Exercise "pull in" for FYS4230 Microsystem modeling Ideal elastic spring + straight plate simplification of doubly clamped beam Electrostatic forces: rough estimation of pull-in voltage

The aim of this exercise is to make a rough estimation of the pull-in voltage for the aluminum beam from the coventorware exercise "doubly clamped aluminum beam".

We want to simplify the elastic and geometric properties of the doubly clamped aluminum beam.

1) The elasticity of the beam is modeled as an ideal linear spring. See page 217 in Senturia for estimation of the spring constant. The described spring constant defines a linear relation between the maximum deflection of the beam and the total force on the beam.

The displacement function u(x) of the doubly clamped beam can be found to be

$$u(x) = \frac{q}{24EI} \left( x^4 - 2x^3L + x^2L^2 \right)$$

At which x-position is the maximum deflection? How does this maximum deflection increase with the total force  $F(F=q^*L)$ ? Find an expression for the constant *k* in the linear force-displacement relation:  $F=k^*u(x)$ .

2) The geometry of the beam is modeled as a straight plate suspended by a linear spring. The area of the plate could be set to the area of the beam A, but e.g. A/2 is a better approximation due to the bending of the beam.



A sketch of the simplified system (see also page 130 in Senturia):

Refer to exercise "aluminum beam", and Coventor tutorial 1: Write up the value of the initial gap  $g_0$  (2  $\mu$ m). Which forces act of the upper, moveable plate? Find the pull-in voltage of this simplified model.