Range of Nuclear Forces in Yukawa's Theory

FOUR years ago, Yukawa, in an attempt to develop a relativistic theory of the interaction of heavy particles in nuclei, was led to predict the existence of charged particles of mass intermediate between those of the electron and the proton[1].

In view of the great interest and hope raised by the striking discovery in cosmic rays of particles having just the desired mass, which one is naturally tempted to identify with Yukawa's particles, it may be desirable to have a derivation as elementary as possible of the fundamental relation :

$$\rho = \frac{h}{mc}, \qquad (1)$$

where ρ is range of the nuclear forces, h is Planck's constant, m is the mass of the 'heavy electron', c is the velocity of light, which led Yukawa to his remarkable prediction.

It may perhaps be of interest, therefore, to point out that the meaning of relation (1) may be simply illustrated by an argument based on Heisenberg's Uncertainty Principle, in close analogy to Bohr's discussion of Gamow's formula and other related problems.

The argument runs as follows: in Yukawa's theory the interaction between heavy particles is carried by the semi-heavy particles, by means of simple emission and absorption processes (much in the same way as the relativistic interaction between two electrons can be described in terms of emission and absorption of light quanta); these are not, of course, actual emission and absorption processes, which would be contrary to the energy principle; they are called, therefore, virtual transitions. Let us see, however, a little closer how it comes about that the energy principle is respected. One might try to show that this is not so by setting up some device which could 'see' the heavy electron whilst it is travelling from one heavy particle to the other. In this case the energy principle can only be saved, as usual, if the uncontrollable energy exchange involved in the operation of the device is so large as to cover the energy excess actually observed, which is at least mc^2. Now the time t employed by the Yukawa particle in travelling from one heavy particle to the other is at least r/c, where r is the distance between the heavy particles. The time of operation of the device must on the other hand be smaller than t (otherwise the system will react as a whole, and the device will not be able to detect the presence of the individual Yukawa particle), but it need not be essentially smaller than this. We see, therefore, that the energy uncertainty will be, at most :

$$\triangle E \sim hc/r.$$

The condition :

$$\triangle E > mc^2$$

actually gives the distance (1) as the limit up to which virtual transitions can make themselves felt without contradiction of the energy principle. It may be remarked that by assuming a velocity of the intermediate particle smaller than c, it is only possible to reduce the energy uncertainty further, so that the consideration of relativistic velocities actually gives the optimum conditions or the upper limit to which the interaction may extend.

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[1] Yukawa, H., *Proc. Phys. Math. Soc. Japan*, **17**, 48 (1935); see also Fröhlich, H., Heitler, W., Kemmer, N., *Proc. Roy. Soc.*, A, **166**, 154 (1938), and several papers quoted there.