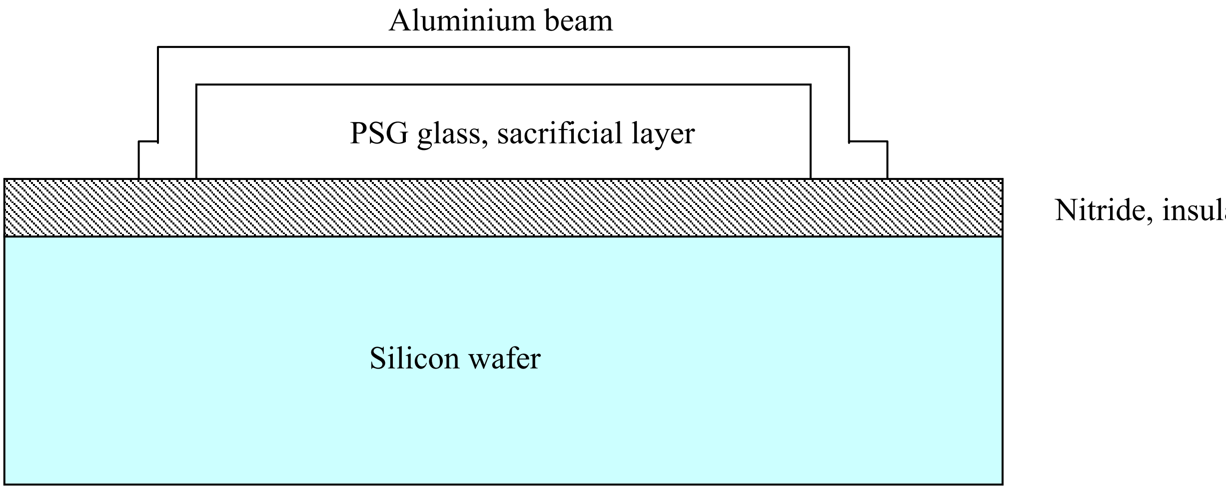
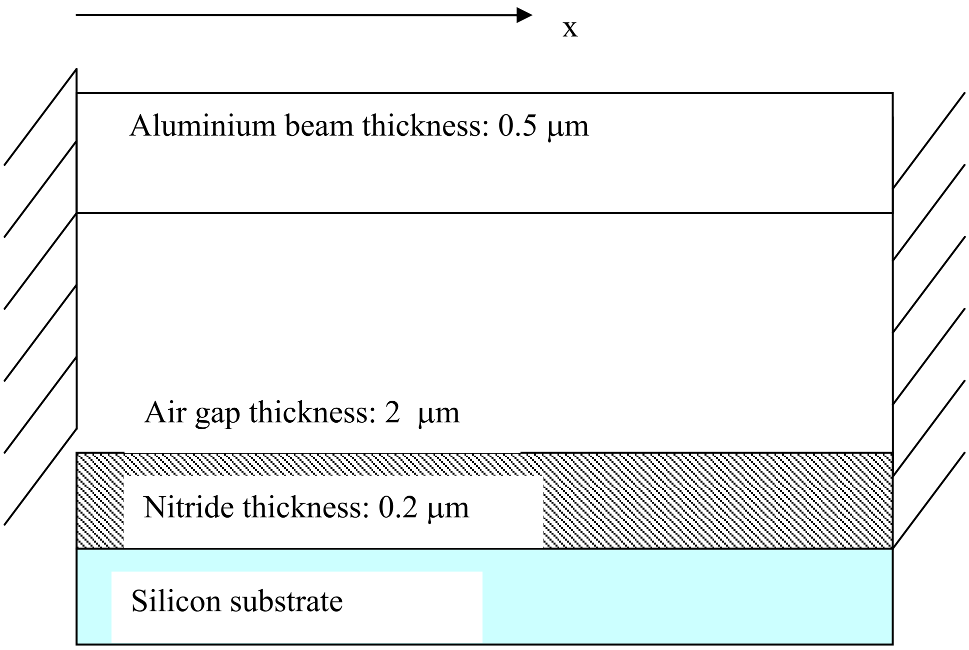
Exercise for FYS4230 Mircosystem modelling
Doubly clamped beam
Capacitance, uniform pressure and electrostatic forces

The aim of this exercise is to perform analytical calculations on a problem that will later be solved with Coventor.



An aluminium beam is manufactured in a surface micromachining process as sketched above.

We disregard the elasticity in the vertical supporting parts of the beam, and treat the beam as a simple fixed-fixed (doubly clamped) beam. A side view of he simplified beam:



NB, some browsers seem to have problems with the "micro" symbol and translates it into m. All lengths in this exercise are expressed in micrometers!

The beam is 80 μm long and 10μm wide.
Aluminium has a Young's modulus (constant of elasticity) of E=7.7 $\cdot 10^4$ MPa.
The density of aluminium is $\rho=2.3 \cdot 10^{-15}$kg/μm^3. (micro cubic meters)
An electric potential U=1V is initially applied between the silicon substrate and the aluminium beam; we consider the electrostatic forces to be negligible.
The permittivity of free space is

$$\varepsilon_0 = 8.8542 \cdot 10^{-12} Fm^{-1}$$

1) Find the capacitance between the aluminium beam and the silicon substrate. You may use the parallel plate approximation.

2) What are the boundary conditions for the beam equation in this case?

3) A pressure of q=0.001 MPa is applied on the top side of the aluminium beam. Find the deflection function. Where is the x-position of the maximum deflection of the beam? What is the maximum deflection? What are the moments and shear forces at the fixed ends?

4) A voltage of U=10V is applied between the aluminium beam and the silicon substrate. What is the maximum deflection if you treat the electrostatic forces similarly to the pressure in 2) and use the parallel plate approximation?

Parallel plate approximation, force F between the plates:

$$F = \frac{A\varepsilon_0}{2G^2}U^2$$

A is the area of the parallel plates and G is the gap between the plates.