Oppgave 1:

a) Vi kaller de norm. vinkelfrekvensene

$\hat{\omega}_1 = \pi$, $\hat{\omega}_2 = 2\pi/3$, $\hat{\omega}_3 = \pi/3$, $\hat{\omega}_4 = \pi/90$

Vi har

$$\hat{\omega} = \frac{2\pi f}{f_s} \Rightarrow f = \frac{\hat{\omega} \cdot f_s}{2\pi}$$

så vi får (for $f_s = 900$ Hz)

$$f_1 = f_s/2 = 450 \text{ Hz}$$

$$f_2 = f_s/3 = 300 \text{ Hz}$$

$$f_3 = f_s/6 = 150 \text{ Hz}$$

$$f_4 = f_s/180 = 5 \text{ Hz}$$

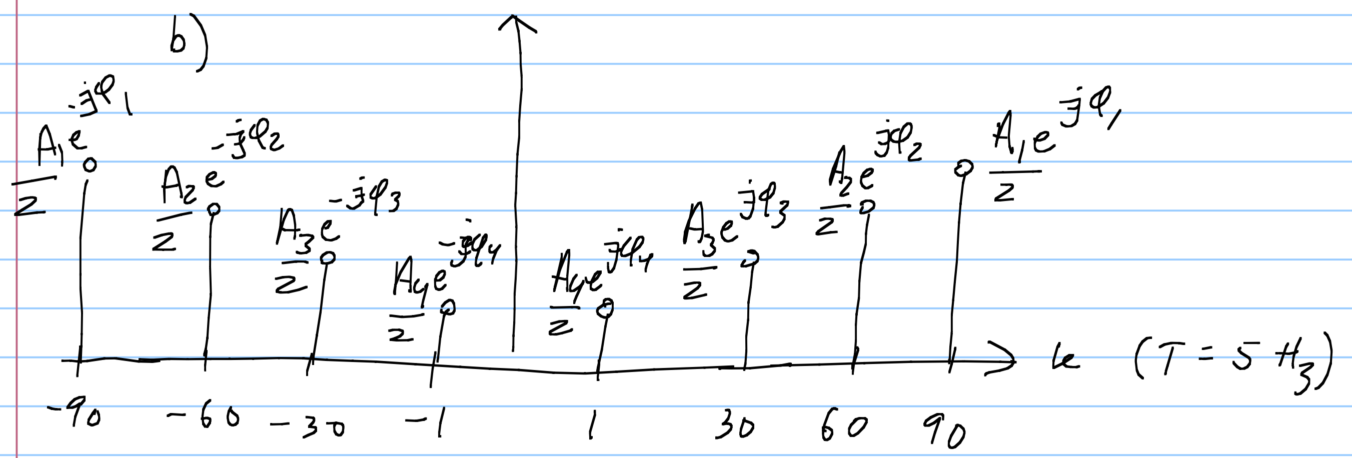
Som gir:

$$x(t) = A_1 \cos(900\pi t + \varphi_1) + A_2 \cos(600\pi t + \varphi_2) + A_3 \cos(300\pi t + \varphi_3) + A_4 \cos(10\pi t + \varphi_4)$$

Signalet er periodisk fordi

$$\gcd(450, 300, 150, 5) = 5$$

b)


$\frac{A_1}{2}e^{-j\varphi_1}$
$\frac{A_2}{2}e^{-j\varphi_2}$
$\frac{A_3}{2}e^{-j\varphi_3}$
$\frac{A_4}{2}e^{-j\varphi_4}$
$\frac{A_4}{2}e^{j\varphi_4}$
$\frac{A_3}{2}e^{j\varphi_3}$
$\frac{A_2}{2}e^{j\varphi_2}$
$\frac{A_1}{2}e^{j\varphi_1}$
-90
-60
-30
-1
1
30
60
90
k
(T = 5 Hz)

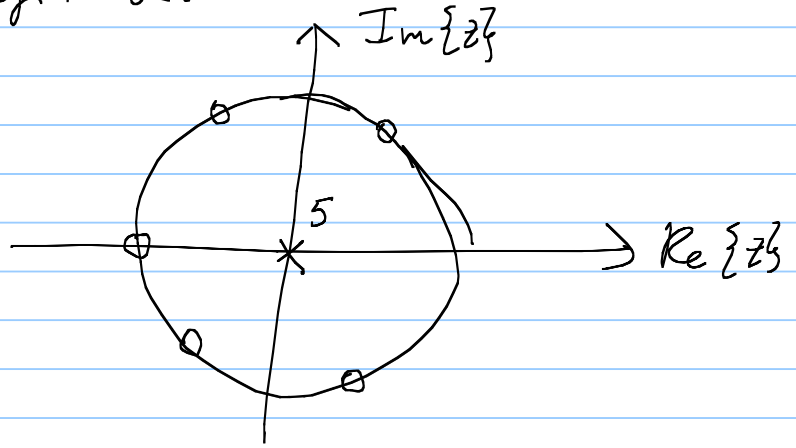
Oppgave 2:

a)
$$y[n] = |H(e^{j\pi})| A_1 \cos(\pi n + \varphi_1 + \measuredangle H(e^{j\pi}))$$
$$+ |H(e^{j2\pi/3})| A_2 \cos\left(\frac{2\pi}{3} n + \varphi_2 + \measuredangle H(e^{j2\pi/3})\right)$$
$$+ |H(e^{j\pi/3})| A_3 \cos\left(\frac{\pi}{3} n + \varphi_3 + \measuredangle H(e^{j\pi/3})\right)$$
$$+ |H(e^{j\pi/90})| A_4 \cos\left(\frac{\pi}{90} n + \varphi_4 + \measuredangle H(e^{j\pi/90})\right)$$

Vi må ha (i det minste):
$$|H(e^{j\pi})| = |H(e^{j2\pi/3})| = |H(e^{j\pi/3})| = 0$$
$$|H(e^{j\pi/90})| \neq 0$$

b) $H(z) = 0$ for $z = e^{j\pi} = -1$
$$= e^{j2\pi/3}$$
$$= e^{j\pi/3}$$

gir oss



NB: Kravet om kausalitet gjør at vi har 5 poler i $z=0$!

c) Vi vet hvor røttene til polynomet $H(z)$ er, så vi kan skrive det på faktorisert form:

$$H(z) = (1-z^{-1}e^{j\pi})(1-z^{-1}e^{j2\pi/3})(1-z^{-1}e^{-j2\pi/3})(1-z^{-1}e^{j\pi/3})(1-z^{-1}e^{-j\pi/3})$$

som kan utvides til

$$H(z) = (1-z^{-1}e^{j\pi})(1-2\underbrace{\cos\left(\tfrac{2\pi}{3}\right)}_{=-1/2}z^{-1}+z^{-2})(1-2\underbrace{\cos(\pi/3)}_{=1/2}z^{-1}+z^{-2})$$

$$= (1+z^{-1})(1+z^{-1}+z^{-2})(1-z^{-1}+z^{-2})$$

$$= (1+z^{-1})(1+z^{-2}+z^{-4})$$

$$= (1+z^{-2}+z^{-4}+z^{-1}+z^{-3}+z^{-5})$$

$$= \underline{\underline{1+z^{-1}+z^{-2}+z^{-3}+z^{-4}+z^{-5}}}$$

Dette er <u>ikke</u> den eneste gyldige systemfunksjonen, fordi $A\,H(z) = 0 \Leftrightarrow H(z) = 0$ for $A \neq 0$ (en konstant).

d) $h[n] = \sum_{k=0}^{5} \delta[n-k]$

Hvis vi velger $A = \frac{1}{6}$ i forrige deloppg., får vi $h[n] = \frac{1}{6}\sum_{k=0}^{5} \delta[n-k]$, dvs. glidende-middel filter m/ 6 koeffisienter.

Oppgave 3:

$$H(e^{j\hat{\omega}}) = \frac{1}{6}\sum_{n=0}^{5} e^{-j\hat{\omega}n} = \frac{1-e^{-j6\hat{\omega}}}{6(1-e^{-j\hat{\omega}})}$$

$$= \underline{\frac{\sin(3\hat{\omega})}{6\sin(\hat{\omega}/2)} e^{-j5\hat{\omega}/2}}$$

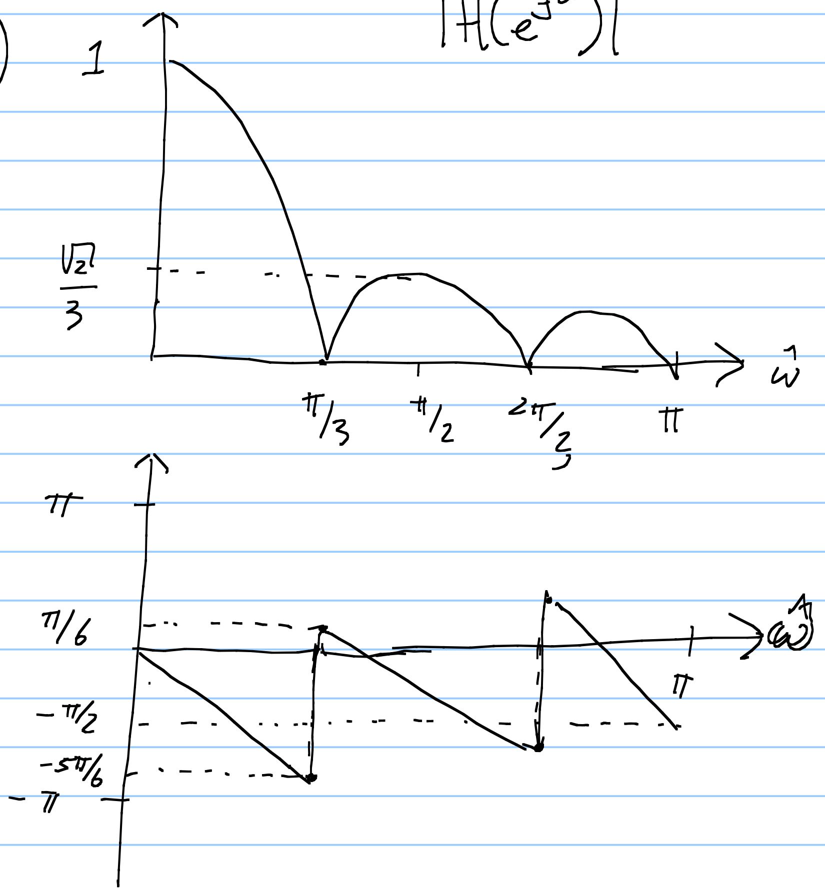
Vi har $\sin(3\hat{\omega}) < 0$ for $\pi/3 < |\hat{\omega}| < \frac{2\pi}{3}$ i intervallet $-\pi < \hat{\omega} < \pi$, så vi får

$$|H(e^{j\hat{\omega}})| = \frac{|\sin(3\hat{\omega})|}{6|\sin(\hat{\omega}/2)|}$$

$$\measuredangle H(e^{j\hat{\omega}}) = \begin{cases} -5\hat{\omega}/2 + \pi & \text{for } \pi/3 < |\hat{\omega}| < \frac{2\pi}{3} \\ -5\hat{\omega}/2 & \text{ellers} \end{cases}$$

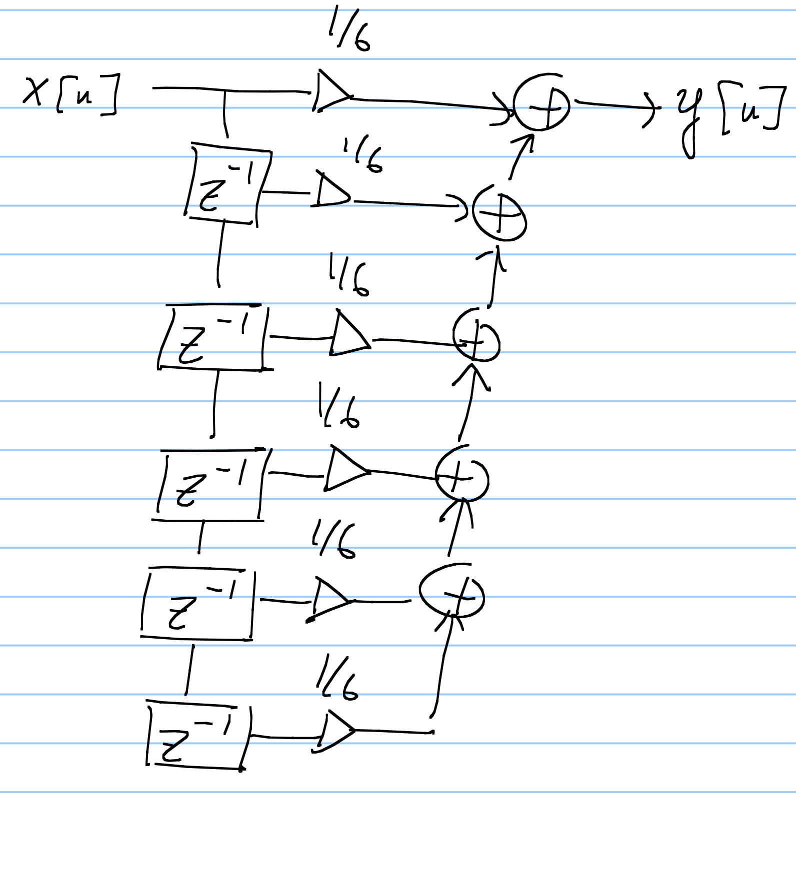
og $H(e^{j\hat{\omega}}) = |H(e^{j\hat{\omega}})| e^{j\measuredangle H(e^{j\hat{\omega}})}$

b)



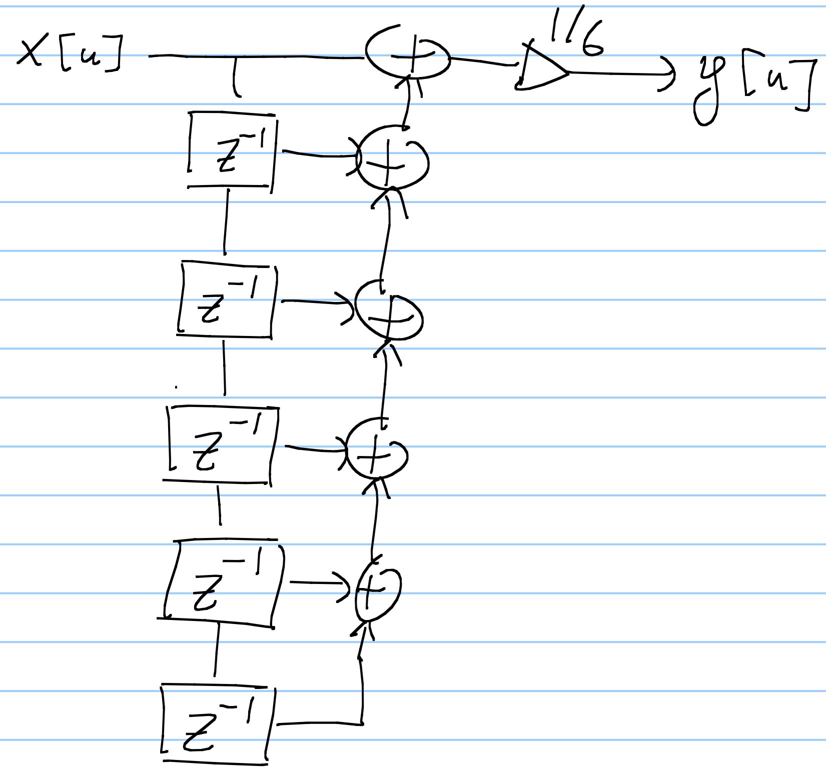
$|H(e^{j\hat{\omega}})|$
1
$\frac{\sqrt{2}}{3}$
$\pi/3$
$\pi/2$
$2\pi/3$
π
$\hat{\omega}$
π
$\pi/6$
$-\pi/2$
$-5\pi/6$
$-\pi$
π
$\hat{\omega}$

c)



$x[n]$
$1/6$
$1/6$
$1/6$
$1/6$
$1/6$
$1/6$
z^{-1}
z^{-1}
z^{-1}
z^{-1}
z^{-1}
$y[n]$

Kan skrives enklere:



Oppgave 4:

$$b_k = \frac{1}{T} \int_{\tau}^{\tau+T} y(t)\, e^{-j 2\pi k t/T}\, dt$$

$$= \frac{1}{T} \int_{\tau}^{\tau+T} x(-t)\, e^{-j 2\pi k t/T}\, dt$$

$$\theta \stackrel{\Delta}{=} -t \Rightarrow t = -\theta \Rightarrow \begin{array}{l} t = \tau \Rightarrow \theta = -\tau \\ t = \tau + T \Rightarrow \theta = -\tau - T \end{array}$$

$$\frac{d\theta}{dt} = -1 \Rightarrow d\theta = -dt$$

vi får

$$b_k = -\frac{1}{T}\int_{-\tau}^{-\tau-T} x(\theta)\, e^{j2\pi k\theta/T}\, d\theta$$

$$= \frac{1}{T}\int_{-\tau-T}^{-\tau} x(\theta)\, e^{j2\pi k\theta/T}\, d\theta$$

$$= \left(\frac{1}{T}\int_{-\tau-T}^{-\tau} x(\theta)\, e^{-j2\pi k\theta/T}\, d\theta \right)^*$$

$$= (a_k)^* = \underline{\underline{a_{-k}}}$$

Så : $\boxed{b_k = a_{-k}}$

<u>Forklaring</u>: Et periodisk signal $x(t)$ kan skrives ved syntese formelen som en sum av sinusoider med gitt amplitude og faseskift. Å reversere signalet betyr å reversere hver av disse sinusoidene. Å reversere sinusoider betyr i praksis å endre fortegnet på fasen. Derfor er det naturlig at Fourier-koeffisientene til $x(-t)$ er like dem som tilhører $x(t)$ med mots. fortegn på fasen!

Oppgave 5:

a) $h[n] \in \mathbb{R} \; \forall n$ gir oss all nødv. info.

i $H(e^{j\hat{\omega}})$ for $0 \leq \hat{\omega} \leq \pi$.

b) Vi har $H(e^{j\hat{\omega}}) = H^*(e^{-j\hat{\omega}})$, så

$$|H(e^{j\hat{\omega}})| \text{ for } -\pi < \hat{\omega} < 0 = |H(e^{j\hat{\omega}})| \text{ for } 0 < \hat{\omega} < \pi$$

og $\measuredangle H(e^{j\hat{\omega}})$ for $-\pi < \hat{\omega} < 0 = -\measuredangle H(e^{j\hat{\omega}})$ for $0 < \hat{\omega} < \pi$

Oppgave 6:

i) $$T\{\alpha x_1[n] + \beta x_2[n]\} = (n+1)(\alpha x_1[n] + \beta x_2[n]) + \alpha x_1[n-1] + \beta x_2[n-1]$$

$$\alpha T\{x_1[n]\} + \beta T\{x_2[n]\} = \alpha((n+1)x_1[n] + x_1[n-1]) + \beta((n+1)x_2[n] + x_2[n-1])$$

$$= (n+1)(\alpha x_1[n] + \beta x_2[n]) + \alpha x_1[n-1] + \beta x_2[n-1]$$

Systemet er lineært.

$$y[n-k] = (n+1-k)\,x[n-k] + x[n-1-k]$$

$$T\{x[n-k]\} = (n+1)\,x[n-k] + x[n-1-k]$$

Systemet er ikke tidsinvariant.

ii) Dette er et FIR-filter, som er LTI. (NB: Vis helst helst på vanlig måte.)

iii) $$T\{\alpha x_1[n] + \beta x_2[n]\} = \frac{\alpha x_1[n] + \beta x_2[n]}{\alpha x_1[n-1] + \beta x_2[n-1]}$$

$$\alpha T\{x_1[n]\} + \beta T\{x_2[n]\} = \alpha \frac{x_1[n]}{x_1[n-1]} + \beta \frac{x_2[n]}{x_2[n-1]}$$

$$= \frac{\alpha x_1[n]\,x_2[n-1] + \beta x_2[n]\,x_1[n-1]}{x_1[n-1]\,x_2[n-1]}$$

Systemet er ikke lineært

$$\left.\begin{aligned} y[n-k] &= \frac{x[n-k]}{x[n-k-1]} \\ T\{x[n-k]\} &= \frac{x[n-k]}{x[n-k-1]} \end{aligned}\right\}$$ Systemet er tidsinvariant