nu_L \)
- Isotropic emission
- Fraction of photoelectric interactions:
  \( P_K \left[ h\nu > (E_b)_K \right] \) and
  \( P_L \left[ (E_b)_L < h\nu < (E_b)_K \right] \)
- Probability for emission: \( Y_K \) and \( Y_L \) (fluorescence yield)
- Energy emitted from the atom:
  \( P_K Y_K h\nu_K + (1-P_K) P_L Y_L h\nu_L \)

Photoelectric cross section

- General formula:
  \[ \tau \propto \frac{Z^n}{(\nu\tau)^m} \quad 4 \leq n \leq 5, \ 1 \leq m \leq 3 \]
- Fraction of energy transferred to photoelectron:
  \[ \frac{T}{h\nu} = \frac{h\nu - E_b}{h\nu} \]
- However: don't forget Auger electron(s)
- Cross section for energy transfer to photoelectron:
  \[ \tau_{\nu} = \frac{(h\nu - P_K Y_K h\nu_K - (1-P_K) P_L Y_L h\nu_L)}{h\nu} \]

Auger effect

- Energy release by ejection of loosely bound electron
- Energy of emitted electron equal to deexcitation energy
- Low Z: Auger dominates
- High Z: characteristic radiation dominates

Pair production 1

- Photon absorption in the nuclear electromagnetic field where an electron-positron pair is created
  \[ T_0 \geq 0 \text{ mm. } < T_0 \]
- Triplet production: in the electromagnetic field of an electron
Pair production

- Conservation of energy:
  \[ h\nu = 2m_e c^2 + T^+ + T^- \]
- Average kinetic energy after absorption:
  \[ \overline{T} = \frac{h\nu - 2m_e c^2}{2} \]
- Estimated electron/positron scattering angle:
  \[ \theta = \frac{m_e c^2}{\overline{T}} \]
- Total cross section:
  \[ \kappa = \alpha \kappa_e Z^2 \overline{T} \]

Discovery of pair production

- In the electromagnetic field from an electron, an electron-positron pair is created
- Energy conservation:
  \[ h\nu = 2m_e c^2 + T^+ + T^- \]
- Average kinetic energy:
  \[ \overline{T} = \frac{h\nu - 2m_e c^2}{3} \]
- Primary electron is also given energy
- Threshold: \( 4m_0c^2 \)

Triplet production

- Pair production dominates:
  \[ \kappa_{pp}/Z^2 \text{ or } \kappa_{tp}/Z \]

Pair- and triplet production
Photonuclear reactions

- Photon (energy above a few MeV) excites a nucleus
- Proton or neutron is emitted
- \((\gamma, n)\) interactions may have consequences for radiation protection
- Example: Tungsten W \((\gamma, n)\)

Summary, interactions

- n \(_v\) atoms per volume = \(\rho(N_A/A)\)
- Number of atoms:
  \[ n = n_v V = n_v S dx \]
- Interaction probability
  \[ p = n \sigma / S = n_v \sigma dx \]
- Probability per unit length:
  \[ \mu = p / dx = n_v \sigma = \rho(N_A/A) \sigma \]
  \(\mu\): linear attenuation coefficient
Attenuation coefficients 2
- \( N_A \): Avogadro’s constant; \( 6.022 \times 10^{23} \) mole\(^{-1} \)
- \( A \): number of grams per mole
- \( N_A/A \): number of atoms per gram
- \( N_A/Z/A \): number of electrons per gram
- Number of atoms per volume: \( r(N_A/A) \)
- Etc.

Attenuation coefficients 3
- Total mass attenuation coefficient:
  \[
  \frac{\mu}{\rho} = \frac{\tau}{\rho} + \frac{\sigma}{\rho} + \frac{\kappa}{\rho} + \frac{\sigma_\text{a}}{\rho}
  \]
- Coefficient for energy transfer:
  \[
  \frac{\mu_\text{e}}{\rho} = \frac{\mu}{\rho} \frac{\tau}{h\nu}
  \]
- Bragg’s rule for mixture of atoms:
  \[
  \left( \frac{\mu}{\rho} \right)_\text{mix} = \sum_{i=1}^{n} f_i \left( \frac{\mu}{\rho} \right)_i, \quad f_i = \frac{m_i}{\sum m_i}
  \]

Attenuation coefficients 4

Attenuation 1
- Beam with \( N \) photons impinge absorber with thickness \( dx \):
  \[
  N \begin{array}{c}
  \text{dx}
  \\
  N-\text{dN}
  \end{array}
  \]
- Probability for interaction: \( \mu dx \)
- Number of photons interacting: \( N\mu dx \)
  \[
  dN = N \mu dx \quad \Rightarrow \quad \int \frac{dN}{N} = \int \mu dx
  \Rightarrow \quad N = N_0 e^{-\mu x}
  \]
Attenuation 2

• Note that $\mu$ is the interaction probability per unit length – not the absorption probability

• $e^{-\mu x}$ corresponds to a narrow beam measurement geometry:

\[
\begin{align*}
\text{Shield} & \quad \text{Attenuator, thickness } dx \\
\text{Scattered photons} & \quad \text{detector}
\end{align*}
\]

Attenuation 3

• 'Probability' for photon not interacting: $e^{-\mu x}$

• Normalized probability

\[
p_{\text{not interacting}} = Ce^{-\mu x}, \quad \int_0^\infty p_{\text{not interacting}} \, dx = 1, \quad \Rightarrow \quad p_{\text{not interacting}} = \mu e^{-\mu x}
\]

• Mean free path:

\[
\langle \lambda \rangle = \int_0^\infty x p_{\text{not interacting}} \, dx = \int_0^\infty x \mu e^{-\mu x} \, dx = \frac{1}{\mu}
\]

Attenuation 4

• 2 MeV photons

\[
Pb: \quad \mu = 0.516 \text{ cm}^{-1} \\
H_2O: \quad \mu = 0.049 \text{ cm}^{-1}
\]

\[
e^{-\mu_{H_2O} x_{H_2O}} = e^{-\mu_{Pb} x_{Pb}} \\
\Rightarrow \quad x_{H_2O} = \frac{\mu_{Pb}}{\mu_{H_2O}} x_{Pb}
\]

• 10 times as much water necessary
Scattered photons

- \( e^{\mu x} \): number of primary photons at a given depth
- What about the scattered photons?
- Monte Carlo simulations

Primary and scattered photons, 100 keV

Primary and scattered photons, 1 MeV