

FYS4630/FYS9630

Assignment #7 Friday October 30, 2015

Problem 1

The discrete ordinate approximation to the radiative transfer equation for collimated beam incidence is given by Eqs (8.9) and (8.10) on page 288 in Thomas and Stamnes. Explain why these two equations can be written as (neglecting the thermal source term):

$$u_i \frac{dI(\tau, u_i)}{d\tau} = I(\tau, u_i) - \frac{a}{2} \sum_{\substack{j=-N \\ j \neq 0}}^N w_j p(u_j, u_i) I(u_j, \tau) - X_0(u_i) e^{-\tau/\mu_0}, \quad i = -N, \dots, N$$

Particular solutions of the above equation are

$$I_p(\tau, u_i) = Z_0(u_i) e^{-\tau/\mu_0}.$$

Show that

$$\sum_{\substack{j=-N \\ j \neq 0}}^N \left[\left(1 + \frac{u_j}{\mu_0}\right) \delta_{ij} - w_j \frac{a}{2} p(u_j, \mu_i) \right] Z_0(u_j) = X_0(u_i) \quad i = -N, \dots, N$$

Problem 2

The quadrature weights (w'_j) for full range Gaussian quadrature are

$$w'_j = \frac{1}{P'_M} \int_{-1}^1 \frac{du \cdot P_M(u)}{u - u_j}$$

Where u_j are the roots of the Legendre polynomial, i.e. $P_M(u) = 0$.

Find the double-Gauss weights, w_j , and the corresponding discrete ordinates, μ_j , for 2-streams and 4-streams. (The Legendre polynomials you need are found on page 178 in Thomas and Stamnes)

Problem 3

Assume a three-layer atmosphere, each layer having an optical thickness of 0.1, and a Rayleigh scattering phase function in each layer. The surface albedo is $A = \pi r_L = 0.05$ (Lambert surface), incoming solar flux at the top of the atmosphere is $F^s = 1.0$, $\mu_0 = 0.51$, single scattering albedo $a = 1.0$ (conservative scattering).

- Use computer simulations to calculate the **diffuse** downward flux at the surface according to the table below. 'exact' in the fifth column is $F^-(\tau^*)$ with 48 streams and δ -M scaling turned on. Comment the results. In the main program (main-disort-2014.f) use 'onlyfl = .true.' and 'usrang = .false.'

N-stream	Code	$F^-_{\text{diffuse}}(\tau^*)$ No δ -M	$F^-_{\text{diffuse}}(\tau^*)$ δ -M	Ratio column 4/exact
4	Disort			
6	Disort			
8	Disort			
10	Disort			
12	Disort			
16	Disort			
48	Disort			1.0
2	Two-stream			

- b) Repeat a) with a cloud layer of optical thickness $\tau=20$. In the simulations replace layer 2 with a pure cloud layer with asymmetry factor $g = 0.95$ and $d\tau_{\text{cloud}}(2) = 20.0$. Single scattering albedo is still $a = 1.0$ for all layers. Comment the results.
- c) Repeat a), but now compute the intensity at the surface for $\mu = -0.4$ (' $\mu(1) = -0.4$ ' in main-disort-2014.f). Use $\varphi_0 = 0$ (' $\phi_0 = 0$ ' in main-disort-2014.f) and $\varphi = 0$ (' $\phi(1) = 0$ ') in main-disort-2014.f. Set ' $\text{usrang} = .\text{true}.$ ' and ' $\text{onlyfl} = .\text{false}.$ ' Comment the results.
- d) Repeat c), but now with a cloud layer as in b). Comment the results.