TEXTURE

Plan for today

- Why texture, and what is it?
- Statistical descriptors
  - First order
  - Second order
    - Gray level co-occurrence matrices
  - Higher order
    - Gray level runlength matrices
    - Laws’ texture energy measures
    - Fourier analysis

What is segmentation?

- A process that splits the image into meaningful regions.
- One of the most important elements of a complete image analysis system.
- Segmentation gives us regions and objects that we may later describe and recognize.
- The simplest case: two classes:
  - Foreground
  - Background

What and where is the object in this image?

Segmentation problems

- Usually several objects in an image.
- Objects are seldom alike, even if they are of same class.
- Often several classes.
- Lighting may vary over image.
- Reflection, color etc. may vary.

- We perceive
  - intensity,
  - color,
  - and texture.
What is texture?

• Intuitively obvious, but no precise definition exists
  – “fine, coarse, grained, smooth” etc
• Texture consists of texture primitives, texels,
  – a contiguous set of pixels with some tonal and/or regional property
• Texture can be characterized by
  – tone, intensity properties of texels
  – structure, spatial relationships of texels
• A texel is the characteristic object that the texture consists of (the “brick in the wall”)
• Textures are highly scale dependent.

How do we segment these images?

Texel what?

Uses for texture analysis

• Segment an image into regions with the same texture, i.e. as a complement to graylevel or color
• Recognize or classify objects in images based on their texture
• Find edges in an image, i.e. where the texture changes
• “shape from texture”
• Object detection, compression, synthesis
• Industrial inspection:
  – find defects in materials

Note that you can define texels in any image scale, and that the best image scale for analysis is problem dependent.
Features

- Features can be found from:
  - Homogeneous regions
    - Mean and variance are useful for describing homogeneous regions
  - Edges
    - Gives the borders between the regions
  - The texture of the local window
    - A feature that describes how the gray levels in a window varies, e.g. roughness, regularity, smoothness, contrast etc.

A simple approach to texture

- To be able to find changes in texture in an image, a simple strategy is to perform texture measurements on a sliding window
- Most texture features can be summed up as scalars, so we can assign features to each of the image pixels corresponding to window centers
- For each pixel, we now have a description of the "texture" in its neighborhood
- Beware of image edges, edge artifacts will occur!

Computing texture images

- Select a window size and a texture measure
- For each pixel \((i,j)\) in the image:
  - Center the window at pixel \((i,j)\)
  - Compute the texture measure
    - One value is computed based on the gray-level variations of pixels inside the image
  - Assign the computed value to the center pixel \((i,j)\) in a new image of the same size
- This is similar to filtering
- Pixels close to the image border can be handled in the same manner as for filtering/convolution

Texture feature image example

Input image
For each pixel, compute a local homogeneity measure in a local window.
Create a new image with the homogeneity values.
Segment the feature image.

Create a new image with the homogeneity values.
Try to get an image where pixels belonging to the same texture type get similar values.
"Texture" – description of regions

- Remember: we estimate local properties (features) to isolate regions similar in an image (segmentation), and later identify these regions (classification), usually with the final goal of object description.
- One can describe the "texture" of a region by:
  - smoothness, roughness, regularity, orientation...
- Problem: we want the local properties to be as "local" as possible.
  - Large region or window:
    - Precise estimate of features
    - Imprecise estimate of location
  - Small window:
    - Precise estimate of location
    - Imprecise estimate of feature values

Texture description is scale dependent

- What is our goal for texture description in the image?
- Scale impacts the choice of texels, or vice versa.
- The curtain can be described as a repetition of single threads, a configuration of meshes or as a repetition of folds.

Example

Original image
Variance feature computed in window of size 3x3
Variance feature computed in window of size 15x15

Statistical texture description

- Describe texture in a region by a vector of statistics (feature vector)
  - First order statistics from grayscale intensity histogram p(i)
    - Mean, variance, 3. and 4. order moment
  - Second order statistics, describing relation between pixel pairs
    - How does the gray levels of pixels i and j at a distance d depend on each other. Are they similar or different?
  - Higher order statistics
    - Quantize region in runs of pixels
First order statistics from greylevel histogram

- Mean (hardly a useful feature)
  \[ \mu = \frac{\sum_{i=0}^{G-1} ip(i)}{\sum_{i=0}^{G-1} p(i)} = \frac{\sum_{i=0}^{G-1} ip(i)}{\sum_{i=0}^{G-1} p(i)} \]

- Variance (a more credible feature, measures region "roughness")
  \[ \sigma^2 = \sum_{i=0}^{G-1} (i - \mu)^2 p(i) \]

- Skewness (are the texel intensities usually darker/lighter than average?)
  \[ m_3 = \sum_{i=0}^{G-1} (i - \mu)^3 p(i) \]

- Kurtosis (how "uniform" is the greylevel distribution?)
  \[ m_4 = \sum_{i=0}^{G-1} (i - \mu)^4 p(i) \]

Using variance estimates

- Variance, \( \sigma^2 \), is directly a measure of "roughness"

- A measure of "smoothness" is
  \[ R = 1 - \frac{1}{1+\sigma^2} \]
  - R is close to 0 for homogenous areas
  - R tends to 1 as \( \sigma^2 \), "roughness", increase

First order statistics from greylevel histogram

- Entropy
  (how normal/nonnormal is the greylevel distribution?)
  \[ H = -\sum_{i=0}^{G-1} P(i) \log_2 P(i) \]

- Energy
  (how much intensity variation is there in the region?)
  \[ E = \sum_{i=0}^{G-1} [P(i)]^2 \]

Using variance estimates

- "Coefficient of variation"
  \[ cv = \frac{\sigma_w(x, y)}{\mu_w(x, y)} \]
  - where \( \sigma_w \) and \( \mu_w \) is dependent on the window, \( w \) centered in \((x, y)\)
  - \( cv \) is scale invariant
    \[ i' = Ai \]
  - but not shift invariant
    \[ i' = i + B \]
  - alternatives
    - use median instead of mean
    - interpercentil-distance instead of standard deviation
Skewness

- Higher order moments can also be used for texture description

- Skewness

\[ \gamma_3 = \frac{1}{\sigma^3} \sum_{i=0}^{G-1} (i - \mu)^3 P(i) = \frac{m_3}{\sigma^3} \]

  - Skewness is a measure of the asymmetry of the probability distribution of a variable
  - Measures if there is a "wider" range of either darker or lighter pixels than average
  - Positive skew: The right tail is longer; the mass of the distribution is concentrated on the left of the figure (more darker pixels than average).
  - Negative skew: The left tail is longer; the mass of the distribution is concentrated on the right of the figure.

Skewness example

Region 1
Region 2
Histogram - Region 1
Histogram - Region 2

Skewness feature
Computed in 15x15 window
Region 1: all gray levels occur
Histogram is fairly symmetric
Skewness is gray (average)
Region 2: bright pixels more frequent. Histogram asymmetric. Skewness is dark.

Kurtosis

- Kurtosis

\[ \gamma_4 = \frac{1}{\sigma^4} \sum_{i=0}^{G-1} (i - \mu)^4 P(i) = \frac{m_4}{\sigma^4} - 3 \]

  - Measure of the "peakedness" of the probability distribution of a variable
  - A high kurtosis distribution has a sharper "peak" and flatter "tails"
  - Low kurtosis distribution has a more rounded peak with wider "shoulders"

First order statistics - Entropy

- Entropy (how uniform is the graylevel distribution?)

\[ H = - \sum_{i=0}^{G-1} P(i) \log_2 P(i) \]

  - We see that \( 0 \leq H \leq \log G \)

- Remember entropy from compression:

  - \( H \) is a lower limit on the average code word length in compression

- Here, we use \( H \) as a texture feature, computed in a local window
Entropy example

Region 1

Region 2

Histogram - Region 1

Histogram - Region 2

Entropy feature
Computed in 15x15 Window

Region 1: high entropy
Region 2: low entropy

First order statistics - Energy

- Energy (how much intensity variation is there in the region?)
  \[ E = \sum_{i=0}^{G-1} [P(i)]^2 \]
- A measure of heterogeneity
- If all \( P(i) \) are equal (histogram is uniform), \( E = 1/G \)
- If the image contains only one gray level:
  \[ E = (G-1)0 + 1 \times 1 = 1 \]
- Thus, \( 1/G \leq E \leq 1 \)

1. order statistics discussion

- 1. order statistics can separate two regions even if \( \mu_1 = \mu_2 \), as long as \( \sigma_1^2 \neq \sigma_2^2 \)
- The statistics of a pixel \((x, y)\) is found in a local window
- Problems:
  - Edges around objects are exaggerated
    - Solution: use adaptive windows
  - 1. order statistics does not describe geometry or context
    - Cannot discriminate between
    - Solution:
      - Calculate 1. order statistics with different resolutions, and obtain indirect information about 2. and higher order statistics.
      - Simply use 2. or higher order statistics.

1. order description example

- Brick wall texture
  - Mean 79.13
  - Variance 42.73
  - CV 0.08
  - Skew 1.37
  - Kurtosis 5.93
- Granite texture
  - Mean 157.08
  - Variance 96.9573
  - CV 0.06
  - Skew -0.73
  - Kurtosis 3.25
Second order statistics

• Spatial frequencies / edge frequency
  – Transform to Fourier domain, integrate over rings or wedges
  – Gabor filters combine estimate of orientation and frequency

• Gray-level Co-Occurrence matrices
  – Intensity change “histograms” as a function of distance and direction
  – By far the most popular texture description method due to its simplicity
  – The co-occurrence matrix is an estimate of the second order joint probability, which is the probability of going from gray level \(i\) to gray level \(j\), given the distance \(d\) between two pixels along a given direction \(\theta\).

Gray Level Cooccurrence Matrices (GLCM)

• Matrix element \(P(m,n)\) in a GLCM is 2. order probability of changing from graylevel \(m\) to \(n\) when moving distance \(d\) in the direction \(\theta\) of the image, or equivalent, \((\Delta x, \Delta y)\)

• From a \(M \times N\) image with \(G\) graylevels, and \(f(m,n)\) is the intensity. Then
  \[ P(i,j | \Delta x, \Delta y) = \frac{1}{W} Q(i,j | \Delta x, \Delta y) = \sum_{m=1}^{N-\Delta y} \sum_{n=1}^{M-\Delta x} A \]
  and
  \[ A = \begin{cases} 1 & \text{if } f(m,n) = i \text{ or } f(m + \Delta x, n + \Delta y) = j \\ 0 & \text{else} \end{cases} \]

• Alternative notation, dependent on direction and distance, \(P(i,j | d, \theta)\)

GLCM

• From one window of size \(w \times w\) we get one GLCM matrix
• The dimension of the co-occurrence matrix is \(G \times G\)
  if we have \(G\) gray-levels in the image.
• Choose a distance \(d\) and a direction \(\theta\)

In this example, \(d=1\) and \(\theta=0\)

• Check all pixel pairs with distance \(d\) and direction \(\theta\) inside the window. \(Q(i,j | d, \theta)\) is the number of pixel pairs where pixel 1 in the pair has pixel value \(i\) and pixel 2 has pixel value \(j\).
GLCM – practical issues

- The matrix must have an "average occupancy level" that is large enough:
  - Reduce number of gray levels (Less precise description if the texture has low contrast)
  - Select $L$ =number of gray levels
  - Rescale the image if necessary to use these levels using (histogram transform)
  - Requantize the scaled image from $G$ to $L$ gray levels before GLCM computation
  - Increase window size (Errors due to changes in texture)

- Heuristics:
  - 16 graylevels is usually sufficient
  - window should be 30 x 30 – 50 x 50 pixels.

Preprocessing examples

GLCM matrices for subregions

- Usually a good idea to reduce the number of $(d, \theta)$ variations evaluated
- Simple pairwise relations:
  - $P(d,0) = P\cdot(d,180)$
  - $P(d,45) = P\cdot(d,225)$
  - $P(d,90) = P\cdot(d,270)$
  - $P(d,135) = P\cdot(d,315)$

- Isotropic matrix by averaging $P(\theta), \theta \in \{0^\circ, 45^\circ, 90^\circ, 135^\circ\}$
  - Beware of differences in effective window size!
- An isotropic texture is equal in all directions
- If the texture has a clear orientation, we select $\theta$ according to this.
Isotropic GLCM example

How to use the GLCM

- Can be used directly to measure statistical distance between textures.
- Usually, used by extracting secondary features
  - Haralick et al. and Conners et al.
  - Features are usually strongly correlated, using more than 4-5 simultaneously is not advisable
  - Need to evaluate several distances $d$
    - Would you perform anti-aliasing filtering for $d>1$?
    - Optimal set of features is problem dependent
- It is also advisable to preprocess by histogram transform to remove effect of absolute gray level.
- Usually, we want to make the features "rotation" invariant by using the isotropic GLCM (remember different weights).

GLCM Features

- Angular Second Moment,
  \[
  ASM = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (P(i,j))^2
  \]
  - ASM is a measure of homogeneity of an image.
  - A homogeneous scene will contain only a few gray levels, giving GLCM with a few but relatively high values of $P(i,j)$.
  - Thus, the sum of squares will be high.
- Entropy,
  \[
  E = -\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} P(i,j) \times \log(P(i,j))
  \]
  - Inhomogeneous scenes have low first order entropy, while a homogeneous scene has a high entropy.
  - Maximum Entropy is reached when all probabilities are equal

GLCM Features

- Correlation,
  - Correlation is a measure of gray level linear dependence between the pixels at the specified positions relative to each other.
  \[
  COR = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} \frac{(i - \mu_i)(j - \mu_j)P(i,j)}{\sigma_i \sigma_j}
  \]
  \[
  \mu_i = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} iP(i,j)
  \]
  \[
  \mu_j = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} jP(i,j)
  \]
  \[
  \sigma_i = \sqrt{\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (j - \mu_j)^2 P(i,j)}
  \]
  \[
  \sigma_j = \sqrt{\sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i - \mu_i)^2 P(i,j)}
  \]
GLCM Features

• Contrast,
  - This measure of contrast or local intensity variation will favor contributions from \( P(i,j) \) away from the diagonal, i.e. \( i \neq j \)
  
  \[
  CTR = \sum_{n=0}^{G-1} n^2 \left( \sum_{i=1}^{G} \sum_{j=1}^{G} P(i,j) \right) \quad |i-j| = n
  \]

• Inverse Difference Moment (also called homogeneity)
  - IDM is also influenced by the homogeneity of the image.
  - Because of the weighting factor \((1+(i-j)^2)^{-1}\), IDM will get small contributions from inhomogeneous areas (\( i \neq j \)).
  - The result is a low IDM value for inhomogeneous images, and a relatively higher value for homogeneous images.
  
  \[
  IDM = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} \frac{1}{1+(i-j)^2} P(i,j)
  \]

GLCM Features

• Sum of Squares, Variance,
  \[
  VAR = \sum_{i=0}^{G-1} \sum_{j=0}^{G-1} (i - \mu)^2 P(i,j)
  \]
  - This feature puts relatively high weights on the elements that differ from the average value of \( P(i,j) \)
  - GLCM variance uses the GLCM, therefore it deals specifically with the dispersion around the mean of combinations of reference and neighbor pixels, i.e., encoding contextual (2. order) information
  - Variance calculated using \( i \) or \( j \) gives the same result, since the GLCM is symmetrical
  - Note that the contextuality is an integral part of this measure; one can measure the variance in pixels in one direction. Thus it is not the same as the 1. order statistic variance.

GLCM feature image examples, \( w = 15 \)

GLCM contrast is
  - negative correlated with IDM
  - positively correlated with variance

GLCM entropy is negatively correlated with ASM
GLCM feature image examples, w= 15

GLCM IDM  GLCM ASM  GLCM correlation

Sum and difference histograms

- The sum histogram $S$ is simply the histogram of the sums of all pixels $dx$ and $dy$ apart
- For example, the gray level at $I(x,y)$ is added to the gray level at $I(x+dx,y+dy)$ and the histogram bin corresponding to that sum is incremented
- The difference histogram $D$ is simply the histogram of the difference of all pixels $dx$ and $dy$ apart

\[ s_{\Delta x,\Delta y}(m, n) = f(m, n) + f(m + \Delta x, n + \Delta y) \]
\[ d_{\Delta x,\Delta y}(m, n) = f(m, n) - f(m + \Delta x, n + \Delta y) \]
- The number of possible values of sum histogram is $2G-1$, whereas for difference histogram it is $G-1$.

Sum and difference histograms

- GLCM features can be derived from $P_s$ and $P_d$
- Example:
  - Contrast from $P_d$
    \[ CON = \sum_{j=0}^{2G-2} j^2 P_d(j | \Delta x, \Delta y) \]
  - Contrast from GLCM
    \[ CON = \sum_{n=0}^{G-1} n^2 \{ \sum_{i=1}^{G} \sum_{j=1}^{G} P(i,j) \}, \quad |i-j| = n \]
- Some of the features mentioned earlier is derived from the histograms
  - Sum Average,
  - Sum Entropy,
  - Difference Entropy,
  - Inverse Difference moment,

Fourier analysis

- The Fourier spectra give direction and frequency for periodic or near periodic 2D patterns
- Local FFT in windows
- Texture with a dominating direction will have peaks in the spectra along a line orthogonal to the texture orientation
- High frequency = fine texture = peaks in the spectra far from the origin
- Thus it is possible to separate fine and coarse spectra
- The spread in image frequencies = width of the peak in Fourier
- Isotropic textures with a defined frequency can be seen as rings in the spectra
- Scalar features can be extracted by integration over rings, wedges or from results of Gabor filtering
Fourier analysis example

Higher order statistics

• Higher order methods include
  – Gray level runlength matrices - “histograms” of graylevel run lengths in different directions (INF 5300)
  – Laws’ texture masks - masks resulting from combinations of 0,1,2 derivatives

Laws’ texture energy measures

• Based on convolution (INF 2310)

• Uses 3×3 or 5×5 separable masks that are symmetric or anti-symmetric

• This results in a new texture image for every convolution mask.

• From the results of convolution find the standard deviation or average of absolute values over a larger window (e.g. 15×15)

• This is a measure of texture energy in some direction depending on the mask chosen.

3×3 Law’s texture estimator

• A set of masks is convolved with the image:

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<th>1 0 1</th>
<th>1 2 -1</th>
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<td>1 2 -1</td>
</tr>
</tbody>
</table>

• The texture measure is found by computing the standard deviation over a larger window for every image convolved with the Law’s masks
5×5 Law’s texture masks

- E5L5
  \[
  \begin{array}{cccccc}
  1 & 0 & 2 & 0 & 1 \\
  -2 & 0 & 1 & 0 & -2 \\
  0 & 0 & 0 & 0 & 0 \\
  2 & 8 & 12 & 8 & 2 \\
  1 & 4 & 6 & 4 & 1 \\
  \end{array}
  \]

- E5S
  \[
  \begin{array}{cccccc}
  1 & 0 & 2 & 0 & 1 \\
  -2 & 0 & 1 & 0 & -2 \\
  0 & 0 & 0 & 0 & 0 \\
  2 & 8 & 12 & 8 & 2 \\
  1 & 4 & 6 & 4 & 1 \\
  \end{array}
  \]

- R5R5
  \[
  \begin{array}{cccccc}
  -1 & -1 & 6 & -1 & 1 \\
  1 & 6 & 24 & 16 & -1 \\
  6 & -24 & 36 & -24 & 6 \\
  6 & 16 & -24 & 16 & -1 \\
  1 & -1 & 6 & -4 & 1 \\
  \end{array}
  \]

- L5S
  \[
  \begin{array}{cccccc}
  -1 & 0 & 2 & 0 & 1 \\
  -4 & 0 & 8 & 0 & -4 \\
  -6 & 0 & 12 & 0 & -6 \\
  -4 & 0 & 8 & 0 & -4 \\
  -1 & 0 & 2 & 0 & 1 \\
  \end{array}
  \]

- E5R5
  \[
  \begin{array}{cccccc}
  -1 & -1 & 6 & -1 & 1 \\
  1 & 6 & 24 & 16 & -1 \\
  6 & -24 & 36 & -24 & 6 \\
  6 & 16 & -24 & 16 & -1 \\
  1 & -1 & 6 & -4 & 1 \\
  \end{array}
  \]

- E3
  \[
  \begin{array}{cccc}
  1 & 2 & 1 \\
  -1 & 0 & 1 \\
  -1 & 2 & -1 \\
  \end{array}
  \]

- S3
  \[
  \begin{array}{cccc}
  -1 & 0 & 2 & 0 \\
  -1 & 2 & -1 \\
  \end{array}
  \]

"Finding rippling texture" - R5R5 example

But how did we create these?

- The masks are separable, zero-sum masks
- The 5×5 masks are generated by convolving a vertical 5-vector with a horizontal 5-vector:
  - \[ L5 = [1 4 6 4 1] \]
  - \[ E5 = [-1 2 0 2 1] \]
  - \[ S5 = [-1 0 2 0 -1] \]
  - \[ R5 = [1 -4 6 -4 1] \]
- Each 5-vector is found by convolving two 3-vectors:
  - \[ L3 = [1 2 1] \]
  - \[ E3 = [-1 0 1] \]
  - \[ S3 = [-1 2 -1] \]

Multiscale texture analysis

- We have ascertained that texture description is scale dependent
- By representing the image in a pyramid structure we can describe image texture on different scales
- A discrete wavelet-transform will give four new images with reduced resolution, called \[ s_{hr}, s_{hr}, s_{lr} \] and \[ s_{ll} \] where \[ a, b, c, d \] are four neighbors in the original image:
  \[
  s_{hr} [n; m] = \left( \frac{a + b + c + d}{2} \right), \quad s_{lr} [n; m] = \left( \frac{a - b + c - d}{2} \right), \quad s_{hb} [n; m] = \left( \frac{a + b - c - d}{2} \right), \quad s_{lb} [n; m] = \left( \frac{a - b - c + d}{2} \right)
  \]
  \[
  a = f [2n; 2m], \quad b = f [2n + 1; 2m], \quad c = f [2n; 2m + 1], \quad d = f [2n + 1; 2m + 1]
  \]
Multiscale texture analysis

- By repeating this decomposition on $s_p$, the result is a hierarchical pyramid structure for different resolutions, both in the lowpass image and the other subbands.
- The image can be represented in multiple resolutions and multiscale GLCM (for example) can be calculated.
- Furthermore, each of the three other subbands can be viewed as edge information for that scale.

Learning goals - texture

- Understand what texture is, and the difference between first order and second order measures.
- Understand the GLCM matrix, and be able to describe algorithm.
- Understand how we go from an image to a GLCM feature image – Preprocessing, choosing $d$ and $\theta$, selecting some features that are not too correlated.
- Understand Law's texture measures and how they are built based on basic filtering operations.
- There is no optimal texture features, it depends on the problem.
- A good tutorial on texture: http://www.fp.ucalgary.ca/mhallbey/tutorial.htm