FYS3120 Classical mechanics and electrodynamics
Mid-term exam – Spring term 2018

Your **candidate number**

March 18, 2018

Important information:

- Your answers are to be submitted electronically as pdf-files, either generated from LaTeX or scanned, using the devilry submission system, at the latest Friday 23rd of March at 16.00 local time (GMT+1).
- This deadline is absolute.
- You **must** clearly mark your answer with the candidate number assigned to you for this course, as the answers are to be kept anonymous during grading. You can find this number on Studentweb. Any submission without a candidate number will result in an automatic fail. We would strongly prefer (as in, it would really annoy us if you did not do this) if the pdf was named `XX.pdf`, where `XX` is your candidate number.
- This mid-term exam counts for roughly 25% of the total grade in FYS3120, and you must receive a passing score on the mid-term in order to pass the course.
- As this is a take-home exam you are free to use any sources of information you may want, and you may collaborate with other students on solving the problems. However, the text of the submitted answers must be your own, and the usual rules of plagiarism apply. (We may check answers for similarities.)
- The best possible score on this exam is 25 points. Up to one point will be given for clear, concise and well presented answers, including appropriate figures and/or diagrams.
- You may give your answers either in English or Norwegian.
- Good luck!

Question 1 Pendulum with a rotating wheel

A pendulum can rotate freely about a horizontal axis A as shown in Fig. 1. The pendulum consists of a rigid rod and attached to this is a wheel which rotates about a point B on the rod. The mass of the wheel is m and the moment of inertia about B is denoted I. We consider the mass of the pendulum rod to be negligible. The distance between the points A and B is b. The gravitational acceleration is g and the angle of the pendulum rod relative to the vertical direction is denoted ϕ. We assume that the pendulum is free to perform full rotations about the axis A.

A motor (not included in the figure) affects the rotation of the wheel by a constant angular acceleration, so that the angular velocity of the wheel measured relative to a fixed direction is $\omega = \dot{\phi} + \alpha t$, where α is the acceleration constant. For simplicity we assume that all other effects of the motor can be neglected and that friction can be disregarded.

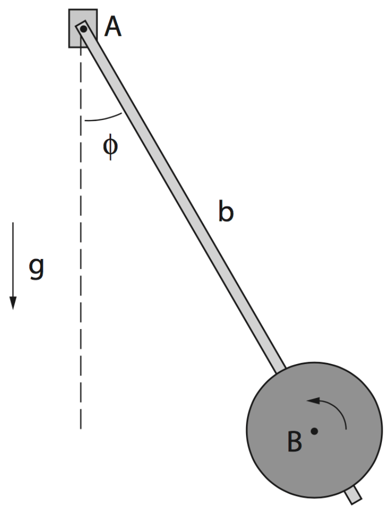

Figure 1: Pendulum with a rotating wheel.

a) Show that the Lagrangian of the system, with ϕ as a coordinate, is

$$L = \frac{1}{2} m b^2 \dot{\phi}^2 + \frac{1}{2} I (\dot{\phi} + \alpha t)^2 + mgb\cos\phi. \tag{1}$$

[1 point]

b) Find Lagrange's equation for ϕ. [2 points]

c) Show that you can rewrite the Lagrangian as

$$L(\phi, \dot{\phi}, t) = L'(\phi, \dot{\phi}) + \frac{d}{dt} f(\phi, t), \tag{2}$$

where

$$f(\phi, t) = I\alpha\phi t + \frac{1}{6}I\alpha^2 t^3. \tag{3}$$

[2 points]

d) What are the equations of motion for the Lagrangian L'? [1 point]

e) Find the canonical momentum p'_ϕ corresponding to the coordinate ϕ of the Lagrangian L' and determine the corresponding Hamiltonian $H'(\phi, p'_\phi)$. [2 points]

f) Explain why H' is a constant of motion. [1 point]

g) Make a two dimensional contour plot of the phase space potential function $H'(\phi, p'_\phi)$, for different values of

$$\lambda = \frac{I}{mgb}\alpha, \tag{4}$$

for example for $\lambda = 0, 0.5, 1.0$. Make sensible choices for the other parameters. Give a qualitative description of the different types of motion that can be read out of the diagrams and comment on how the situation changes with increasing λ. [3 points]

Question 2 Two-body decays

In particle physics we are often interested in two-body decays, where one heavy particle decays into two other lighter particles, for example the top quark t can decay into a W-boson and a bottom quark b, a process that we symbolise as $t \to bW$. Here we would like to study generic two-body decays of the form $B \to aA$.

a) Below we will use the concept of *invariant mass*. For two particles a and b the invariant mass m_{ab} is given by

$$m_{ab}^2 c^2 = (p_a + p_b)^2 = (p_a + p_b)^\mu (p_a + p_b)_\mu, \tag{5}$$

where p_a and p_b are the four-momenta of the particles. Explain why the invariant mass does not change between reference frames. [1 point]

b) For the decay $B \to aA$, find the magnitude of the relativistic momentum of particle a in the rest frame of A expressed in terms of the masses of the particles (and c). [3 points]

c) In the two sequential two-body decays $C \to bB$ and $B \to aA$, using four-vectors and invariants, find the square of the invariant mass of a and b, m_{ab}^2, expressed by the particle masses and the angle θ between a and b in the rest frame of B. To simplify the calculation you may assume $m_a = m_b = 0$. [5 points]

d) For a chain of four sequential two-body decays, $E \to dD$, $D \to cC$, $C \to bB$ and $B \to aA$, write a code to numerically find the distribution of invariant masses of all possible combinations of pairs of particles a, b, c and d. Plot these distributions in the same figure. For masses you should choose $m_E = 600$ GeV/c^2, $m_D = 500$ GeV/c^2, $m_C = 200$ GeV/c^2, $m_B = 150$ GeV/c^2, $m_A = 100$ GeV/c^2, $m_d = m_c = 0$ and $m_b = m_a = 1.8$ GeV/c^2. You can assume that all the decays are *isotropic*, *i.e.* that the direction of the decay products in the rest frame of the decaying particle is uniformly distributed on a sphere.[1] It may be wise to use recursive function calls for the decays. No points will be given for submitted code. [3 points]

e) Find a general expression for the invariant mass distribution for any two particles in a chain of n two-body decays. [0 points, but if you can do this, please tell me and we can write a paper.[2]]

[1] Note that there is a very nasty but educational trap here.

[2] Analytical results are known for $n = 2, 3$, and some properties of the solution is known for $n = 4$.