

FYS3410 - Vår 2010 (Kondenserte fasers fysikk)

<http://www.uio.no/studier/emner/matnat/fys/FYS3410/index-eng.xml>

Based on Introduction to Solid State Physics by Kittel

Course content

- Periodic structures, understanding of diffraction experiment and reciprocal lattice
- Crystal binding, elastic strain and waves
- Imperfections in crystals: point defects and diffusion
- Crystal vibrations: phonon heat capacity and thermal conductivity
- Free electron Fermi gas: density of states, Fermi level, and electrical conductivity
- Electrons in periodic potential: energy bands theory classification of metals, semiconductors and insulators
- Semiconductors: band gap, effective masses, charge carrier distributions, doping, pn-junctions
- Metals: Fermi surfaces, temperature dependence of electrical conductivity

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FYS3410 lecture schedule and exams: Spring 2010

M/18/1/2010:	Introduction and motivation. Periodicity and lattices	2h
W/20/1/2010:	Index system for crystal planes. Crystal structures	1h
M/25/1/2010:	Reciprocal space, Laue condition and Ewald construction	2h
W/27/1/2010:	Brillouin Zones. Interpretation of a diffraction experiment	1h
M/01/2/2010:	Crystal binding, elastic strain and waves	2h
W/03/2/2010:	Elastic waves in cubic crystals; defects in crystals	1h
M/08/2/2010:	Defects in crystals; case study - vacancies	2h
W/10/2/2010:	Diffusion	1h
M/15/2/2010:	Crystal vibrations and phonons	2h
W/17/2/2010:	Crystal vibrations and phonons	1h
M/22/2/2010:	Lattice heat capacity: Dulong-Petit and Einstein models	2h
W/24/2/2010:	Phonon density of states (DOS) and Debye model	1h
M/01/3/2010:	General result for DOS; role of anharmonic interactions	2h
W/03/3/2010:	Thermal conductivity	1h
M/08/3/2010:	Free electron Fermi gas in 1D and 3D – ground state	2h
W/10/3/2010:	Density of states, effect of temperature – FD distribution	1h
M/15/3/2010:	Heat capacity and thermal conductivity of FEEG	2h
W/17/3/2010:	Repetition	1h
22/3/2010:	Mid-term exam	

M/12/4/2010:	Drude model and the idea of energy bands	2h
W/14/4/2010:	Nearly free electron model; Kronig - Penny model	2h
M/19/4/2010:	no lectures	
W/21/4/2010:	Empty lattice approximation; number of orbitals in a band	2h
M/26/4/2010:	Semiconductors, effective mass method, intrinsic carriers	2h
W/28/4/2010:	Impurity states in semiconductors and carrier statistics	2h
M/03/5/2010:	p-n junctions and heterojunctions	2h
W/05/5/2010:	surface structure, surface states, Schottky contacts	1h
M/10/5/2010:	Metals and Fermi surfaces	2h
W/12/5/2010:	no lectures	
W/19/5/2010:	no lectures	
W26/5/2010:	Repetition	2h
27-28/5/2010:	Final Exam (sensor: Prof. Arne Nylandsted Larsen at the Aarhus University, Denmark, http://person.au.dk/en/anl@phys.au.dk)	

Lecture 24: p-n junctions and heterojunctions

- **Repetition: Carrier charge density in semiconductors**
- **p-n junctions in thermal equilibrium**
- **forward and reverse bias in p-n junctions**
- **illumination of p-n junctions**
- **heterostructures for enhancing efficiency of solar cells**

Lecture 24: p-n junctions and heterojunctions

- Repetition: Carrier charge density in semiconductors
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Carrier charge density in semiconductors

Previously, the density of states is given by $D(E) = \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}}$
Take a unit volume i.e. $V=1$

For conduction band $E_{\vec{k}} - E_c = \frac{\hbar^2 k^2}{2m_n^*}$ where E_c is the conduction band minimum

Now, the density of states

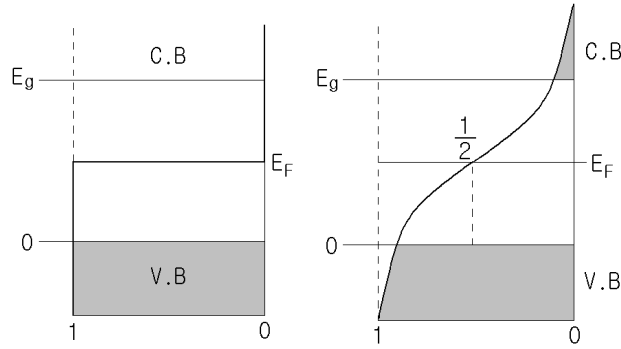
$$D_c(E) = \frac{1}{2\pi^2} \left(\frac{2m_n^*}{\hbar^2}\right)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}}$$

How this states are going to be filled is termed but a purely statistical process, governed by a proper distribution function, in particular for electrons, Fermi-Dirac

$$f(E) = \frac{1}{e^{(E-E_F)/k_B T} + 1} \approx e^{-\frac{(E-E_F)}{k_B T}}$$

Carrier charge density in semiconductors

T = 0K
 f_{FD} is a step function.
 By symmetry, it is reasonable
 That $E_F = \frac{1}{2}E_g$



T > 0K
 when $D_C(E) = D_V(E)$
 It is true that E_F remains at $\frac{1}{2}E_g$

$$N_e = n = \int_{E_C}^{\infty} D_C(E) f_{FD}(E) dE$$

$$= \int_{E_C}^{\infty} dE \frac{1}{2\pi^2} \left(\frac{2m_n^*}{\hbar^2} \right)^{\frac{3}{2}} (E - E_C)^{\frac{1}{2}} e^{-\frac{(E - E_F)}{k_B T}}$$

for electrons

$$= \frac{1}{2\pi^2} \left(\frac{2m_n^*}{\hbar^2} \right)^{\frac{3}{2}} e^{-\frac{(E_C - E_F)}{k_B T}} \int_0^{\infty} (k_B T)^{\frac{1}{2}} x^{\frac{1}{2}} e^{-x} dx (k_B T) = 2 \left(\frac{m_n^* k_B T}{2\pi \hbar^2} \right)^{\frac{3}{2}} e^{-\frac{(E_C - E_F)}{k_B T}} = N_{eff}^C e^{-\frac{(E_C - E_F)}{k_B T}}$$

Carrier charge density in semiconductors

$$= 2 \left(\frac{m_p^* k_B T}{2\pi \hbar^2} \right)^{\frac{3}{2}} e^{\frac{(E_V - E_F)}{k_B T}} = N_{eff}^V e^{\frac{(E_V - E_F)}{k_B T}} \quad \text{for holes}$$

$$np = N_{eff}^C N_{eff}^V e^{-\frac{E_g}{k_B T}}$$

The conductivity of the semiconductor including holes and electrons

$$\sigma = ne\mu_e + pe\mu_p$$

Carrier charge density in semiconductors

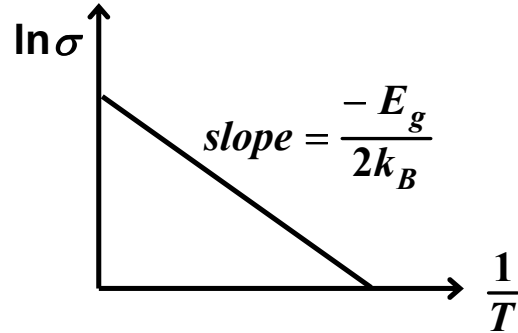
In intrinsic semiconductor, $n=p$

$$\begin{aligned}\sigma &= n(\mu_e + \mu_p)e \\ &= (\mu_e + \mu_p)e 2 \left(\frac{2\pi k_B T}{h^2} \right)^{\frac{3}{2}} (m_p^* m_n^*)^{\frac{3}{4}} e^{-E_g/2k_B T}\end{aligned}$$

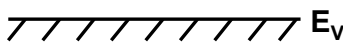
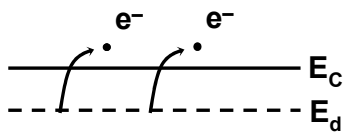
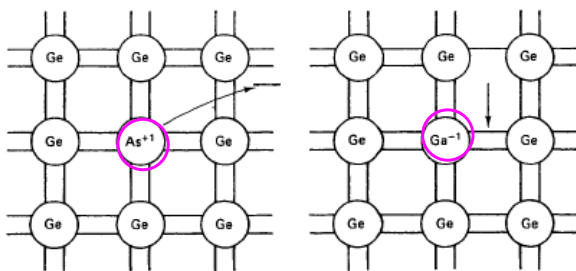
As T increase, σ increases

As E_g increase, σ decreases

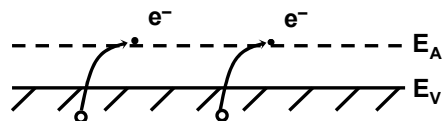
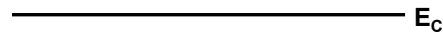
$$\ln \sigma \sim \frac{-E_g}{2k_B T} + \frac{3}{2} \ln T$$



Carrier charge density in semiconductors



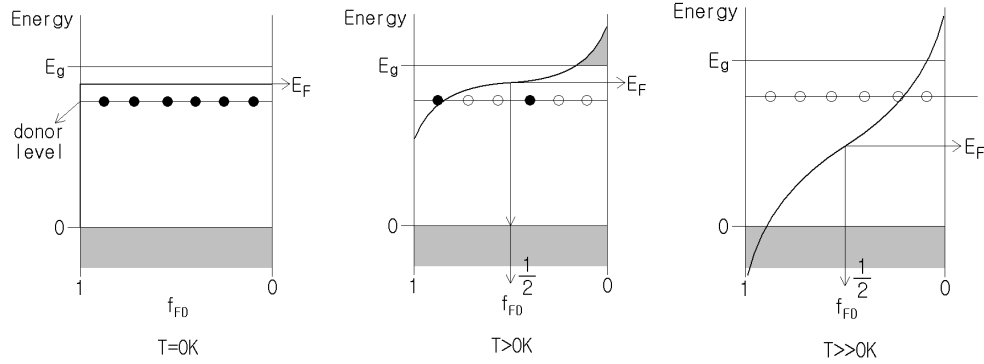
$$n = N_d^+$$



$$p = N_a^-$$

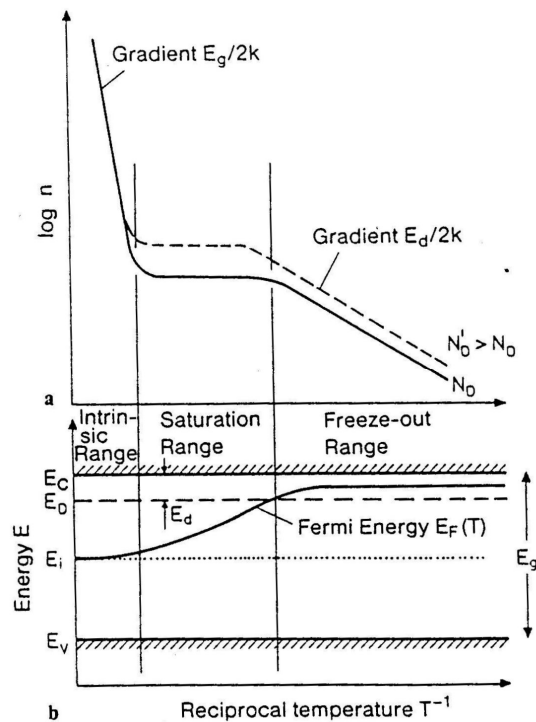
Carrier charge density in semiconductors

n-type semiconductors



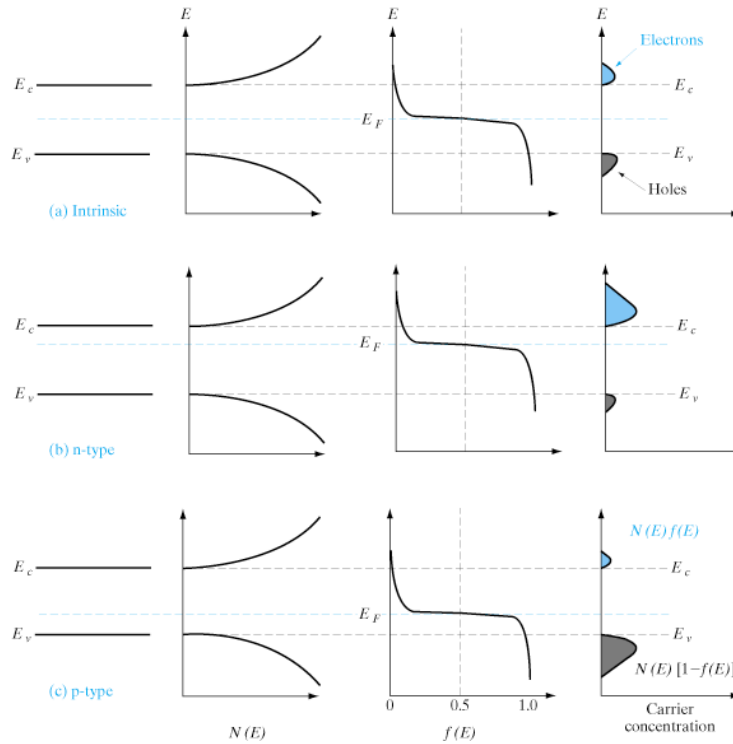
T = 0K All states up to the donor levels are occupied and the conduction band is empty
 E_F lies somewhere between the donor level and band edge

Carrier charge density in semiconductors



Carrier charge density in semiconductors

Schematic band diagram, density of states, Fermi–Dirac distribution, and the carrier concentrations for



(a) intrinsic,

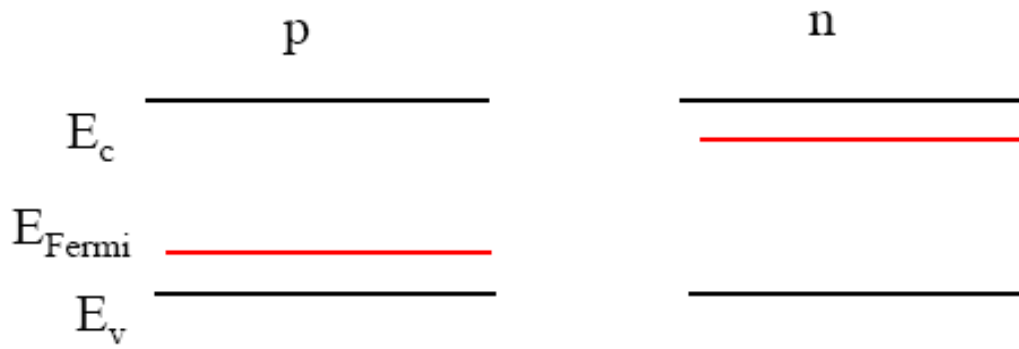
(b) n-type

(c) p-type semiconductors at thermal equilibrium.

Lecture 24: p-n junctions and heterojunctions

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- illumination of p-n junctions
- heterostructures for enhancing efficiency of solar cells

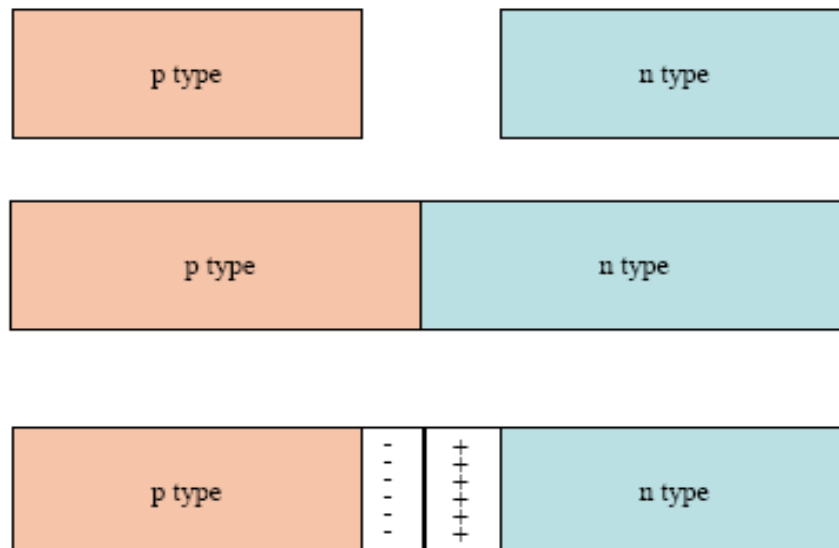
p-n junctions: two identical materials having different movable charge particle on each side



What happens when we bring the two together?



p-n junctions: two identical materials having different movable charge particle on each side

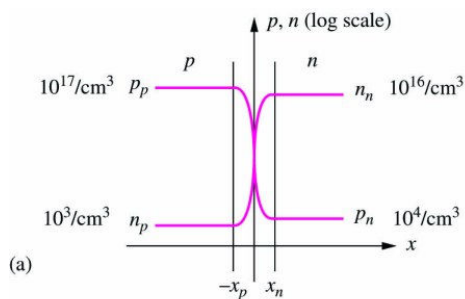


p-n junctions: microscopic scenario

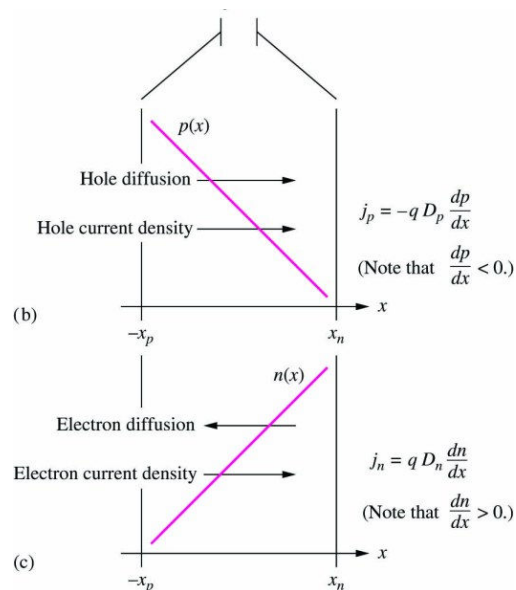
When junction is formed, electrons from n-type and holes from p-type are free movable charges and will diffuse leaving behind ionized (charged) dopant atoms. Remember, the dopant atoms are fixed in the lattice sites and do not move. Effectively, electrons diffused from the n-type leave behind positively charged donors while holes diffused from the p-type leave behind negatively charged acceptors. Electron/hole diffusion – as long as charged particles are involved – causes corresponding "diffusion" currents.

The net result is a build up of uncompensated charge (called space charge or depletion region) and, consequently, an electric field that is directed from "+" to "-" charged parts of the semiconductor in the vicinity of the interface, i.e. from the n-type to p-type. The application of this field on electrons/holes causes "drift" currents of the same charge carriers directed in the opposite way to the diffusion currents and at a certain value of the electric field diffusion is no longer possible. This state is called thermal equilibrium of the junction when the net current through the junction is zero.

p-n junctions: diffusion current



Donor and acceptor concentration on either side of the junction. Concentration gradients give rise to diffusion currents.



p-n junctions: drift current

- Diffusion currents lead to uncompensated charge density distribution in the vicinity of the *p-n* interface.
- Gauss' law predicts an electric field due to the charge distribution:

$$\nabla \cdot E = \frac{\rho_c}{\epsilon_s}$$

- Assuming constant permittivity,

$$E(x) = \frac{1}{\epsilon_s} \int \rho(x) dx$$

- Resulting electric field gives rise to a drift current. With no external circuit connections, drift and diffusion currents cancel each other. There is no actual drift current if fact, rather the electric field cancels the diffusion current "tendency".

p-n junctions: finding built-in potential considering a balance between the drift and diffusion currents

$$J_p(x) = q \left[\mu_p p(x) \cdot E(x) - D_p \frac{dp(x)}{dx} \right] = 0$$

$$\left. \begin{aligned} \frac{\mu_p}{D_p} E(x) &= \frac{1}{p(x)} \frac{dp(x)}{dx} \\ E(x) &= -\frac{dV(x)}{dx} \\ \frac{\mu_p}{D_p} &= \frac{q}{k_B T} \end{aligned} \right\} -\frac{q}{kT} \frac{dV(x)}{dx} = \frac{1}{p(x)} \frac{dp(x)}{dx}$$

p-n junctions: finding built-in potential considering a balance between the drift and diffusion currents

$$-\frac{q}{kT} \frac{dV(x)}{dx} = \frac{1}{p(x)} \frac{dp(x)}{dx}$$

$$-\frac{q}{kT} \int_{V_p}^{V_n} dV = \int_{p_p}^{p_n} \frac{dp}{p}$$

$$-\frac{q}{kT} (V_n - V_p) = \ln p_n - \ln p_p$$

$$\frac{q}{kT} V_0 = \ln \frac{p_p}{p_n}$$

$$n_i^2 = n_n \cdot p_n = N_d \cdot p_n$$

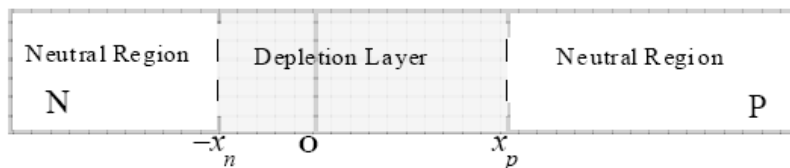
$$\frac{p_p}{p_n} = e^{\frac{qV_0}{kT}}$$

$$p_n = \frac{n_i^2}{N_d}$$

$$p_p = N_a$$

$$V_0 = \frac{kT}{q} \ln \frac{N_a \cdot N_d}{n_i^2}$$

p-n junctions: depletion region

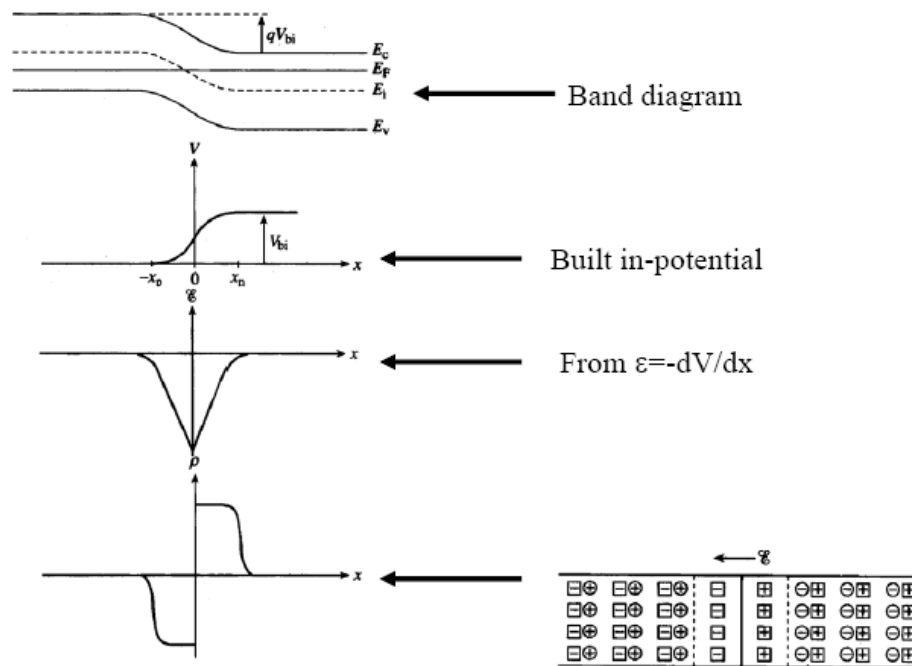


The electric field is continuous at $x = 0$.

$$N_a x_p = N_d x_n$$

one-sided junction is called a ***N⁺P junction*** or ***P⁺N junction***

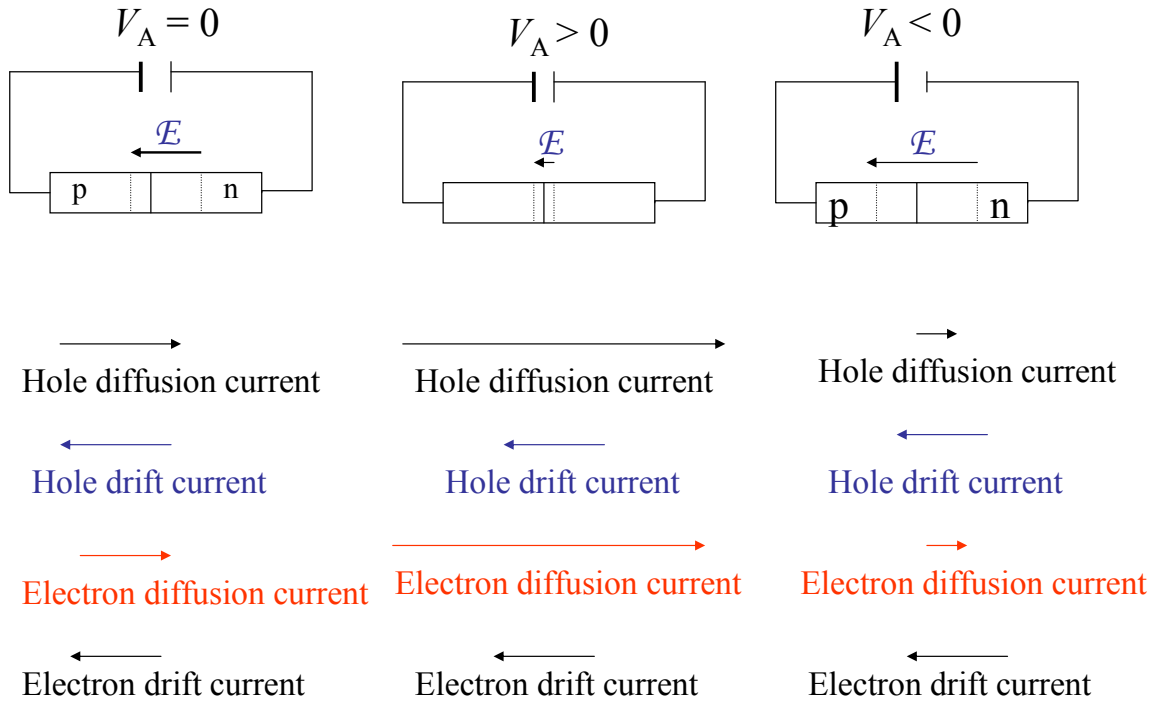
p-n junctions: electrostatics



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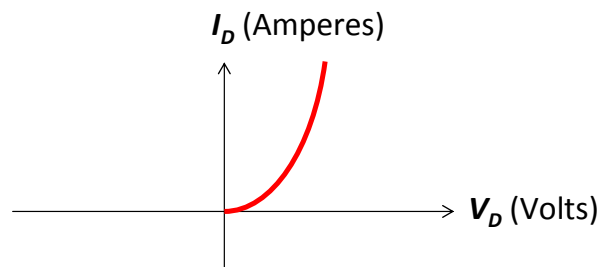
p-n junctions: forward/reverse bias



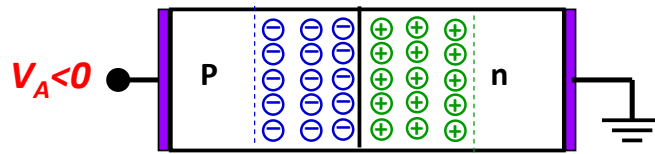
p-n junctions: forward bias



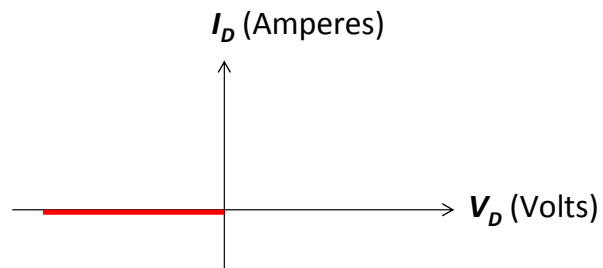
$$\frac{p_p}{p_n} = e^{\frac{q(V_0 - V_A)}{kT}}$$



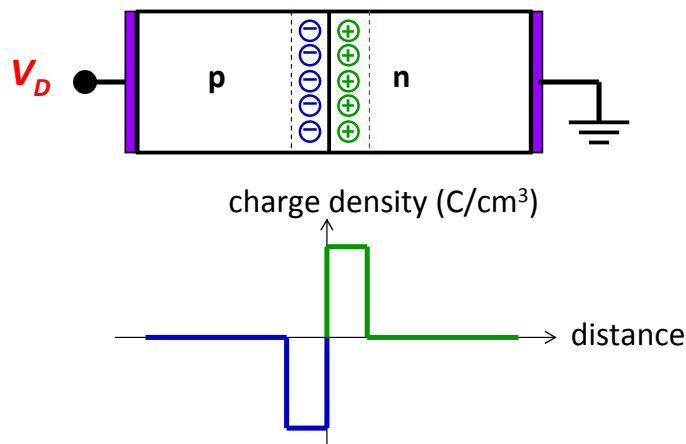
p-n junctions: reverse bias



$$\frac{p_p}{p_n} = e^{\frac{q(V_0 - V_A)}{kT}}$$



p-n junctions: capacitance



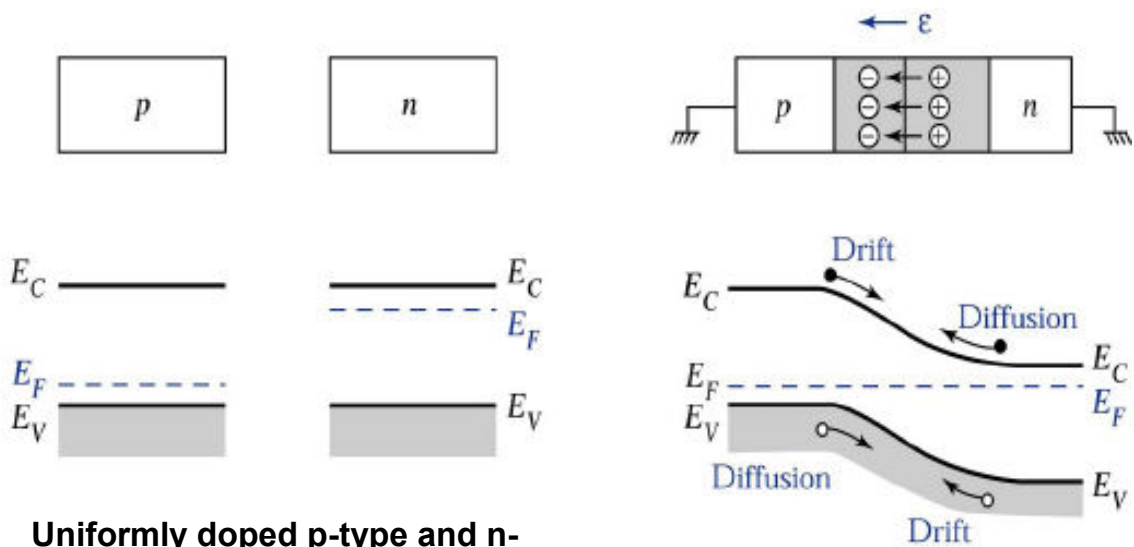
- The charge stored in the depletion region changes with applied voltage. This is modeled as junction capacitance

$$C_j = \frac{A_D \epsilon_{Si}}{W_j}$$

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p-n junctions: solar cells

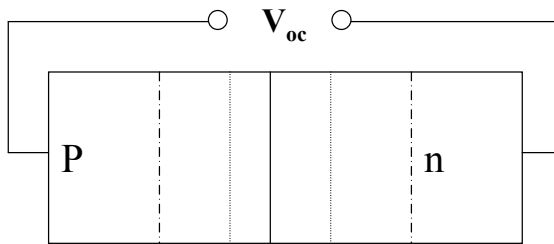


Uniformly doped p-type and n-type semiconductors before the junction is formed.

Internal electric-field occurs in a depletion region of a p-n junction in thermal equilibrium

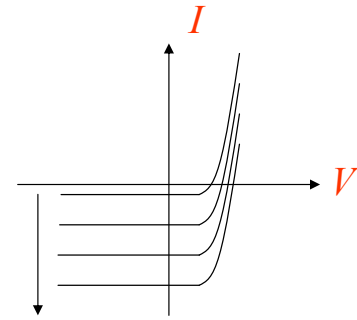
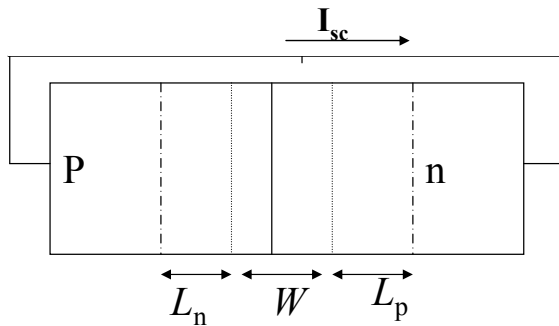
p-n junctions: solar cells

V_{oc} = the open circuit voltage



$$\frac{p_p}{p_n} = e^{\frac{qV_0}{kT}}$$

I_{sc} = current when device is short circuited



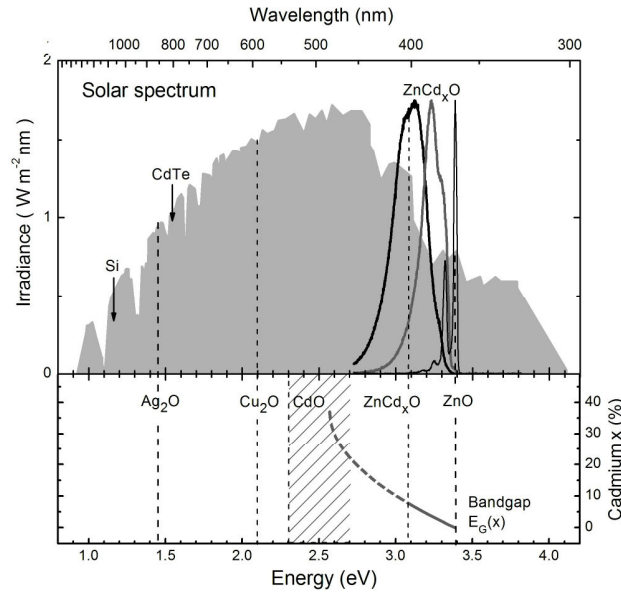
Increasing
light intensity

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p-n junctions: solar cells

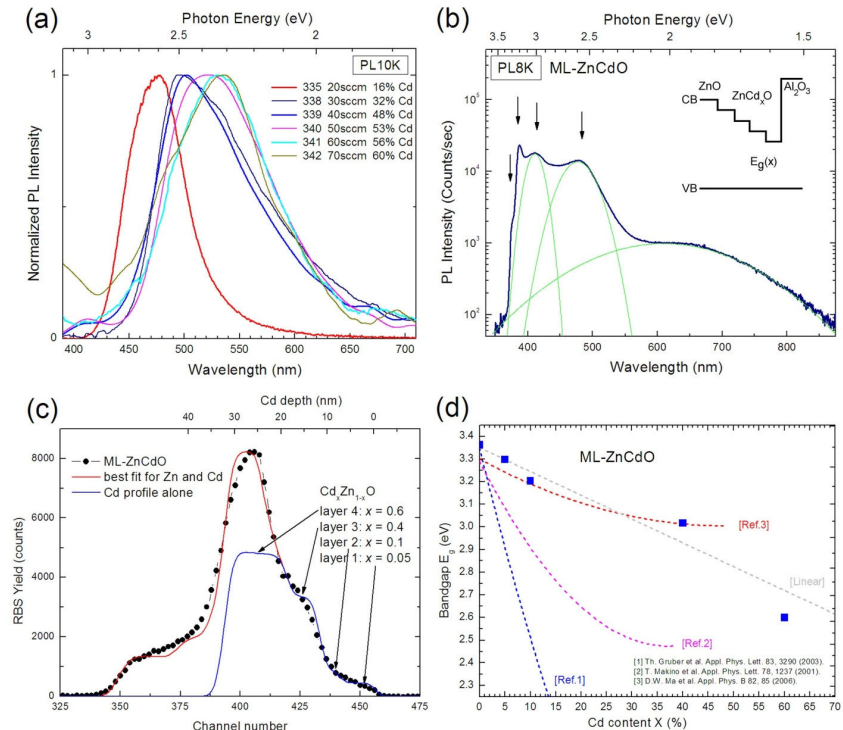
$\eta = \text{power conversion efficiency} \sim (I_{sc} V_{oc})/P_{in}$



Schematic of a “perfect absorber”

ZnO
Zn _{1-x} Cd _x O x: min
Zn _{1-x} Cd _x O
Zn _{1-x} Cd _x O
Zn _{1-x} Cd _x O x: max
C-Al ₂ O ₃

p-n junctions: solar cells



[1] Th. Gruber et al. Appl. Phys. Lett. 83, 3290 (2003)
 [2] T. Makino et al. Appl. Phys. Lett. 78, 1237 (2001)
 [3] D.W. Niu et al. Appl. Phys. B 62, 85 (2006)