Relevant GEO exercises for MAT-IN1105L

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Week 0 (formulas)

Exercise 2.7: Density of the earth (*from GEO-KJM1040*)
The mass M of the earth is $5.9737 \cdot 10^(24)$kg and the radius r is 6371km. Compute the avergae density ρ of the earth and give your answer in kg/m^3. You may employ the formulas

$$\rho = \frac{M}{V}\ , \qquad V = \frac{4\pi r^3}{3}$$

Here V denotes the volume of the earth. *Remark: The given radius is not completely accurate since the earth is not sphere, but rather slightly compressed at the poles. However the given radius is an average and gives a good estimate.*

Week 1 (loops, lists)

Exercise 4.7 Salt in a water tank (*from GEO-KJM1040, extended with b)*)
A water tank contains 10 litres of unsalted water on day 0. Related to an experiment in oceanography, 1dl of salt is added to the tank each day starting from day 1. Also each day 2dl of unsalted water evaporates from the tank.

a) Compute how the ratio salt/water evolves during the first week and print out the ratio for each day.

a) Repeat **a)**, but use a `while`-loop this time if you used a `for`-loop in **a)**. Otherwise use a `for`-loop.

Exercise 5.6: Pressure in the atmosphere (from GEO-KJM1040, *extended with b)*)
The pressure p at a given height z in the atmosphere can be expressed by

$$p(z) = p_0 e^{-z/H}$$

where $p_0 \approx 1013$hPa is the pressure at sea level and $H \approx 7800$m is a characteristic height.

a) Create a list `z` with heights ranging from 0 to 20km with intervals of 400m.

b) Find the pressure for each height in your `z` list and store it in new list `p`.

c) Make a nicely formatted table with heights and corresponding pressures.

Exercise *NEW*: Should I move? (*Completely new*)
Assume a probability (based on historical data for example) of 0.1% that there will occur a dangerous earthquake in the area you live within one year.

a) How many years do you have to live in this area before you have at least a 50% chance of experiencing a dangerous earthquake? Would you consider this a safe area for you as a person? (*Hint: the probability that one or the other, of two independent events, will occur, is equal to the sum of the individual probabilities).*

b) Say you decided to move anyway and the car you drove away with, uses fuel at a rate of 0.0001L/kg/km times the mass of your car (which decreases each km because you use fuel). Assuming the initial mass of your car (with full fuel) is 1500kg, how much fuel do you use for driving your 100km trip to your potential new home?

c) Compare the fuel consumption in **b)** to the case where the mass of the car is constant 1500kg throughout the 100km trip.

Week 2 (functions, if-tests)

Exercise 2.2: Make some conversion tables (*from INF1100, extended with b)*)
Here you will make conversion tables for two different types of physical entities:

a) The formula for converting Fahrenheit degrees (F) to Celsius (C) is

$$C = \frac{5}{9}(F - 32) \tag{1}$$

but many people also use an approximate formula for quickly converting Fahrenheit to Celsius degrees:

$$C \approx \hat{C} = \frac{F - 30}{2} \tag{2}$$

Consider the Fahrenheit degrees $0, 10, 20, ..., 100$ and write a program that prints three columns: F, the corresponding Celsius C, and the approximate value $\hat{C}$.

b) Repeat **a)**, but now consider rock densities; find the densities (in g/cm^3) of four rocks on the web and convert them to kg/m^3. Print out a similar table as in **a)** and make sure each row in the table is identified by the rock names you choose.

Exercise 3.4: Fahrenheit-Celsius conversion functions (*from INF1100*)
Write a function C(F) that implements the formula (1). Also write the inverse function F(C) for going from Celsius to Fahrenheit degrees. How can you test that the two functions work? (*Hint:* `C(F(c))` *should result in* `c` *and* `F(C(f))` *should result in* `f`).

Exercise 10.11: Earthquake energies (*from GEO-KJM1040*)
Below is a series of observations from different earthquakes:

```
richter = [5.5, 5.3, 2.9, 3.6, 4.1, 5.2, 3.7, 4.4, 6.9, 5.8, 4.0]
```

The values are measurements of earthquake energies on the Richter scale (which is a scale often used when dealing with earthquakes).

a) Use if-tests to count the number of earthquakes in the following three categories:

Light shake: $m \leq 4$
Medium shake: $4 \leq m \leq 5.5$
Heavy shake: $m \geq 5.5$

Here m represents an energy value on the Richter scale.

b) Print to screen every time there is a medium or heavy quake and how many such quakes there have been earlier.

c) The energy difference between two quakes can be described by $\Delta E = 10^{2(m_1 - m_2)/3}$. How many times more energy is there in the largest quake in the series compared to the smallest?

Week 3 (user input, error handling)

Exercise 5.8: Geological data entry (*new, but inspired by GEO-KJM1040*)
This exercise involves registration of geological data and handling possible input errors made by the user of the program.

a) Write a function which takes 2 arguments; 1. A question to ask the user, 2. A datatype to convert user input to. Let the function ask the user for input (you can use the `input()` function) and then attempt to convert the provided input to the datatype specified by the second argument. Keep asking the user until the conversion is successful and return the converted variable.

b) You are going to support data entry of various rocks/minerals and their densities. Let the user be able to enter as many different rocks as she/he wants and their densities (in g/cm^3). If the user enters an invalid density, i.e. one which cannot be converted to a float, let she/he know and ask again until valid entry. (*Hint: Here your function from **a)** can be used and modified*).

c) Compute corresponding densities in kg/m^3 and print out a nicely formatted table with rock names, g/cm^3 densities and kg/m^3 densities. Also compute the min, max and average densities and print them out to the screen.

Week 4 (arrays, plotting)

Exercise 5.40: Plot the velocity profile for pipeflow (*from INF1100*)
A fluid that flows through a (very long) pipe has zero velocity on the pipe wall and a maximum velocity along the centerline of the pipe. The velocity v varies through the pipe cross section according to the following formula:

$$v(r) = \left(\frac{\beta}{2\mu_0}\right)^{1/n} \frac{n}{n+1}\left(R^{1+1/n} - r^{1+1/n}\right)$$

where R is the radius of the pipe, β is the pressure gradient (the force that drivesthe flow through the pipe), μ_0 is a viscosity coefficient (small for air, larger for water and even larger for toothpaste), n is a real number reflecting the viscous properties of the fluid ($n = 1$ for water and air, $n < 1$ for many modern plastic materials), and r is a radial coordinate that measures the distance from the centerline ($r = 0$ is the centerline, $r = R$ is the pipe wall)

a) Make a Python function that evaluates $v(r)$.

b) Plot $v(r)$ as a function of $r \in [0, R]$, with $R = 1, \beta = 0.02, \mu_0 = 0.02$ and $n = 0.1$.

c) Make an animation of how the $v(r)$ curves varies as n goes from 1 and down to 0.01. Because the maximum value of $v(r)$ decreases rapidly as n decreases, each curve can be normalized by its $v(0)$ value such that the maximum value is always unity.

Exercise 6.3: Two volcanoes (*from GEO-KJM1040*)
We can imagine a volcano as a cylinder and describe its magma emission rate (in m^3/s) by

$$Q = \frac{\pi R^4 P}{8\mu L}$$

where L is the length in meters, R the radius in meters, μ the viscosity of the magma in Pascel per second and P the pressure in the volcano given in Pascal. For a volcano on Hawaii and Mount Helen we can have typical values of

$$\begin{aligned} L_H &= 5000 & L_{St} &= 7000 \\ \mu_H &= 100 & \mu_{St} &= 2 \cdot 10^6 \\ R_H &= 10 & R_{St} &= 50 \\ P_H &= 5 \cdot 10^6 & P_{St} &= 3 \cdot 10^7 \end{aligned}$$

a) Which volcano emits the most magma under typical conditions?

b) What pressure would be necessary in the Hawaii volcano for the two volcanoes to emit equal amounts of magma?

a) Plot Q as a function of radii and lengths close to the given typical values. What are the most important factors in how much magma is emitted?

Week 5 (difference equations)

Exercise 6.11: Three animal groups (*from GEO-KJM1040, but extended and slightly modified*)
In this exercise we will study the evolution of an animal population divided amongst three cohorts (youngest, middle and oldest). We assume 40% of the animals in the youngest cohort live on to the next year, while 70% of the middle cohort live on to the next year. No animals live more than three years. An individual in the middle cohort becomes, on average, parent to 1.5 individuals that are born the next year. An individual in the oldest cohort becomes, on average, parent to 1.4 individuals that are born the next year.

a) Let x_n, y_n, z_n be the number of animals after n years in the youngest, middle and oldest cohort, respectively. Explain why the following set of coupled linear first order difference equations describe the evolution of the animal population:

$$\begin{aligned} x_{n+1} &= 1.5y_n + 1.4z_n \\ y_{n+1} &= 0.4x_n \\ z_{n+1} &= 0.7y_n \end{aligned}$$

b) Write a function which takes the initial conditions x_0, z_0, y_0 and max year N as arguments, solves the system of difference equations for $1 \leq n \leq N$ and returns a tuple of the three computed arrays x_n, y_n, z_n.

c) Assume that there are 100 animals of each cohort the first year and use $N = 100$ years. Call your function from **b)** and plot all three curves in the same window. Generate another window where you plot the relative populations

$$x'_n = \frac{x_n}{x_n + y_n + z_n}\,, \qquad y'_n = \frac{y_n}{x_n + y_n + z_n}\,, \qquad z'_n = \frac{z_n}{x_n + y_n + z_n}$$

d) Repeat **b)** with different initial values, e.g. $x_0 = 300, y_0 = z_0 = 0$. Compare the results with what you got in **b)**. Try again for yet another set of initial conditions. Do you spot a pattern?

e) Often we like to vectorize Python code in order to make it more efficient. Why is it not so straightforward to vectorize the process of solving the difference equations in this exercise?