FYS4230 – Høst 2005

Report – oblig II

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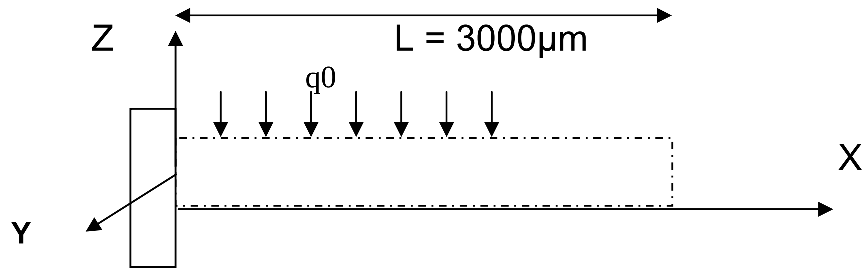
Exercise: Silicon Beam Accelerometer



For a clamped cantilever silicon beam the deflection function can be calculated by integrating the general differential equation for a beam and applying appropriate boundary conditions:

$$\omega(x) = \frac{q_0 . x^2}{24EI}(.x^2 + 6L^2 - 4LX)$$

Here, q_0 is a uniformly distributed load (per unit length along the beam), E is Young's modulus, I is the moment of inertia, and L is the length of the beam. Note that the beam is clamped at x = 0, with a free end at x = L. The cross section of the beam has rectangular shape:

$$-b/2 \leq y \leq b/2 , \quad -h/2 \leq z \leq h/2$$

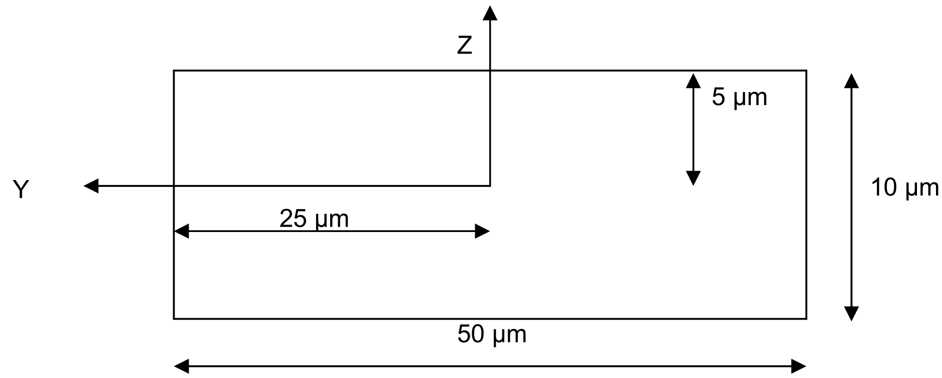
where the z axis points in the opposite direction of the force q0. I is then $bh^3/12$. An important stress measure is the normal stress on a surface perpendicular to the axis of the beam:

$$\sigma_{xx} = zE\omega''(x)$$

The beam is a part of a system that is subjected to an acceleration a.

Suitable data for the problem are a = 50g, g = 9.81 m/s2, L = 3 mm, b = 50 µm, h = 10 µm, E = 168 GPa. The density can be taken as ρ = 2300 kg/m3.

(a) Draw the cross section in the y; z plane.



(b) What is the force q0 acting on the beam per unit length when the external acceleration is a? (Hint: The acceleration can be treated as a fictitious force in an accelerated coordinate system in which the deflected beam is at rest.)

V=(b)*(L)*(h) = 1.5 pm^3 (volume)

M=D*V = 2.3 *10^6 g/m^3 * 1.5* 10^{-12} = 3.45 µg_r (weight)

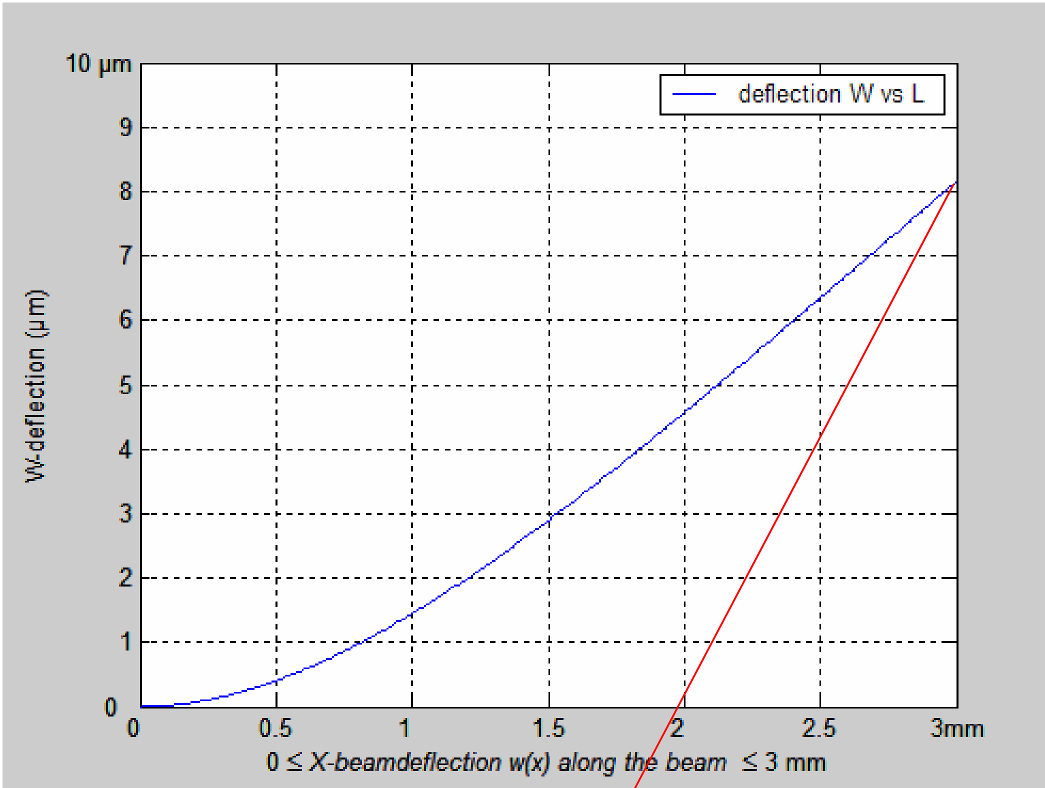
a = 50 * g = 50 * 9.81 = 490.5 m/S^2 (acceleration)

F = M.a = 3.45*10^{-6} * 490.5 = 1.6922 m.g_r/S^2 (force)

F =1.6922 µN

q_0 = F/L = 1.6922 (µN) / 3*10^{-3}= 564 µN/m (force per unit length)

(c) Plot the deflection w(x) along the beam (0 ≤x≤ L).



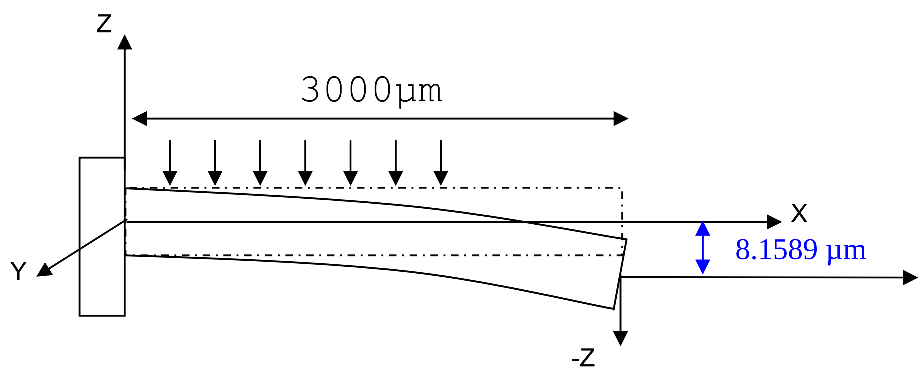
(d) Where is the maximum deflection, and how large is this deflection?

ω_{max} would be at the end of the beam and in the -Z direction

$$L = 3000\ \mu m$$

$$\omega_{max}(L) = \frac{q_0 . L^2}{24EI}(L^2 + 6L^2 - 4L^2) = \frac{q_0 . L^4}{8EI}$$

$$\boldsymbol{\omega_{max(L)} = 8.1589\ \mu m}$$



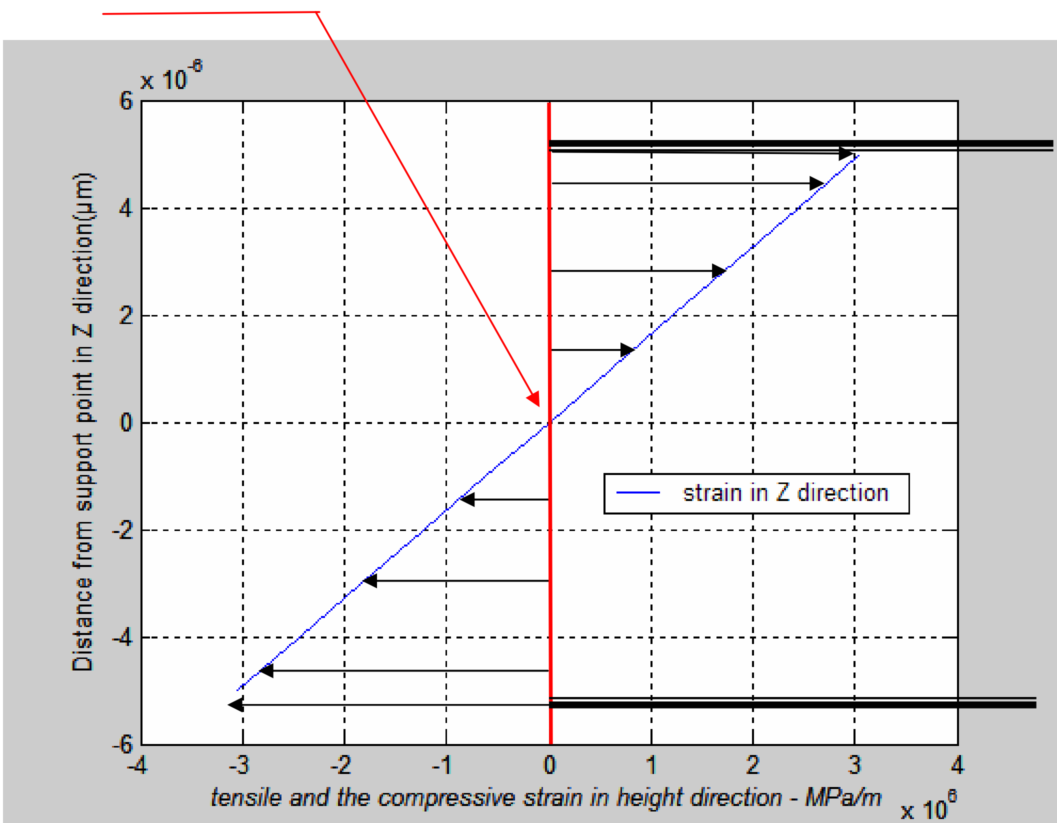
(e) Draw σ_{xx} as function of z for a = 50g and x =0.

$\sigma_{xx} = zE\omega''(x)$

$$\sigma_{xx} = \frac{Z.q_0}{2I}(x^2 - 2Lx^2 + L^2)$$

X= 0 $\sigma_{xx-\max} = \frac{Z.q_0}{2I}(L^2)$ $\sigma_{xx-\max} = \pm 3.046 * 10^6 \frac{P_a}{m}$

Middle of Support point

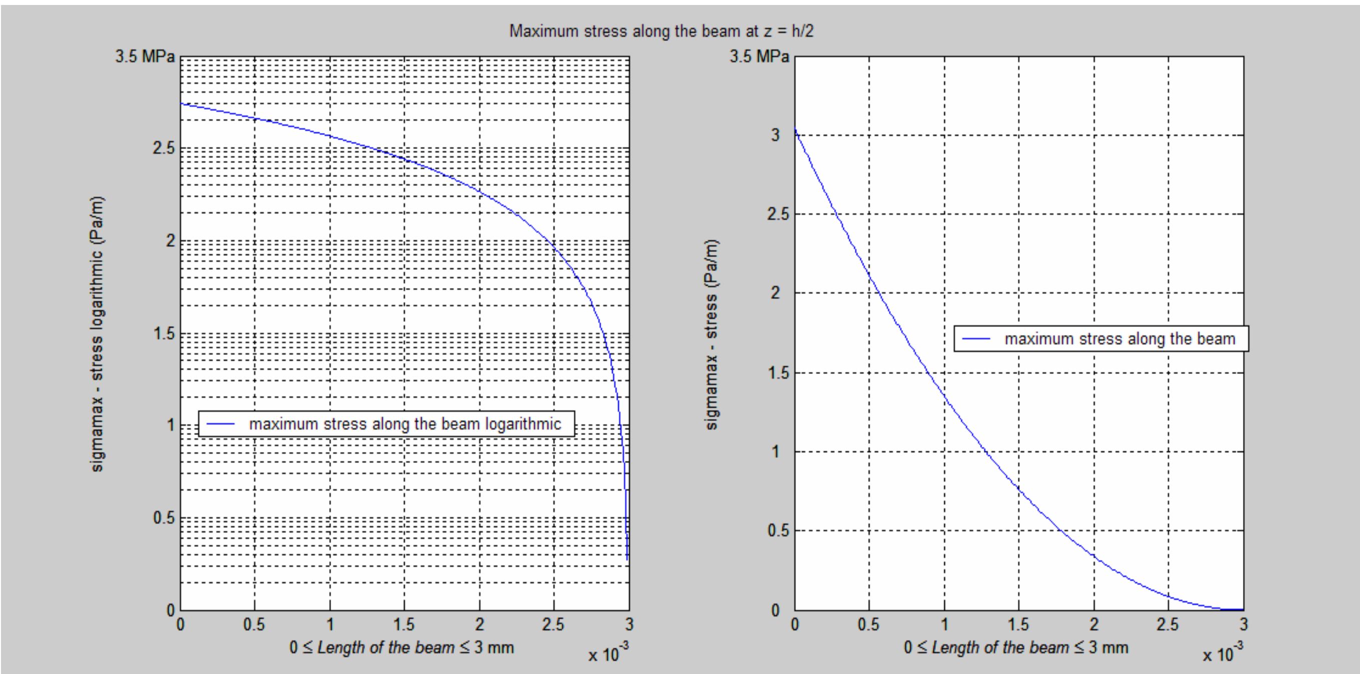


(f) What is the physical interpretation of the quantity σ_{xx} ?

The maximum tensile strain ε is at top surface and the maximum compressive strain will be at the bottom surface of the X, Y plain (x direction). **The maximum stress on the surface of X, Y is identified σ_{xx} as (- X, X direction).**

Plot σ_{xx} as a function of x for the z value which gives the largest stress in an arbitrary cross section.

The graph shows the stress σ_{xx} around top surface at h = +Z/2 on the XZ plain (on the surface XY as tensile strain) and along the X direction to maximum X at L = 3 mm. It would be the same as h = -Z/2 but as compressive strain.



Find where σ_{xx} reaches its maximum value.

$$\sigma_{xx-\max} = \pm 3.046005 \frac{MP_a}{m}$$

σ_{xx} reaches its maximum at 2 points. First at maximum strain ($\sigma_{xx-\max} = -3.046005 \frac{MP_a}{m}$) at the top surface of x,y at X, Z (0, h/2 = + 5μm). Secondly it reaches its maximum compressive strain ($\sigma_{xx-\max} = +3.046005 \frac{MP_a}{m}$) at the bottom plain of xy at X,Z (0, -h/2 = 5μm).

(g) Plot the max deflection as a function of a when 0≤a≤50g and x = L.

Deflection w(x) would be linear at L proportional to a in the -Z direction.