Ch.5: Array computing and curve plotting (Part 1)

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Plan for week 38

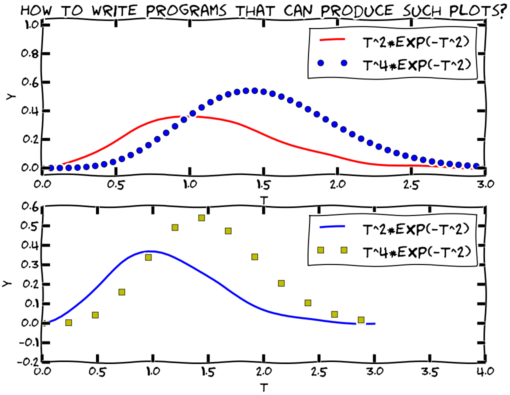
Wednesday 20 september

- Live programming of ex 4.4, 4.5, 4.6, 4.7
- Intro to plotting and NumPy arrays

Friday 22 september

- Live programming of ex 5.7, 5.9, 5.10, 5.11, 5.13
- Making movies and animations from plots
- (Making your own Python modules)

Goal: learn to visualize functions



We need to learn about a new object: array

- Curves $y = f(x)$ are visualized by drawing straight lines between points along the curve
- Need to store the coordinates of the points along the curve in lists or *arrays* `x` and `y`
- Arrays $\approx$ lists, but computationally much more efficient
- To compute the `y` coordinates (in an array) we need to learn about *array computations* or *vectorization*
- Array computations are useful for much more than plotting curves!

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The minimal need-to-know about vectors

- Vectors are known from high school mathematics, e.g., point (x, y) in the plane, point (x, y, z) in space
- In general, a vector v is an n-tuple of numbers: $v = (v_0, \ldots, v_{n-1})$
- Vectors can be represented by lists: v_i is stored as `v[i]`, but we shall use arrays instead

Vectors and arrays are key concepts in this chapter. It takes separate math courses to understand what vectors and arrays really are, but in this course we only need a small subset of the complete story. A learning strategy may be to just start using vectors/arrays in programs and later, if necessary, go back to the more mathematical details in the first part of Ch. 5.

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The minimal need-to-know about arrays

Arrays are a generalization of vectors where we can have multiple indices: $A_{i,j}$, $A_{i,j,k}$

Example: table of numbers, one index for the row, one for the column

$$
\begin{bmatrix} 0 & 12 & -1 & 5 \\ -1 & -1 & -1 & 0 \\ 11 & 5 & 5 & -2 \end{bmatrix}
\qquad
A = \begin{bmatrix} A_{0,0} & \cdots & A_{0,n-1} \\ \vdots & \ddots & \vdots \\ A_{m-1,0} & \cdots & A_{m-1,n-1} \end{bmatrix}
$$

- The no of indices in an array is the *rank* or *number of dimensions*
- Vector = one-dimensional array, or rank 1 array
- In Python code, we use Numerical Python arrays instead of nested lists to represent mathematical arrays (because this is computationally more efficient)

Storing (x,y) points on a curve in lists

Collect points on a function curve $y = f(x)$ in lists:

```
>>> def f(x):
...     return x**3
...
>>> n = 5                       # no of points
>>> dx = 1.0/(n-1)              # x spacing in [0,1]
>>> xlist = [i*dx for i in range(n)]
>>> ylist = [f(x) for x in xlist]

>>> pairs = [[x, y] for x, y in zip(xlist, ylist)]
```

Turn lists into Numerical Python (NumPy) arrays:

```
>>> import numpy as np          # module for arrays
>>> x = np.array(xlist)        # turn list xlist into array
>>> y = np.array(ylist)
```

Make arrays directly (instead of lists)

The pro drops lists and makes NumPy arrays directly:

```
>>> n = 5                              # number of points
>>> x = np.linspace(0, 1, n) # n points in [0, 1]
>>> y = np.zeros(n)          # n zeros (float data type)
>>> for i in range(n):
...     y[i] = f(x[i])
...
```

Arrays are not as flexible as list, but computational much more efficient

- List elements can be *any* Python objects
- Array elements can only be of *one object type*
- Arrays are very efficient to store in memory and compute with if the element type is `float`, `int`, or `complex`
- Rule: use arrays for sequences of numbers!

We can work with entire arrays at once - instead of one element at a time

Compute the sine of an array:

```
from math import sin

for i in range(len(x)):
    y[i] = sin(x[i])
```

However, if `x` is array, `y` can be computed by

```
y = np.sin(x)                    # x: array, y: array
```

The loop is now inside `np.sin` and implemented in very efficient C code.

Vectorization gives:

- shorter, more readable code, closer to the mathematics
- much faster code

A function `f(x)` written for a number `x` usually works for array `x` too

```python
from numpy import sin, exp, linspace

def f(x):
    return x**3 + sin(x)*exp(-3*x)

x = 1.2                         # float object
y = f(x)                        # y is float

x = linspace(0, 3, 10001)       # 10000 intervals in [0,3]
y = f(x)                        # y is array
```

Note: `math` is for numbers and `numpy` for arrays

```python
>>> import math, numpy
>>> x = numpy.linspace(0, 1, 11)
>>> math.sin(x[3])
0.29552020666133962
>>> math.sin(x)
...
TypeError: only length-1 arrays can be converted to Python scalars
>>> numpy.sin(x)
array([ 0.     ,  0.09983,  0.19866,  0.29552,  0.38941,
        0.47942,  0.56464,  0.64421,  0.71735,  0.78332,
        0.84147])
```

Very important application: vectorized code for computing points along a curve

$$f(x) = x^2 e^{-\frac{1}{2}x} \sin(x - \frac{1}{3}\pi), \quad x \in [0, 4\pi]$$

Vectorized computation of $n + 1$ points along the curve

```
from numpy import *

n = 100
x = linspace(0, 4*pi, n+1)
y = 2.5 + x**2*exp(-0.5*x)*sin(x-pi/3)
```

New term: vectorization

- *Scalar*: a number
- *Vector* or *array*: sequence of numbers (vector in mathematics)
- We speak about scalar computations (one number at a time) versus vectorized computations (operations on entire arrays, no Python loops)

- *Vectorized functions* can operate on arrays (vectors)
- *Vectorization* is the process of turning a non-vectorized algorithm with (Python) loops into a vectorized version without (Python) loops
- Mathematical functions in Python without `if` tests automatically work for both scalar and vector (array) arguments (i.e., no vectorization is needed by the programmer)

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Small quiz:

What is output from the following code? Why?

```
import numpy as np

l = [0,0.25,0.5,0.75,1]
a = np.array(l)

print(l*2)
print(a*2)
```