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More on ML & Types

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Based on John C. Mitchell's slides (Stanford U.) , adapted by Gerardo Schneider, UiO.

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ML lectures

1. 04.09: The Algol Family and ML (Mitchell's chap. 5 + more)
2. **11.09: More on ML & types (chap. 5 and 6)**
3. 18.09: More on Types, Type Inference and Polymorphism (chap. 6)
4. 02.10: Control in sequential languages, Exceptions and Continuations (chap. 8)

Outline

- More recursive examples
- More on higher-order functions
- Something about equality
- Something on the ML module system
- Types in programming
- Type safety

More on list functions

- Writing a recursive function is not difficult, but what about efficiency?

- Example: Reverse a list
  (remember [1,2] @ [3,4] = [1,2,3,4])

```
fun  reverse [] = []
   |  reverse (x::xs)  = (reverse xs) @ [x] ;
```

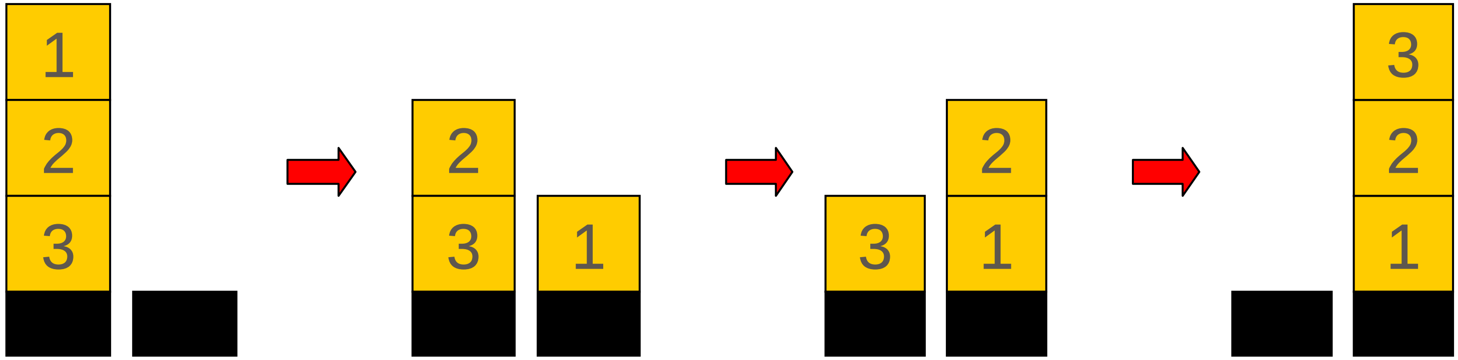
- Questions
  - How efficient is reverse?
  - Can you do this with only one pass through list?

More efficient reverse function

```
fun revAppend ([],ys) = ys
  | revAppend (x::xs,ys) = revAppend(xs,(x::ys)) ;

fun rev xs = revAppend(xs,[]);
```

Tail recursive function!



Two factorial functions

- Standard recursion

```
fun fact n =
    if n = 0 then 1 else n * fact(n-1) ;
```

- Tail recursive (iteritative)

```
fun facti(n,p) =
    if n = 0 then p else facti(n-1,n*p) ;
fun fact n = facti(n,1) ;
```

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Higher-order functions (functionals)

- Functions are computational values, hence they can be passed as an argument to another function

A *functional* is a function that operates on other functions

- Program are more concise and clear when using functionals
- Functionals on lists have been very popular in Lisp

Curried functions

- A function can have only one argument
  - tuples are used for more than one argument
- Multiple arguments may be realized by giving a function as a result
  - *Currying* -> after the logician Haskell B. Curry
- A function over pairs has type

  ’a * ’b -> ’c

  while a curried function has type

  ’a -> (’b -> ’c)
- A curried function allows *partial application*: applied to its 1st argument (of type ’a), it results in a function of type ’b -> ’c

Curried functions

- Example: function to add two numbers

```
- fun  pluss(x,y) = x + y ;
val pluss = fn : int * int -> int
- pluss(2,3) ;
val it = 5 : int
```

- Curried version of the same function

```
- fun cPluss x y = x + y ;
val cPluss = fn : int -> int -> int
- cPluss 2 3 ;
val it = 5 : int
- val addTwo = cPluss 2 ;
val addTwo = fn : int -> int
- addTwo 5 ;
val it = 7 : int
```

Curried functions

- Curry and uncurry

```
- fun curry f x y = f(x,y) ;
val curry = fn : ('a * 'b -> 'c) -> 'a -> 'b -> 'c

- fun uncurry f (x,y) = f x y  ;
val uncurry = fn : ('a -> 'b -> 'c) -> 'a * 'b -> 'c
```

Example: the map function

- Recall that map can be defined as

```
fun map (f, nil) = nil
   | map (f, x::xs) = f(x) :: map (f,xs);
   val map = fn : ('a -> 'b) * 'a list -> 'b list

   - map (fn x => x+1, [1,2,3]);
   val it = [2,3,4] : int list
```

- By currying it, we can define map as

```
fun map f nil = nil
   | map f (x::xs) = (f x) :: map f xs;
 val map = fn : ('a -> 'b) -> 'a list -> 'b list

 - map (fn x => x+1)  [1,2,3];
   val it = [2,3,4] : int list
```

More on the map function

- We can have a function having as argument a function which has another function as an argument
- Thanks to currying, we can combine functionals to work on lists of lists

  Example:

```
- map (map (fn x => x+1))  [[1], [1,2], [1,2,3]];
```

  What does it give as a result?

```
val it = [[2],[2,3], [2,3,4]] : int list list
```

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Equality

- Equality in (S)ML is defined for many types but not all – E.g., it is defined for:
  - Integers
  - Booleans
  - Strings
  - Characters
- What about floating points (reals), compund types (tuples, records, lists), functions, abstract data types, etc?

Equality on ”reals”

- In old versions of SML/NJ it was possible to compare floating points (reals) equality but not anymore

- Example

```
- 4.343 = 4.234234;
Error: operator and operand don't agree [equality type required]
  operator domain: ''Z * ''Z
  operand:         real * real
  in expression 4.343 = 4.234234
```

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Equality

- When are two expressions equal?
  - The so-called *Leibniz's Principle of the Identity of Indiscernables*:

"e1 and e2 are equal iff they cannot be distinguished by any operation in the language"

"e1 and e2 are distinct iff there is some way to tell them apart"

- What is difficult about Leibniz's Principle?

Problems with Equality

- Equality, as defined by Leibniz's principle, is undecidable

In general, there is no program which determines whether two expressions are equal in Leibniz's sense

Also:

- Problems with reference cells (aliasing)
- Polymorphic equality complicates the compiler

Equality Types

- An equality type is a type admiting equality test
- Types admiting equality in (S)ML
  - *int*, *bool*, *char*, *string*
  - *tuples* and *records*, if all their components admit equality
  - *datatypes*, if every constructor's parameter admits equality

    Ex: *lists* admit equality if the underlying element type admits equality. Moreover, two lists are equal if they have the same length and the same elements in corresponding positions

Equality Types (cont.)

- Do **not** admit equality in (S)ML
  - *reals*
  - *functions*
  - *tuples, records and datatypes* not mentioned in the previous slide
  - *abstract data types*
- Equality type variable: ’ ’ a
  - fun equals (x,y) = if x = y then true else false ;

  stdIn:7.25 Warning: calling polyEqual

  val equals = fn : ''a * ''a -> bool

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Equality: Examples

- Equality tests on functions is not computable since

  $$f = g \quad \text{iff} \quad \text{for all } x,\ f(x) = g(x)$$

- There is no ”standard” notion of equality for an abstract type
  - What is supposed to be the equality on *trees*? Is it defined structurally? Is it over the list of their elements? By DFS or BFS?
- Mitchell doesn’t cover the material presented on Equality – Check, for instance, Section 2.9 of Pucella’s notes

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Modularity: Basic Concepts

- Component
  - Meaningful program unit
    - Function, data structure, module, ...
- Interface
  - Types and operations defined within a component that are visible outside the component
- Specification
  - Intended behavior of component, expressed as property observable through interface
- Implementation
  - Data structures and functions inside component

Example: Function Component

- **Component**
  - Function to compute square root
- **Interface**
  - function sqrt (float x) returns float
- **Specification**
  - If x>1, then sqrt(x)*sqrt(x) ≈ x.
- **Implementation**

```
float sqroot (float x){
   float y = x/2; float step=x/4; int i;
   for (i=0; i<20; i++){if ((y*y)<x) y=y+step; else y=y-step; step = step/2;}
   return y;
}
```

Something on ML Modules

- Signatures and structures are part of the standard *ML module system*
- An ML structure is a module, which is a collection of:
  - Types
  - Values
  - Structure declarations
- Signatures are module interfaces
  - Kind of ”type” for a structure

Example: Point

- Signature definition (Interface)

```
signature POINT =
 sig
   type point
   val mk_point : real * real -> point  (*constructor*)
   val x_coord : point -> real          (*selector*)
   val y_coord : point -> real          (*selector*)
   val move_p : point * real * real -> point
 end;
```

Example: Point (cont.)

- Structure definition (Implementation)

```
structure pt : POINT =
 struct
   type point = real * real
   fun mk_point(x,y) = (x,y)
   fun x_coord(x,y) = x
   fun y_coord(x,y) = y
   fun move_p((x,y):point,dx,dy) = (x+dx, y+dy)
 end;
```

- To be able to use the implementation:

```
  - open pt;
```

Example: Point (cont.)

- Tests:

```
- val p1 = mk_point(4.3, 6.56);
  val p1 = (4.3,6.56) : point

- y_coord (p1);
  val it = 6.56 : real

- move_p (p1, 3.0, ~1.0);
  val it = (7.3,5.56) : point
```

Remarks – Further reading

- *signatures* and *structures* are part of ML Module system. Modules, in general, will be developed later on this course. For the present lecture you might want to read Section 9.3.2 of Mitchell's book

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Type

A type is a collection of computational entities sharing some common property

- Examples
  - Integers
  - [1 .. 100]
  - Strings
  - int $\rightarrow$ bool
  - (int $\rightarrow$ int) $\rightarrow$bool
- "Non-examples"
  - {3, true, 5.0}
  - Even integers
  - {f:int $\rightarrow$ int | if x>3 then f(x) > x*(x+1)}

Distinction between types and non-types is language dependent.

Uses for types

- Program organization and documentation
  - Separate types for separate concepts
    - E.g., customer and accounts (banking program)
  - Types can be checked, unlike program comments
- Identify and prevent errors
  - Compile-time or run-time checking can prevent meaningless computations such as 3 + true - "Bill"
- Support optimization
  - Short integers require fewer bits
  - Access record component by known offset

Type errors

- Hardware error
  - Function call x() (where x is not a function) may cause jump to instruction that does not contain a legal op code
    - If x = 512, executing x() will jump to location 512 and begin execute “instructions” there

- Unintended semantics
  - int_add(3, 4.5): Not a hardware error, since bit pattern of float 4.5 can be interpreted as an integer

General definition of type error

- A *type error* occurs when execution of program is not faithful to the intended semantics
- Type errors depend on the concepts defined in the language; not on ***how*** the program is executed on the underlying software
- All values are stored as sequences of bits
  - Store `4.5` in memory as a floating-point number
    - Location contains a particular bit pattern
  - To interpret bit pattern, we need to know the type
  - If we pass bit pattern to integer addition function, the pattern will be interpreted as an integer pattern
    - Type error if the pattern was intended to represent 4.5

Subtyping

- Subtyping is a relation on types allowing values of one type to be used in place of values of another
  - ***Substitutivity:*** If A is a subtype of B (A<:B), then any expression of type A may be used without type error in any context where B may be used
- In general, if f: A -> B, then f may be applied to x if x: A
  - Type checker: If f: A -> B and x: C, then C = A
- In languages with subtyping
  - Type checker: If f: A -> B and x: C, then C <: A

Remark: No subtypes in ML!

Monomorphism vs. Polymorphism

- *Monomorphic* means ”having only one form”, as opposed to *Polymorphic*
- A type system is monomorphic if each constant, variable, etc. has unique type
- Variables, expressions, functions, etc. are polymorphic if they ”allow” more than one type

Example. In ML, the *identity* function `fn x => x` is polymorphic: it has infinitely many types!

```
- fn x => x
val it = fn : 'a -> 'a
```

Warning! The term ”polymorphism” is used with different specific technical meanings (more on that later)

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Type safety

- A Prog. Lang. is *type safe* if no program can violate its type distinction (e.g. functions and integer)
- Examples of not type safe language features:
  - Type casts (a value of one type used as another type)
    - Use integers as functions (jump to a non-instruction or access memory not allocated to the program)
  - Pointer arithmetic
    - *(p) has type A if p has type A*
    - x = *(p+i) what is the type of x?
  - Explicit deallocation and dangling pointers
    - Allocate a pointer p to an integer, deallocate the memory referenced by p, then later use the value pointed to by p

Relative type-safety of languages

- **Not safe**: BCPL family, including C and C++
  - Casts; pointer arithmetic
- **Almost safe**: Algol family, Pascal, Ada.
  - Explicit deallocation; dangling pointers
    - No language with explicit deallocation of memory is fully type-safe
- **Safe**: Lisp, ML, Smalltalk, Java
  - Lisp, Smalltalk: dynamically typed
  - ML, Java: statically typed

Compile-time vs. run-time checking

- Lisp uses run-time type checking

  (car x) check first to make sure x is list

- ML uses compile-time type checking

  f(x) must have $f : A \rightarrow B$ and $x : A$

- Basic tradeoff
  - Both prevent type errors
  - Run-time checking slows down execution (compiled ML code, up-to **4** times faster than Lisp code)
  - Compile-time checking restricts program flexibility

    Lisp list: elements can have different types

    ML list: all elements must have same type

Compile-time type checking

- *Sound* type checker: no program with error is considered correct
- *Conservative* type checker: some programs without errors are considered to have errors
- Static typing always conservative

```
if   (possible-infinite-run-expression)
     then  (expression-with-type-error)
     else  (expression-with-type-error)
```

Cannot decide at compile time if run-time error will occur

(from the undecidability of the Turing machine's halting problem)