Ch.9: Object-oriented programming

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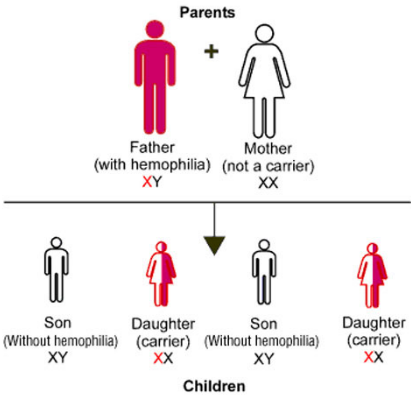
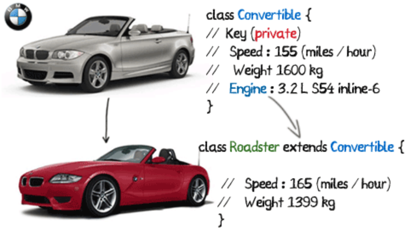
1. Inheritance

Inheritance





The chapter title *Object-oriented programming* (OO) may mean two different things

1. Programming with classes (better: object-*based* programming)
2. Programming with class hierarchies (class families)

New concept: collect classes in families (hierarchies)

What is a class hierarchy?

- A family of closely related classes
- A key concept is *inheritance*: child classes can inherit attributes and methods from parent class(es) - this saves much typing and code duplication

As usual, we shall learn through examples!

OO is a Norwegian invention by Ole-Johan Dahl and Kristen Nygaard in the 1960s - one of the most important inventions in computer science, because OO is used in all big computer systems today!

Warning: OO is difficult and takes time to master

- Let ideas mature with time
- Study many examples
- OO is less important in Python than in C++, Java and C#, so the benefits of OO are less obvious in Python
- Our examples here on OO employ numerical methods for $\int_a^b f(x)dx$, $f'(x)$, $u' = f(u,t)$ - make sure you understand the simplest of these numerical methods before you study the combination of OO and numerics
- Our goal: write general, reusable modules with lots of methods for numerical computing of $\int_a^b f(x)dx$, $f'(x)$, $u' = f(u,t)$

A class for straight lines

Problem:

Make a class for evaluating lines $y = c_0 + c_1 x$.

Code:

```
class Line:
    def __init__(self, c0, c1):
        self.c0, self.c1 = c0, c1

    def __call__(self, x):
        return self.c0 + self.c1*x

    def table(self, L, R, n):
        """Return a table with n points for L <= x <= R."""
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class Parabola:
    def __init__(self, c0, c1, c2):
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Observation:

This is almost the same code as class `Line`, except for the things with c2

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Class Parabola as a subclass of Line; principles

- Parabola code = Line code + a little extra with the c_2 term
- Can we utilize class Line code in class Parabola?
- This is what inheritance is about!

Writing

```
class Parabola(Line):
    pass
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makes Parabola inherit all methods and attributes from Line, so Parabola has attributes c0 and c1 and three methods

- Line is a *superclass*, Parabola is a *subclass* (parent class, base class; child class, derived class)
- Class Parabola must add code to Line's constructor (an extra c2 attribute), __call__ (an extra term), but table can be used unaltered
- The principle is to reuse as much code in Line as possible and avoid duplicating code

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Class Parabola as a subclass of Line; code

A subclass method can call a superclass method in this way:

```
superclass_name.method(self, arg1, arg2, ...)
```

Class Parabola as a subclass of Line:

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class Parabola(Line):
    def __init__(self, c0, c1, c2):
        Line.__init__(self, c0, c1)  # Line stores c0, c1
        self.c2 = c2

    def __call__(self, x):
        return Line.__call__(self, x) + self.c2*x**2
```

What is gained?

- Class Parabola just adds code to the already existing code in class Line - no duplication of storing c0 and c1, and computing $c_0 + c_1 x$
- Class Parabola also has a table method - it is inherited
- `__init__` and `__call__` are *overridden* or *redefined* in the subclass

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Class Parabola as a subclass of Line; code

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Class `Parabola` as a subclass of `Line`:

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What is gained?

- Class `Parabola` just adds code to the already existing code in class `Line` - no duplication of storing `c0` and `c1`, and computing $c_0 + c_1 x$
- Class `Parabola` also has a `table` method - it is inherited
- `__init__` and `__call__` are *overridden* or *redefined* in the subclass

Class Parabola as a subclass of Line; demo

```
p = Parabola(1, -2, 2)
p1 = p(2.5)
print p1
print p.table(0, 1, 3)
```

Output:

```
8.5
           0             1
         0.5           0.5
           1             1
```

```python
class Line:
    def __init__(self, c0, c1):
        self.c0, self.c1 = c0, c1

    def __call__(self, x):
        return self.c0 + self.c1*x

    def table(self, L, R, n):
        """Return a table with n points for L <= x <= R."""
        s = ''
        for x in linspace(L, R, n):
            y = self(x)
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        return s

class Parabola(Line):
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    def __call__(self, x):
        return Line.__call__(self, x) + self.c2*x**2

p = Parabola(1, -2, 2)
print p(2.5)
```

(Visualize execution)

We can check class type and class relations with `isinstance(obj, type)` and `issubclass(subclassname, superclassname)`

```
>>> from Line_Parabola import Line, Parabola
>>> l = Line(-1, 1)
>>> isinstance(l, Line)
True
>>> isinstance(l, Parabola)
False

>>> p = Parabola(-1, 0, 10)
>>> isinstance(p, Parabola)
True
>>> isinstance(p, Line)
True

>>> issubclass(Parabola, Line)
True
>>> issubclass(Line, Parabola)
False

>>> p.__class__ == Parabola
True
>>> p.__class__.__name__   # string version of the class name
'Parabola'
```

Line as a subclass of Parabola

- Subclasses are often special cases of a superclass
- A line $c_0 + c_1x$ is a special case of a parabola $c_0 + c_1x + c_2x^2$
- Can `Line` be a subclass of `Parabola`?
- No problem - this is up to the programmer's choice
- Many will prefer this relation between a line and a parabola

Code when Line is a subclass of Parabola

```
class Parabola:
    def __init__(self, c0, c1, c2):
        self.c0, self.c1, self.c2 = c0, c1, c2

    def __call__(self, x):
        return self.c2*x**2 + self.c1*x + self.c0

    def table(self, L, R, n):
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        s = ''
        for x in linspace(L, R, n):
            y = self(x)
            s += '%12g %12g\n' % (x, y)
        return s

class Line(Parabola):
    def __init__(self, c0, c1):
        Parabola.__init__(self, c0, c1, 0)
```

Note: `__call__` and `table` can be reused in class `Line`!

Recall the class for numerical differentiation from Ch. 7

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

```
class Derivative:
    def __init__(self, f, h=1E-5):
        self.f = f
        self.h = float(h)

    def __call__(self, x):
        f, h = self.f, self.h      # make short forms
        return (f(x+h) - f(x))/h

def f(x):
    return exp(-x)*cos(tanh(x))

from math import exp, cos, tanh
dfdx = Derivative(f)
print dfdx(2.0)
```

There are numerous formulas numerical differentiation

$$f'(x) = \frac{f(x+h) - f(x)}{h} + \mathcal{O}(h)$$

$$f'(x) = \frac{f(x) - f(x-h)}{h} + \mathcal{O}(h)$$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} + \mathcal{O}(h^2)$$

$$f'(x) = \frac{4}{3}\frac{f(x+h) - f(x-h)}{2h} - \frac{1}{3}\frac{f(x+2h) - f(x-2h)}{4h} + \mathcal{O}(h^4)$$

$$f'(x) = \frac{3}{2}\frac{f(x+h) - f(x-h)}{2h} - \frac{3}{5}\frac{f(x+2h) - f(x-2h)}{4h} + \frac{1}{10}\frac{f(x+3h) - f(x-3h)}{6h} + \mathcal{O}(h^6)$$

$$f'(x) = \frac{1}{h}\left(-\frac{1}{6}f(x+2h) + f(x+h) - \frac{1}{2}f(x) - \frac{1}{3}f(x-h)\right) + \mathcal{O}(h^3)$$

How can we make a module that offers all these formulas?

It's easy:

```
class Forward1:
    def __init__(self, f, h=1E-5):
        self.f = f
        self.h = float(h)

    def __call__(self, x):
        f, h = self.f, self.h
        return (f(x+h) - f(x))/h

class Backward1:
    def __init__(self, f, h=1E-5):
        self.f = f
        self.h = float(h)

    def __call__(self, x):
        f, h = self.f, self.h
        return (f(x) - f(x-h))/h

class Central2:
    # same constructor
    # put relevant formula in __call__
```

What is the problem with this type of code?

All the constructors are identical so we duplicate a lot of code.

- A general OO idea: place code common to many classes in a superclass and inherit that code
- Here: inhert constructor from superclass,
  let subclasses for different differentiation formulas implement their version of `__call__`

Class hierarchy for numerical differentiation

Superclass:

```
class Diff:
    def __init__(self, f, h=1E-5):
        self.f = f
        self.h = float(h)
```

Subclass for simple 1st-order forward formula:

```
class Forward1(Diff):
    def __call__(self, x):
        f, h = self.f, self.h
        return (f(x+h) - f(x))/h
```

Subclass for 4-th order central formula:

```
class Central4(Diff):
    def __call__(self, x):
        f, h = self.f, self.h
        return (4./3)*(f(x+h)   - f(x-h))  /(2*h) - \
               (1./3)*(f(x+2*h) - f(x-2*h))/(4*h)
```

Use of the differentiation classes

Interactive example: $f(x) = \sin x$, compute $f'(x)$ for $x = \pi$

```
>>> from Diff import *
>>> from math import sin
>>> mycos = Central4(sin)
>>> # compute sin'(pi):
>>> mycos(pi)
-1.000000082740371
```

`Central4(sin)` calls inherited constructor in superclass, while `mycos(pi)` calls `__call__` in the subclass `Central4`

```python
class Diff:
    def __init__(self, f, h=1E-5):
        self.f = f
        self.h = float(h)

class Forward1(Diff):
    def __call__(self, x):
        f, h = self.f, self.h
        return (f(x+h) - f(x))/h

dfdx = Diff(lambda x: x**2)
print dfdx(0.5)
```

(Visualize execution)

A flexible main program for numerical differentiation

Suppose we want to differentiate function expressions from the command line:

```
Terminal> python df.py 'exp(sin(x))' Central 2 3.1
-1.04155573055

Terminal> python df.py 'f(x)' difftype difforder x
f'(x)
```

With `eval` and the `Diff` class hierarchy this main program can be realized in a few lines (many lines in C# and Java!):

```
import sys
from Diff import *
from math import *
from scitools.StringFunction import StringFunction

f = StringFunction(sys.argv[1])
difftype = sys.argv[2]
difforder = sys.argv[3]
classname = difftype + difforder
df = eval(classname + '(f)')
x = float(sys.argv[4])
print df(x)
```

Investigating numerical approximation errors

- We can empirically investigate the accuracy of our family of 6 numerical differentiation formulas
- Sample function: $f(x) = \exp(-10x)$
- See the book for a little program that computes the errors:

```
.    h          Forward1           Central2          Central4
6.25E-02 -2.56418286E+00  6.63876231E-01 -5.32825724E-02
3.12E-02 -1.41170013E+00  1.63556996E-01 -3.21608292E-03
1.56E-02 -7.42100948E-01  4.07398036E-02 -1.99260429E-04
7.81E-03 -3.80648092E-01  1.01756309E-02 -1.24266603E-05
3.91E-03 -1.92794011E-01  2.54332554E-03 -7.76243120E-07
1.95E-03 -9.70235594E-02  6.35795004E-04 -4.85085874E-08
```

Observations:

- Halving h from row to row reduces the errors by a factor of 2, 4 and 16, i.e, the errors go like h, h^2, and h^4
- `Central4` has really superior accuracy compared with `Forward1`

Alternative implementations (in the book)

- *Pure Python functions*
  downside: more arguments to transfer, cannot apply formulas twice to get 2nd-order derivatives etc.
- *Functional programming*
  gives the same flexibility as the OO solution
- *One class and one common math formula*
  applies math notation instead of programming techniques to generalize code

These techniques are beyond scope in the course, but place OO into a bigger perspective. Might better clarify what OO is - for some.

Formulas for numerical integration

There are numerous formulas for numerical integration and all of them can be put into a common notation:

$$\int_a^b f(x)dx \approx \sum_{i=0}^{n-1} w_i f(x_i)$$

w_i: weights, x_i: points (specific to a certain formula)

The Trapezoidal rule has $h = (b-a)/(n-1)$ and

$$x_i = a + ih, \quad w_0 = w_{n-1} = \frac{h}{2},\ w_i = h\ (i \neq 0, n-1)$$

The Midpoint rule has $h = (b-a)/n$ and

$$x_i = a + \frac{h}{2} + ih, \quad w_i = h$$

More formulas

Simpson's rule has

$$
\begin{aligned}
x_i &= a + ih, \quad h = \frac{b-a}{n-1} \\
w_0 &= w_{n-1} = \frac{h}{6} \\
w_i &= \frac{h}{3} \text{ for } i \text{ even}, \quad w_i = \frac{2h}{3} \text{ for } i \text{ odd}
\end{aligned}
$$

Other rules have more complicated formulas for w_i and x_i

Why should these formulas be implemented in a class hierarchy?

- A numerical integration formula can be implemented as a class: a, b and n are attributes and an `integrate` method evaluates the formula
- All such classes are quite similar: the evaluation of $\sum_j w_j f(x_j)$ is the same, only the definition of the points and weights differ among the classes
- Recall: code duplication is a bad thing!
- The general OO idea: place code common to many classes in a superclass and inherit that code
- Here we put $\sum_j w_j f(x_j)$ in a superclass (method `integrate`)
- Subclasses extend the superclass with code specific to a math formula, i.e., w_i and x_i in a class method `construct_rule`

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- All such classes are quite similar: the evaluation of $\sum_j w_j f(x_j)$ is the same, only the definition of the points and weights differ among the classes
- Recall: code duplication is a bad thing!
- The general OO idea: place code common to many classes in a superclass and inherit that code
- Here we put $\sum_j w_j f(x_j)$ in a superclass (method `integrate`)
- Subclasses extend the superclass with code specific to a math formula, i.e., w_i and x_i in a class method `construct_rule`

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The superclass for integration

```
class Integrator:
    def __init__(self, a, b, n):
        self.a, self.b, self.n = a, b, n
        self.points, self.weights = self.construct_method()

    def construct_method(self):
        raise NotImplementedError('no rule in class %s' % \
                                  self.__class__.__name__)

    def integrate(self, f):
        s = 0
        for i in range(len(self.weights)):
            s += self.weights[i]*f(self.points[i])
        return s

    def vectorized_integrate(self, f):
        # f must be vectorized for this to work
        return dot(self.weights, f(self.points))
```

A subclass: the Trapezoidal rule

```
class Trapezoidal(Integrator):
    def construct_method(self):
        h = (self.b - self.a)/float(self.n - 1)
        x = linspace(self.a, self.b, self.n)
        w = zeros(len(x))
        w[1:-1] += h
        w[0] = h/2;  w[-1] = h/2
        return x, w
```

Another subclass: Simpson's rule

- Simpson's rule is more tricky to implement because of different formulas for odd and even points
- Don't bother with the details of w_i and x_i in Simpson's rule now - focus on the class design!

```
class Simpson(Integrator):

    def construct_method(self):
        if self.n % 2 != 1:
            print 'n=%d must be odd, 1 is added' % self.n
            self.n += 1

        <code for computing x and w>
        return x, w
```

About the program flow

Let us integrate $\int_0^2 x^2 dx$ using 101 points:

```
def f(x):
    return x*x

method = Simpson(0, 2, 101)
print method.integrate(f)
```

Important:

- `method = Simpson(...)`: this invokes the superclass constructor, which calls `construct_method` in class `Simpson`
- `method.integrate(f)` invokes the inherited `integrate` method, defined in class `Integrator`

```python
class Integrator:
    def __init__(self, a, b, n):
        self.a, self.b, self.n = a, b, n
        self.points, self.weights = self.construct_method()

    def construct_method(self):
        raise NotImplementedError('no rule in class %s' % \
                                  self.__class__.__name__)

    def integrate(self, f):
        s = 0
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            s += self.weights[i]*f(self.points[i])
        return s

class Trapezoidal(Integrator):
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        w = zeros(len(x))
        w[1:-1] += h
        w[0] = h/2;  w[-1] = h/2
        return x, w

def f(x):
    return x*x

method = Trapezoidal(0, 2, 101)
print method.integrate(f)
```

Applications of the family of integration classes

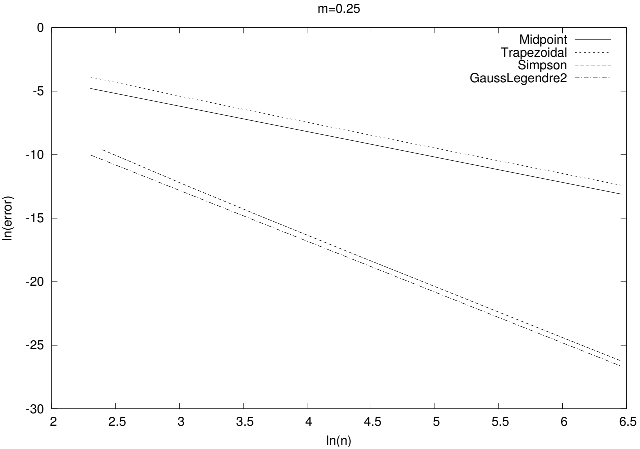
We can empirically test out the accuracy of different integration methods `Midpoint`, `Trapezoidal`, `Simpson`, `GaussLegendre2`, ... applied to, e.g.,

$$\int_0^1 \left(1 + \frac{1}{m}\right) t^{\frac{1}{m}} dt = 1$$

- This integral is "difficult" numerically for $m > 1$.
- Key problem: the error in numerical integration formulas is of the form Cn^{-r}, mathematical theory can predict r (the "order"), but we can estimate r empirically too
- See the book for computational details
- Here we focus on the conclusions

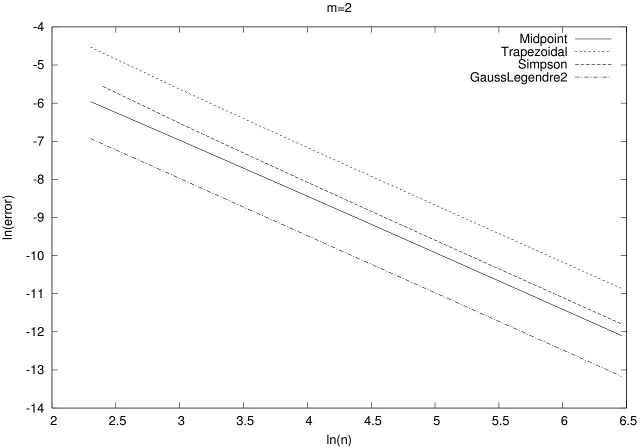
Convergence rates for $m < 1$ (easy case)

Simpson and Gauss-Legendre reduce the error faster than Midpoint and Trapezoidal (plot has ln(error) versus ln n)



Convergence rates for $m > 1$ (problematic case)

Simpson and Gauss-Legendre, which are theoretically "smarter" than Midpoint and Trapezoidal do not show superior behavior!



Summary of object-orientation principles

- A subclass inherits everything from the superclass
- When to use a subclass/superclass?
  - if code common to several classes can be placed in a superclass
  - if the problem has a natural child-parent concept
- The program flow jumps between super- and sub-classes
- It takes time to master *when* and *how* to use OO
- Study examples!

Recall the class hierarchy for differentiation

Mathematical principles:

Collection of difference formulas for $f'(x)$. For example,

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

Superclass `Diff` contains common code (constructor), subclasses implement various difference formulas.

Implementation example (superclass and one subclass)

```
class Diff:
    def __init__(self, f, h=1E-5):
        self.f = f
        self.h = float(h)

class Central2(Diff):
    def __call__(self, x):
        f, h = self.f, self.h
        return (f(x+h) - f(x-h))/(2*h)
```

Recall the class hierarchy for integration (1)

Mathematical principles:

General integration formula for numerical integration:

$$\int_a^b f(x)dx \approx \sum_{j=0}^{n-1} w_i f(x_i)$$

Superclass `Integrator` contains common code (constructor, $\sum_j w_i f(x_i)$), subclasses implement definition of w_i and x_i.

Recall the class hierarchy for integration (2)

Implementation example (superclass and one subclass):

```
class Integrator:
    def __init__(self, a, b, n):
        self.a, self.b, self.n = a, b, n
        self.points, self.weights = self.construct_method()

    def integrate(self, f):
        s = 0
        for i in range(len(self.weights)):
            s += self.weights[i]*f(self.points[i])
        return s

class Trapezoidal(Integrator):
    def construct_method(self):
        x = linspace(self.a, self.b, self.n)
        h = (self.b - self.a)/float(self.n - 1)
        w = zeros(len(x)) + h
        w[0] /= 2;  w[-1] /= 2  # adjust end weights
        return x, w
```

A summarizing example: Generalized reading of input data

Write a table of $x \in [a, b]$ and $f(x)$ to file:

```
outfile = open(filename, 'w')
from numpy import linspace
for x in linspace(a, b, n):
    outfile.write('%12g  %12g\n' % (x, f(x)))
outfile.close()
```

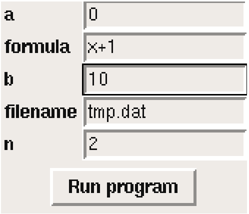
We want flexible input:

Read `a`, `b`, `n`, `filename` and a formula for `f` from...

- the command line
- interactive commands like `a=0`, `b=2`, `filename=mydat.dat`
- questions and answers in the terminal window
- a graphical user interface
- a file of the form

```
a = 0
b = 2
filename = mydat.dat
```

Graphical user interface



First we write the application code

Desired usage:

```
from ReadInput import *

# define all input parameters as name-value pairs in a dict:
p = dict(formula='x+1', a=0, b=1, n=2, filename='tmp.dat')

# read from some input medium:
inp = ReadCommandLine(p)
# or
inp = PromptUser(p)      # questions in the terminal window
# or
inp = ReadInputFile(p)   # read file or interactive commands
# or
inp = GUI(p)             # read from a GUI

# load input data into separate variables (alphabetic order)
a, b, filename, formula, n = inp.get_all()

# go!
```

About the implementation

- A superclass `ReadInput` stores the dict and provides methods for getting input into program variables (`get`, `get_all`)
- Subclasses read from different input sources
- `ReadCommandLine`, `PromptUser`, `ReadInputFile`, `GUI`
- See the book or `ReadInput.py` for implementation details
- For now the ideas and principles are more important than code details!