INF-GEO 3310

Introduction to seismic processing and imaging
Chapter 1: Introduction to rays and waves

1.1 P- and S-waves (body waves)

Body waves propagate within a medium. There are usually two types:

- **Primary (P or compressional) waves** travel through all types of materials including solids, liquids, and gases. Within the earth, P-waves travel at speeds between 1 and 14 km/s, the precise velocity depending on the rock type. The motion produced by a P-wave is an alternating compression and expansion of the material. The ground is deformed along the direction that the wave is travelling:

- **Shear (S or secondary) waves** travel through solids but not through liquids and gases. These waves travel at speeds between 1 and 8 km/s within the earth, the precise velocity depending on the rock type. S-waves vibrate the ground in a shearing motion, with movement perpendicular to the direction that the wave is travelling:

P- and S-waves are called **elastic waves** because they deform the earth elastically, and the rock returns to its original shape and position after the wave passes through. An example of a non-elastic wave is a shock wave. This type of wave fundamentally changes the medium through which it propagates. Elastic waves are also called **seismic waves**. Elastic or seismic waves can be generated from a controlled source like the airgun employed in seismic marine acquisition, or from various other sources as shown in Fig. 1.1.
1.2 Wavelength and period

A monochromatic (P or S) wave is a periodic disturbance. If we measure it at a given fixed place, the wave motion will be periodic in time. The time for one cycle is called a period and the frequency is the reciprocal of the period (cf. Fig. 1.2a). Note that frequency has the unit of one over time (sec⁻¹) or Hz (Hertz). Moreover, if we measure the monochromatic wave at a given instant in time, the wave motion will be periodic in space or distance. The distance between successive similar points on two adjacent cycles is called the wavelength. It has the units of length (m) (cf. Fig. 1.2b).

The amplitude is a measure of the height of the wave. It has units of displacement (m).

In most cases a wave will be composed of many different frequencies. This range of frequencies is sometimes called the band and thus we speak of the bandwidth. As an example take light, which is usually a multiple-frequency wave or signal. The different frequencies will then correspond to what we call colours (cf. Fig. 1.3).

1.3 Rays and waves

An impact or initial disturbance like an explosion or an earthquake will transfer energy by means of propagating waves. A wavefront represents a constant-traveltimetime curve and will be spherical for a ‘point-source’ in a homogenous medium. A ray represents the direction that energy will follow as shown in Fig. 1.4. All rays will be perpendicular to a given wavefront if the medium has the same
physical properties regardless of the direction in which they are measured. Such a medium is called **isotropic**.

If we know the wavefront at a certain time, a later wavefront can be constructed using **Huygen's principle**, which states that the original wavefront can be thought of being built up by unit (secondary) sources and that all these sources are fired at the same time. The next wavefront can then be constructed as the resulting envelope. In a homogeneous medium a plane wavefront will remain plane and a circular wavefront remain circular. Fig. 1.5 shows a more general wavefront and we can see that due to constructive and destructive interference between the source elements the energy follows the ray directions.

**1.4 Ray paths in layered isotropic media**

At an interface between two rock layers there is generally a change in propagation velocity (as well as density) resulting from the difference in physical properties of the two layers. When an elastic wave hits such an interface, the energy within the incident wave is partitioned into transmitted and reflected waves. The relative amplitudes of the transmitted and reflected pulses depend on the velocities ($V$) and densities ($\rho$) and the angle of incidence. The simplest case is that of a **normal incidence** wave (cf. Fig. 1.6), where all waves will be either P or S. The total energy of the transmitted and reflected rays must be equal to the energy of the
incident ray. If we introduce the acoustic impedance, $Z$, as the product of wave speed and density in a material, i.e. $Z = \rho V$, the reflection coefficient $R$ and the transmission coefficient $T$ can be written as

$$R = \frac{A_1}{A_2} = \frac{(\rho_2 V_2 - \rho_1 V_1)}{(\rho_2 V_2 + \rho_1 V_1)} = \frac{(Z_2 - Z_1)}{(Z_2 + Z_1)},$$
$$T = \frac{A_2}{A_c} = \frac{2Z_1}{(Z_2 + Z_1)}$$

(1.1)

A negative value of $R$ signifies a phase change of 180 degrees in the reflected ray. Note that in Eq.(1.1) the term amplitude implies displacement amplitude. If we however consider a P-wave, we can alternatively consider the pressure amplitude. The formula for the transmission coefficient will then read: $T = \frac{2Z_2}{Z_1(Z_2 + Z_1)}$. From a physical point of view we recognize that for a P-wave propagating into a denser material the displacement will decrease and the pressure will correspondingly increase to compensate for the energy balance. Hence, these differences in the transmission coefficient formulas.

![Diagram](FIG. 1.6: Reflection and transmission of normally incident rays.)

If the elastic wave is incident an angle, both reflected P- and S-waves and transmitted P- and S-waves will be generated at the interface between the two elastic media (cf. Fig. 1.7. However, at a fluid-solid interface like the seafloor, shear waves will not exist in the fluid part. The reflection and transmission coefficients are now given by the so-called Zoeppritz’ equations.
1. 5 Snell’s law

When a wave crosses a boundary between two isotropic media, the wave changes direction such that the ratio between sine of the ray-angle and the medium velocity is preserved. This is **Snell’s law** and it applies to both S- and P-waves, and to both transmitted and reflected waves. For the ray shown in Fig. 1.8, Snell’s law becomes:

\[
\frac{\sin \theta_1}{V_1} = \frac{\sin \theta_2}{V_2} = p
\]

This ratio is also called the ray parameter \( p \).

If an incident elastic body wave hits an interface and the deeper medium has the highest velocity, a **refracted wave or head wave** can be generated. Such a wave behaves like an interface wave, propagating in the high-velocity medium and parallel to the bedding. As it propagates along the interface it leaks out energy that propagates away as free body waves as shown in Fig. 1.9.
A head wave will not be generated unless the angle of incidence is equal to the critical angle. This angle can be computed from Snell’s law as follows

\[
\sin(i_c) = \frac{\sin(90^\circ)}{V_2} = \frac{V_1}{V_2}
\]

(1.3)

If a wave hits the termination of one or more reflectors as with faults (cf. Fig. 1.10), the wave energy will no longer follow Snell’s law, but be spread out in all directions. Such events are identified as diffracted energy on a time recording, and are characterized by distinctive curved alignments.

1.6 Fresnel zone and resolution

Consider a wave that is reflected from a horizontal interface as shown in Fig. 1.11. This reflection process can be described in two different ways. The trajectories of the wave energy are described by the rays obeying the Snell’s law, and will be vertical in the case of coinciding source and receiver. These rays are called specular rays and perfectly describe the reflection phenomenon in the theoretical limit of infinite frequencies. However, since the wave is a band-limited energy-packet, the reflection will also involve a considerable volume around the specular point rather than just the point itself as described by the specular ray. This zone is called the Fresnel zone and is defined as the portion of a reflector from which
energy can reach the detector within one-half the wavelength of the first reflected energy.

We have seen that reflections that can be considered as almost coincident in time at the receiver come form a region called the Fresnel zone. If the wavelength is large then the zone over which the reflected returns come from is larger and the horizontal resolution is lower. In general, horizontal resolution depends on the frequency and the wave velocity. For a plane reflecting interface and coincident source and receiver (cf. Fig. 1.12) the Fresnel zone will be circular and with a radius $R_f$ given as:

\[ R_f = \sqrt{\frac{\lambda Z}{2}} \]  

(1.4)

where $\lambda$ is the dominant wavelength and $Z$ is the depth down to the target surface. If we introduce the centre frequency $f_c$ of the pulse (i.e. representing the most energetic part), we have that $\lambda \equiv V / f_c$ with $V$ being the wave velocity. Hence, we can rewrite the formula for the Fresnel zone as follows:

\[ R_f = \sqrt{\frac{VZ}{2 f_c}} \]  

(1.5)
Like horizontal resolution also **vertical resolution** limits how well features of a given model can be detected. Vertical resolution is the ability to separate two features that are close together. An elastic wave can be considered as a propagating energy-packet or pulse. If such a wave is being reflected from the top and bottom of a bed (defined by red lines in Fig. 1.13), the result will depend on the interaction of closely spaced pulses. The limiting case of resolution is when the thickness of the bed is equal to **one-quarter** of the dominant wavelength of the pulse, which is called the tuning thickness. For smaller bed thicknesses, the limit of visibility is reached and positional uncertainties are introduced.

**FIG. 1.13 : Example of a thinning bed (wedge)**

### 1.7 The effect of absorption

The earth acts as a natural filter removing the higher frequencies more readily than the lower frequencies. In effect the deeper the source of reflections, the lower the frequencies we can receive from those depths and therefore the lower resolution we happen to have from great depths.

As an example, if we again introduce the centre frequency $f_c$ of the energy pulse (disturbance) we obtain the following simple relationship between the dominant wavelength $\lambda$, the wave velocity $V$ and the centre frequency:

$$\lambda \approx \frac{V}{f_c}.$$  

(1.6)

Typical values for the dominant wavelength are:

- **Shallow earth**: $\lambda = \frac{2000 \text{ m/s}}{50 \text{ Hz}} = 40 \text{ m}$.
- **Deep earth**: $\lambda = \frac{5000 \text{ m/s}}{20 \text{ Hz}} = 250 \text{ m}$.
For a point-source in a homogenous medium a frequently used absorption model is as follows (where $A$ is the wave amplitude a distance $r$ from the source point, and $A_0$ is the amplitude at a reference point with distance $r_0$ from the source point):

$$\frac{A}{A_0} = \frac{r_0}{r} e^{-\alpha (r-r_0)} , \quad \alpha = \pi f/QV .$$

(1.7)

In Eq. (1.7) $\alpha$ is the absorption factor, $f$ is the frequency, $V$ is the wave velocity and $Q$ is the seismic quality factor. The main assumptions behind this model is that the absorption is proportional to the frequency. The quality factor $Q$ is often diagnostic of rock type. In general, the smaller $Q$ value the larger absorption. From laboratory studies some typical ranges of the quality factor has been obtained:

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>Quality Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Granite</td>
<td>40 - 230</td>
</tr>
<tr>
<td>Sandstone</td>
<td>7 - 70</td>
</tr>
<tr>
<td>Shale</td>
<td>7 - 150</td>
</tr>
<tr>
<td>Limestone</td>
<td>45 - 190</td>
</tr>
</tbody>
</table>

Chapter 2: Acquisition of seismic data

2.1 Marine surveying

Offshore (marine) seismic surveying represents the far most important geophysical exploration method on the Norwegian continental shelf. Marine acquisition of seismic data is generally accomplished using large ships with one or multiple airgun arrays for sources. **Airguns** are deployed behind the seismic vessel and generate a seismic signal by focusing highly pressurized air into the water. Receivers are towed behind the ship in one or several long **streamer(s)** that are several kilometres in length (cf. Fig. 2.1). Marine receivers are composed of piezoelectric **hydrophones**, which respond to changes in water pressure.

![Schematics of marine surveying.](Copyright Leiv-J. Gelius/GeoCLASS - 2007)
2.2 The airgun source

The latest version of the airgun is the **sleeve-gun** which has the ability of producing a much more spherical bubble than the earlier generations of airguns. A single airgun produces a pulse of energy (or signature) as shown in Fig. 2.2. The upper part of the plot shows the actual time-function and the lower part shows the frequency content (spectrum) of the signal. We can easily see that this signal is not very much compressed in time, and its spectrum contains many notches due to the oscillatory nature of the signature.

**FIG. 2.2: Signature of a single airgun.**

Although the initial energy burst is reasonable, a complex pressure interaction between the air bubble and the water causes the bubble to oscillate as it floats towards the surface. This effect produces the extraneous bursts of energy following the initial burst.

The **period of the bubble oscillations** is given approximately by the modified Rayleigh-Willis formula:

\[
T = k \frac{P^{1/3}V^{1/3}}{[P_{atm} + \rho gD]^{5/6}}
\]

(2.1)

where \(P\) is the gun pressure, \(V\) is the gun volume, \(P_{atm}\) is the atmospheric pressure, \(\rho\) is the density of water, \(g\) is gravitational acceleration, \(D\) is depth of the gun, and \(k\) is a constant whose value depends on the units. From the bubble period for one gun of known volume, pressure, depth, and bubble period, it is possible to determine the constant \(k\). It follows directly from this formula that the bigger the capacity of the gun fired, the longer the period of oscillation. For example a 7.4 l (465 cu.in.) gun at a depth of 15m, would have a bubble period of about 85ms.

If we build an **array of guns**, made of different chamber sizes (cf. Fig. 2.3), and fire these simultaneously, we gain several advantages:

- We obviously increase the total amount of energy being directed into the ground for one ‘shot’.
- The difference chamber sizes will produce different bubble responses, and these will tend to cancel out.
• We improve the directivity of the source. Other than directly below the source array, some frequencies will be attenuated by the spatial design of the array.

![FIG. 2.3: Typical design of a gun-array.](image)

A typical output from a gun array is shown in Fig. 2.4 (far-field). The figure shows both the time (upper) and frequency (lower) response of the entire array. When the entire array is fired, the bubbles 'cancel-out' (more or less), and the frequency content is much flatter over the range of typical seismic frequencies, than in the case of a single airgun shown in Fig. 2.2. We can also see that the signature now is much closer to an ideal source, and it is very repeatable. A gun array needs to be very reliable since shots are normally fired roughly every 5-10 seconds, which gives a possibility of up to 10,000 shots in 24 hours!

![FIG. 2.4: Signature from a gun array.](image)

### 2.3 The hydrophone and the seismic streamer

A **hydrophone** (see Fig. 2.5) is a piezoelectric device that can measure the pressure. It measures pressure variations with the aid of piezoelectric material, which generates a voltage upon deformation. The two piezoelectric elements in one hydrophone are connected and polarized so that voltages due to pressure waves (returning signal) adds and voltages due to one directional acceleration will cancel. In this way the influence of movements due to currents, wave action and so on will be minimized.

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The receiver cable towed by the boat in a marine data acquisition is called a **streamer** (cf. Fig. 2.6) in which the receiver elements are hydrophones. A typical length of streamer is about 4 km. Apart from attached equipment, a single streamer basically consists of four components: **the lead-in cable, the stretch sections, the active sections** and **the data acquisition modules**. The lead-in cable is the front-end system and protects it against propeller-wash and the blast of the airguns. The stretch sections connected to the lead-in contain nylon ropes embedded in kerosene for neutral buoyancy and are protected by a neoprene skin. The nylon ropes serve to absorb the jerk from the pitching vessel on the streamer in order to minimize the jerk-noise on the recorded data. The active sections contain the hydrophones and communication coils for the attached devices. A single section within the active sections is typically 75 m long and contains 96 hydrophones which are **grouped in arrays** of a pre-defined length, mostly 12.5 or 25.0 m. Finally, the output from the hydrophones are digitized in data acquisition modules (see Fig. 2.7) which are positioned between the active sections.
2.4 2-D acquisition versus 3-D acquisition

In 2-D data acquisition the vessel is towing a single streamer, hence the data collection occurs along a line of receivers. The resultant image represents only a section below the line. Unfortunately, this method does not always produce a clear image of the geology. 2-D data can often be distorted with diffractions and events produced from offline geologic structures, making accurate interpretations difficult. Because seismic waves travel along expanding wavefronts they have a surface area. A truly representative image of the subsurface is only obtained when the entire wavefield is sampled. A 3-D seismic survey (cf. Fig. 2.8) where a vessel is towing many streamers (up to 20) at the same time (and with multiple arrays of airguns) is more capable of accurately imaging reflected waves because it utilizes multiple points of observations.

FIG. 2.8: 3-D seismic acquisition.

Chapter 3: Basic processing of seismic data

3.1 Seismic exploration

The first step in seismic exploration is the acquisition of measurement data, which in most cases is carried out from the surface (see Fig. 3.1). These raw data are then processed employing advanced methods within signal processing and wave-theory to give images of the subsurface. The seismic images or sections represent slices through the geological model, which can be input to advanced workstations where the actual interpretation can take place. Based on the seismic interpretations one will decide if an area is a possible prospect for oil or gas. If the answer is positive and initial drilling program will be performed, and in the final end be replaced by a production phase if the target area is a commercial reservoir.
The seismic reflection method is the most extensively used geophysical exploration method. A seismic source emits a pulse (e.g. airguns in marine seismic) that propagates through the sedimentary layers. Reflections from the layer interfaces in the subsurface are then measured at receivers (e.g. streamer in marine seismic) as shown schematically in Fig. 3.2.

![FIG. 3.2: Schematics of reflection method.](image)

The purpose and overall goal of seismic processing is then to produce an accurate image of the earth (cf. Fig. 3.3).

![FIG. 3.3: Example of a geological model and a seismic section.](image)

### 3.2 CMP or CDP sorting

The seismic data acquisition is carried out in the source-receiver \((s,g)\)-domain. Introduce now *offset* as the distance between source and receiver for a given trace (*trace* is a seismic time measurement corresponding to one source-receiver pair). The data is then resorted on a midpoint-offset form (cf. Fig. 3.4), i.e. described by the midpoint-offset \((y,h)\)-domain. Note that:

\[
y = \frac{(g + s)}{2}, \quad h = \frac{(g - s)}{2}
\]

Traces with common midpoint between the sources and receivers are gathered in groups. These groups are denoted **CMP (common midpoint) gathers**. The main idea is that for a horizontal layered earth model (see Fig. 3.4) the same subsurface point is imaged many times, which will enhance the signal-to-noise ratio. **Fold** is the number of traces representing the same subsurface point, which during seismic processing will be averaged to give a single stacked trace as output.
However as shown in Fig. 3.5, the method will only work perfectly in the case of a horizontally layered earth model. In the literature CMP-sorting is also known as CDP-sorting, where CDP stands for common depth point.

3.3 Amplitude correction and muting

Muting means removing parts of the data from each CMP-gather. Fig. 3.6 shows an example of a CMP-family before mute, where the data is partly contaminated.
The strong unwanted events in Fig. 3.6 are caused by both the direct wave and the refracted wave associated with the seabottom (see Fig. 3.7).

After having muted these two contributions the CMP-gather will look like the one shown in Fig. 3.9a. We easily see that the amplitudes are much weaker for larger traveltimes. These seismic amplitude losses are caused by three major factors:

- **Geometrical spreading**: progressive diminution of amplitude (proportional to the inverse of propagation distance) caused by the increase in wavefront area (cf. Fig. 3.8a).
- **Intrinsic attenuation**: energy losses due to internal friction (cf. Fig. 3.8b).
- **Transmission losses**: reduction in wave amplitude due to reflection at interfaces.

**Amplitude recovery** attempts to correct for the amplitude losses that are unrelated to reflection coefficient, such as wave attenuation and source variations. Both statistical and deterministic methods exist. One deterministic approach frequently employed is the so-called t-square method, where the data is multiplied with the square of the two-way traveltime. It is based on the following assumptions:

- Multiplication with $t$ to compensate for geometrical spreading (spherical wave and uniform medium).
• An attenuation model of the type where the total losses are given as an integration over all frequencies:

\[
\int_{0}^{\infty} \exp\left[-\pi \frac{f t}{Q}\right] df = \frac{Q}{\pi t}
\]

implying another multiplication with \( t \) to compensate.

Application of the t-square method to the CMP-gather shown in Fig. 3.9a gives the result shown in Fig. 3.9b which is seen to be significantly improved.

![CMP-gather before and after t-square compensation](image)

**FIG. 3.9:** CMP-gather (a) before and (b) after t-square compensation.

### 3.4 Deconvolution (pulse shaping)

To further improve the seismic data, pulse shaping is also often applied. Such filtering techniques are denoted deconvolution and are used both to compress the pulse (increase vertical resolution) and more importantly to remove multiples. Multiples are internal reflections in a layer, which occur when exceptionally large reflection coefficients are present. As shown in Fig. 3.10, we distinguish between multiples on the surface and multiples between two interfaces of a layer (‘peg-leg’ multiple).

![Diagram of different classes of multiples](image)

**FIG. 3.10:** Different classes of multiples.

In marine seismic the water-bottom multiples normally dominates. Fig. 3.11 shows a schematics of the primary seafloor reflection and its two lowest order multiples.
Conceptually we can now consider each subsurface reflection as the result obtained employing a pulse as shown in Fig. 3.12:

**Predictive deconvolution** is then a technique that compresses the long pulse in Fig. 3.12 and aims to replace it with its primary only. Fig. 3.13a shows an example of a seismic section where water-bottom multiples associated with both the seafloor and two other reflectors (R1 and R2) are clearly visible. After predictive deconvolution the result shown in Fig. 3.13b is obtained, where the multiples now have been significantly weakened.

3.5 **NMO – normal moveout**
At this stage we assume that each CMP-gather has been conditioned employing the techniques described in the previous sections. In addition, possible filtering of random noise could also have been applied. We assume a horizontally layered (e.g. stratigraphic) earth model. For such a model each trace in a CMP-gather will contain measurements from the same depth (image) points as shown in Fig. 3.14. The NMO (normal moveout) is defined as the difference between the travelt ime for a certain offset (x) and the vertical (zero-offset) travelt ime $t_0$.

![FIG. 3.14: Common reflection point (horizontal reflector).](image)

To find an explicit expression for NMO in a simple one-layer model as in Fig. 3.14, we introduce the mirror source $S'$ as shown in Fig. 3.15. It follows then directly from the Pythagorean theorem that:

$$
(v't)^2 = (v't_0)^2 + x^2
$$

(3.3)

Introduce then the definition of NMO, e.g.:

$$
t = t_0 + \Delta t
$$

(3.4)

Combination of Eqs.(3.3) and (3.4) gives the final result

$$
t = t_0 + \Delta t = \sqrt{t_0^2 + x^2 / v^2} \Rightarrow \Delta t = \sqrt{t_0^2 + x^2 / v^2} - t_0
$$

(3.5)
The result in Eq.(3.5) can be generalized to the case of many horizontal layers as shown in Fig. 3.16. The NMO-equation now reads:

$$\Delta t = t(x) - t_0 = \sqrt{\frac{x^2}{v_0^2} - t_0^2}$$

(3.6)

When compared with Eq.(3.5) we see that the constant velocity $v$ now has been replaced by the so-called NMO-velocity defined as:

$$v_{NMO} = \sqrt{\frac{\sum_i v_i^2 \Delta t_{i0}}{\sum_i \Delta t_{i0}}}$$

(3.7)

In Eq.(3.7) $v_i$ is the interval velocity of layer $i$, and $\Delta t_{i0}$ represent the vertical traveltime inside the same layer. When computing Eq.(3.7) one has to consider all layers above the interface considered.
The NMO-correction as described by Eq.(3.6) is **non-linear**. Hence, for a finite-bandwidth pulse it will change its dominant period (or frequency) when this correction is applied as indicated in Fig. 3.17

![FIG. 3.17: NMO-stretch.](image)

This effect is called **NMO-stretch**. It can be quantified through the formula (cf. Fig. 3.17):

\[
\text{NMO-stretch: } \frac{\Delta f_p}{f_p} = \frac{(f_p - f_{p0})}{f_p} \approx \frac{\Delta t}{t_0}
\]

From Eq.(3.8) it follows that the NMO-stretch is significant for **large offsets** (i.e. NMO-correction \( \Delta t \) will be large) and/or **shallow depths** (i.e. the zero-offset traveltime \( t_0 \) will be small). Fig. 3.17a shows an example of a CMP-gather after NMO-correction. We can easily recognize the severe distortions due to the NMO-stretch. **After muting** the distorted areas, we obtain the clean CMP-gather as shown in Fig. 3.17b, which is the version to be used.

![FIG. 3.17: NMO-corrected CMP-gather (a) before and (b) after mute.](image)

Up til now we have assumed that the NMO-velocities for a given earth model are **known**. In practice, a **velocity analysis** has to be carried out, where the main purpose is to determine the velocity-distribution as a function of time which will give the most accurate NMO-corrections. After these corrections have been applied we want the **primary reflections** in a CMP-gather to be ‘flat’.
3.6 Velocity analysis

The most common type of velocity analysis is to repeat the procedure of correcting and stacking (adding together) CMP-data for many different velocities and within discrete time-windows as illustrated in Fig. 3.18.

![Fig. 3.18: Test of different velocities for NMO-correction.](image)

By measuring the average absolute value of the data (or more precisely the energy employing the so-called semblance) within time-windows for different test-velocities and plot these results in a time versus velocity histogram (cf. Fig. 3.19), it is possible to interpret the velocity information. This type of plot (see Fig.19) is denoted a velocity spectrum (or semblance plot). In general, the interpreted velocity function is picked so that it goes through areas with highest values.

![Fig. 3.19: Example of velocity spectrum and picked velocity function.](image)

3.7 Stacking

Stacking represents summation of NMO-corrected traces in a CMP-family ($M$ being total number of traces and $f_{i,t(i)}$ is the amplitude value for trace $i$ at two-way time $t(i)$)
The stacking process has two major advantages (see Fig. 3.20):

- It increases the signal-to-noise \((S/N)\) ratio (if the noise is Gaussian or ‘white’ the improvement of \(S/N\) is proportional to the square root of \(M\))
- It amplifies primary energy relative multiple energy.

This last point depends on a **good velocity analysis**. In the case of an accurate velocity model, stacking is the most efficient multiple removal method.

\[
s_t = \sum_{i=1}^{M} f_i s(i), \quad t(i) = \sqrt{t^2(0) + \chi_i^2 / \chi_n^2}
\]  

3.8 Noise filtering

We have already discussed the case of non-linear coherent noise caused by multiples. In addition, the stacked data can be contaminated by **linear coherent noise** caused by diffractions (f.ex. due to irregularities in the bottom of the ocean), refractions and streamer noise. Such type of noise can be removed employing **FK-filtering**, which is a 2D-filter designed in the double Fourier Transform domain. Fig. 3.21 gives an example of a FK-filtered seismic section.

The stacked data can also be distorted by **ambient noise**. Such noise is independent of explosion and produce by rain, wind, tides, vibrations of machinery, passing vehicles, platforms etc. (in general these movements are of high frequencies). Ambient noise is removed by employing **low-pass** and **bandpass** filters. In the Fourier domain this filtering operation can be written as:
In Eq. (3.10) $X(\omega)$ and $M(\omega)$ represent the Fourier Transform of the stacked trace before and after filtering respectively ($\omega$ is the angular frequency, i.e. $\omega = \pi f$). Moreover, $F(\omega)$ represents the transfer function of the filter. Fig. 3.22 gives examples (schematically) of such transfer function for zero-phase filters.

3.9 Plotting convention

The wiggle trace display place the centre of each (stacked) trace at the position along the x-axis corresponding to the mid-point location. Deviation of the wiggle trace to the right or left represents positive or negative pressure (or surface velocity in case of land data), respectively as detected by the recording instruments. In most cases (cf. Fig. 3.23) positive areas of the trace are shaded (variable area) (and the plot scale is often chosen so that the maximum trace value slightly overlaps the adjacent trace). A local maximum positive excursion is a peak and a local minimum is a trough.

When plotting the stacked data, a plot equalization is also applied. The purpose is to make sure that weaker reflections are also visible on the seismic section. The scaling will be time-dependent and the most frequently used technique is AGC (automatic gain control). A small operator (filter) length will enhance local
variations, whereas the larger operator length the less time-variant scaling is introduced (cf. Fig. 3.24).

The following example shows the effect of varying the operator-length of the AGC:

FIG. 3.24: Effect of AGC (plot scaling).

Fig. 3.25 shows an example of a stacked seismic section from the Gulf of Mexico, including a strong seismic anomaly (bright spot) associated with a gas sand.

FIG. 3.25: Example of a stacked section.

Chapter 4: Imaging/migration of seismic data
4.1 Time versus depth model

In a true **zero-offset section** each seismic trace represents a time measurement with coincident source and receiver. A **stacked section** is assumed to be close to a zero-offset section. However, the stacking process is founded on the concept of **common depth points (CDP)**, which says that:

![Diagram showing a non-vertical reflection point.](image)

As shown in Fig. 4.1 this assumption is violated in case of a dipping layer.

Due to the fact that every stacked trace is assumed to be a zero-offset measurement associated with a stratigraphic earth model, plotting them **vertically** in time will not be correct for general geological models. Figs. 4.2a – 4.2c show examples of differences between time and depth models.

![Diagram showing differences between time and depth models.](image)

**Migration** is an imaging method which aims to correct for differences between time and depth models, i.e.:

- Diffractions are collapsed to points.
- Dipping reflections move up dip and become steeper.
• Triplications (‘bow ties’, cf. Fig. 4.2a) associated with synforms are unwrapped.

In order for migration to work correctly, the seismic (background) wave velocities **must be known**. Often, velocities determined previously from stacking velocities are being used. In the flowing sections we will see how to construct the migration method based on the wave-equation.

## 4.2 Exploding reflector (ER) model

As shown in Fig. 4.3, a zero-offset geometry involves many experiments (one for each surface position of the coincident source/receiver unit), and also waves travelling in both upward and downward directions.

![FIG. 4.3: Schematics of a zero-offset geometry](image)

The true experiments in Fig. 4.3 can replaced by one thought experiment denoted the **exploding reflector model**. The main idea is that receivers are placed at the surface and the surface sources are replaced by secondary sources placed at the reflectors with source strength equal to the local reflectivity and fired at \( t=0 \). Hence, only one experiment is needed and only upward travelling waves are considered (cf. Fig. 4.4). Note that according to Huygen’s principle the outgoing ray directions of the secondary source fields will be perpendicular to each interface. Also since the exploding reflector model involves one-way traveltimes data only, a **half-velocity model** is introduced.

![FIG. 4.4: Exploding reflector model.](image)

## 4.3 Main principles of migration

The main assumptions behind migration are summarised in Fig. 4.5:
The migration procedure consists of two well-defined steps:

- **Downward extrapolation**: simulation process (based on the one-way wave equation), where the receivers are moved from the surface down to an arbitrary depth.
- **Imaging**: for each downward extrapolated depth a new seismic section can be formed. From each of these sections only the part close to $t=0$ is being kept. A **migrated section** is now formed by combining these strips.

### 4.4 Downward extrapolation

The acoustic wave-equation (neglecting density variations) reads in 2-D:

$$
\frac{\partial^2}{\partial z^2} p(x, z, t) = \left[ \frac{4}{c^2(x, z)} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} \right] p(x, z, t)
$$

where $x$ is the lateral coordinate, $z$ is the depth coordinate, $t$ represents time and $P$ is the pressure field. Finally, $c(x, z)$ represents the space-variant (wave)velocity field. Note that $c/2$ has been introduced in Eq.(4.1) in accordance with the exploding reflector (ER) model. The acoustic wave-equation will handle the general case of both upward and downward travelling waves. However, since us in the ER-model only deal with upward travelling waves, we will look for alternative equations tailored for 1-way wave fields. Assuming $c = c(x)$, the acoustic wave-equation can be split as follows:
In Eq. (4.2) a plus sign corresponds to upward travelling waves. Employing Eq. (4.2), downward extrapolation can now be carried out as follows:

\[
\frac{\partial}{\partial z} p(x, z, t) = \sqrt{\frac{4}{c^2(x)} \frac{\partial^2}{\partial t^2} \frac{\partial^2}{\partial x^2} p(x, z, t)} \\
\Rightarrow p(z + \Delta z) \approx p(z) + \frac{\partial p}{\partial z} \Delta z
\]

(4.3)

Downward extrapolation can be implemented in two ways (cf. Fig. 4.6):

- **Non-recursive**: the downward extrapolation is always carried out between the surface (e.g. measurement data) and an arbitrary depth level.
- **Recursive**: the downward extrapolated ‘measurements’ form the starting point for the computations in the next (neighbouring) depth level. A recursive formulation allows small depth increments, hence lateral velocity variations can be better handled.

FIG. 4.6: Recursive and non-recursive method.

Based on different implementations of Eq. (4.3) we can discriminate between three different main algorithms:

- **FK or Stolt migration**: (non-recursive), can handle steep dips and moderate lateral velocity variations.
- **Kirchhoff migration**: very much the same characteristics as FK.
- **Finite-difference (FD) migration**: (recursive), includes dip approximation but ability to handle strong lateral velocity changes.
4.5 Imaging

At each depth level a downward extrapolated section can be computed (cf. section 4.4). From each of these sections the part close to \( t=0 \) is kept, which is assumed to give the secondary source strength at that depth level (which is proportional to the reflectivity). The collection of such strips placed in depth makes the migrated section as shown in Fig. 4.7.

![Fig. 4.7: Downward extrapolation and imaging.](image)

Fig. 4.8 shows an example of migrated field data.

![Fig. 4.8: Example of migrated field data.](image)

4.6 Errors in migration velocity

Migration is sensitive to velocity errors. To illustrate this we consider the simple test model shown in Fig. 4.9:
In this model the layer contrast is due to density variations, so that it is effectively a **constant-velocity model**. Synthetic zero-offset data were generated as shown in Fig. 4.10. As expected the horizontal reflector is horizontal also in time but the scattering points is now represented by a diffraction curve in time.

Migrating the data in Fig. 4.10 employing the correct velocity gives the result shown in Fig. 4.11, which is seen to be give a good image of the original depth model in Fig. 4.9:

Finally, by employing a too low or too high velocity we obtain distorted images as shown in Figs. 4.12a and b:
FIG. 4.12: Migrated images obtained employing (a) too low velocity and (b) too high velocity.