# Sampling switches, charge injection, Nyquist data converter fundamentals 

Tuesday, February 8th, 9:15-11:35

Snorre Aunet (sa@ifi.uio.no)<br>Nanoelectronics group<br>Department of Informatics<br>University of Oslo

## Last week - Tuesday, February 1st © $\mathfrak{f}$

### 11.1 General considerations

11.2 Supply-independent biasing
11.3 Temperature-independent References ${ }^{\text {® }}$ 11.3.1 Negative TC-voltage 11.3.2 Positive TC-voltage 11.3.3 Bandgap reference
11.4 PTAT Current generation
11.5 Constant-Gm Biasing
12.2 Sampling Switches
12.3 Switched Capacitor amplifiers


Figure 4 Output reference Verf vs. temperature.


Figure 5 Output reference Vref vs. power supply Vdd A High Precision Curvature Compensated Bandgap Reference without Resistors

## Today, February the 8th



- 12.2 Sampling switches
- 12.2.1 MOSFETS as Switches
- 12.2.2 Speed considerations

- 12.2.3 Precision considerations
- 12.2.4 Charge injection cancellation
- 12.3 Switched-Capacitor Amplifiers
- 12.3.1 Unity-Gain Sampler / buffer
- 12.3.2 Noninverting amplifier
- 12.3.3 Precision Multiply-by-Two Circuit
- 12.4 Switched-Capacitor Integrator
- 12.5 SC common-mode feedback
- Data converter fundamentals ("Maloberti"++)



## Track (/sample-) and Hold capabilities of a sampling circuif <br> 12.2 in "Razavi")


(b)

Figure 12.10 Track and hold capabilities of a sampling circuit.


Figure 2.1 Simple view of a MOS device.

## $\mathrm{S} / \mathrm{H}$ signals (clk, $\mathrm{V}_{\text {in }}, \mathrm{V}_{\mathrm{CH}}, \mathrm{V}_{\text {out }}$ )


(b)

(b) sampling circuit followed by unity-gain buffer.


## Track (/sample-) and Hold capabilities of a sampling circuit

 1/2 (ch. 12.2 in "Razavi")
(a)


Figure 2.15 Saturation of drain current.


- CK goes high at $t=t_{0} . V_{\text {in }}, 0$ for $t \geq 0 . C_{H}$ initially has a voltage equal to $\mathrm{V}_{\mathrm{dd}}$.
- $A t t=t_{0} M_{1}$ operates in saturation, but falls into the triode region after some time, when $V_{\text {out }}=V_{d d}-V_{T H}$.
Discharging continues until $V_{\text {out }}$ approaches zero.
- Current when in saturation:

$$
I_{D 1}=\left(\mu_{n} C_{o x} / 2\right)(W / L)\left(V_{D D}-V_{T H}\right)^{2}
$$

If $V_{D S} \leq V_{G S}-$

Track (/sample-) and Hold capabilities of a sampling circuit II/II (ch. 12.2 in "Razavi")


- CK goes high at $\mathrm{t}=\mathrm{t}_{0}$, when $\mathrm{V}_{\text {out }}=$ 0 V , and $\mathrm{V}_{\mathrm{dd}}=3 \mathrm{~V}$.
- $M_{1}$ 's source is connected to $\mathrm{C}_{\mathrm{H}}$, and the transistor turns on with $V_{G S}=3 \mathrm{~V}$, but $\mathrm{V}_{\mathrm{DS}}=1 \mathrm{~V}$. Thus, M1 operates in the triode ("linear") region, charging $\mathrm{C}_{\mathrm{H}}$ until $\mathrm{V}_{\text {out }}$ appoaches 1 V .


## A couple of observations regarding the MOS switch

 (ch. 12.2 in "Razavi")- We have seen that a MOS switch

(b)

Figure 12.10 Track and hold capabilities of a sampling circuit.
can conduct current in either direction simply by exchanging the role of the source and drain terminals.

- When the switch is on, $\mathrm{V}_{\text {out }}$ follows $V_{\text {in }}$.
- When the switch is off, $\mathrm{V}_{\text {out }}$ remains constant (Fig 12.10 b)).


## (Ex. 12.2 in "Razavi") Vout as as a function of time, for Fig. 12. 9 " ${ }^{\circ}{ }^{\circ}$ <br> ( $\lambda=0$ ) <br> After


(a)

## Solution

Before $V_{\text {out }}$ drops below $V_{D D}-V_{T H}, M_{1}$ is saturated and we have:

$$
\begin{aligned}
V_{o u t}(t) & =V_{D D}-\frac{I_{D 1} t}{C_{H}} \\
& =V_{D D}-\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left(V_{D D}-V_{T H}\right)^{2} \frac{t}{C_{H}}
\end{aligned}
$$

$$
\begin{equation*}
t_{1}=\frac{2 V_{T H} C_{H}}{\mu_{n} C_{o x} \frac{W}{L}\left(V_{D D}-V_{T H}\right)^{2}} \tag{12.9}
\end{equation*}
$$

$M_{1}$ enters the triode region, yielding a time-dependent current. We therefore write:

$$
\begin{aligned}
C_{H} \frac{d V_{\text {out }}}{d t} & =-I_{D 1} \\
& =-\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left[2\left(V_{D D}-V_{T H}\right) V_{\text {out }}-V_{\text {out }}^{2} \text { big }\right] \quad t>t_{1}
\end{aligned}
$$

Rearranging (12.11), we have

$$
\begin{equation*}
\frac{d V_{o u t}}{\left[2\left(V_{D D}-V_{T H}\right)-V_{o u t}\right] V_{o u t}}=-\frac{1}{2} \mu_{n} \frac{C_{o x}}{C_{H}} \frac{W}{L} d t \tag{12.12}
\end{equation*}
$$

which, upon separation into partial fractions, is written as

$$
\begin{equation*}
\left[\frac{1}{V_{o u t}}+\frac{1}{2\left(V_{D D}-V_{T H}\right)-V_{\text {out }}}\right] \frac{d V_{\text {out }}}{V_{D D}-V_{T H}}=-\mu_{n} \frac{C_{o x}}{C_{H}} \frac{W}{L} d t . \tag{12.13}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
\ln V_{\text {out }}-\ln \left[2\left(V_{D D}-V_{T H}\right)-V_{\text {out }}\right]=-\left(V_{D D}-V_{T H}\right) \mu_{n} \frac{C_{O x}}{C_{H}} \frac{W}{L}\left(t-t_{1}\right) \tag{12.14}
\end{equation*}
$$

that is,

$$
\begin{equation*}
\ln \frac{V_{\text {out }}}{2\left(V_{D D}-V_{T H}\right)-V_{o u t}}=-\left(V_{D D}-V_{T H}\right) \mu_{n} \frac{C_{o x}}{C_{H}} \frac{W}{L}\left(t-t_{1}\right) \tag{12.15}
\end{equation*}
$$

Taking the exponential of both sides and solving for $V_{\text {out }}$, we obtain

$$
V_{\text {out }}=\frac{2\left(V_{D D}-V_{T H}\right) \exp \left[-\left(V_{D D}-V_{T H}\right) \mu_{n} \frac{C_{o x}}{C_{H}} \cdot \frac{W}{L}\left(t-t_{1}\right)\right]}{1+\exp \left[-\left(V_{D D}-V_{T H}\right) \mu_{n} \frac{C_{o x}}{C_{H}} \cdot \frac{W}{L}\left(t-t_{1}\right)\right]}
$$

## Maximum output of NMOS S/H (ch. 12.2 in "Razavi", pp. 412)



Figure 12.11 Maximum output level in an NMOS sampler.


- Assume $\mathrm{V}_{\text {in }}=\mathrm{V}_{\text {dd }}$ (for the circuitefor Fig. 12.9 b))
, $\mathrm{Vgs}=\mathrm{Vdd}$, initially. $\mathrm{V}_{\mathrm{DS}}=\mathrm{V}_{\mathrm{DD}} \rightarrow$ $V_{D S} \geq V_{G S}-V_{T H}$; saturation
, T goes, $\mathrm{V}_{\text {out }} \rightarrow \mathrm{V}_{\mathrm{DD}}-\mathrm{V}_{\mathrm{TH}}$. (since the "overdrive" voltage vanishes and the current available for charging $\mathrm{C}_{\mathrm{H}}$ go to negligible values)
- But: Given enough time, $\mathrm{V}_{\text {out }}$ will approach $\mathrm{V}_{\mathrm{DD}}$, due to subthreshold currents conducted by the transistor.
- Serious limitation: If the input signal is close to $\mathrm{V}_{\mathrm{DD}}$, the output provided by an NMOS switch cannot track the input (fast enough - remember subthreshold conduction).
- Similar problem with PMOS.


## Ex. 12.3 $\mathrm{R}_{\text {on }}$ variation in sampling switch (ch. 12.2 in "Razavi")

- Time for the output to settle within a given accuracy (ex. 0.1 \%) depends on the input voltage ( $\mathrm{T}=\mathrm{R}_{\mathrm{ON}} \mathrm{C}$ ).

Example 12.3
In the circuit of Fig. 12.12, calculate the minimum and maximum on-resistance of $M_{1}$. Assume
$\mu_{n} C_{o x}=50 \mu \mathrm{~A} / \mathrm{V}^{2}, W / L=10 / 1, V_{T H}=0.7 \mathrm{~V}, V_{D D}=3 \mathrm{~V}$, and $\gamma=0$.


Solution
We note that in the steady state, $M_{1}$ remains in the triode region because the gate voltage is higher than both $V_{\text {in }}$ and $V_{\text {out }}$ by a value greater than $V_{T H}$. If $f_{\text {in }}=10 \mathrm{MHz}$, we predict that $V_{\text {out }}$ tracks $V_{i n}$ with a negligible phase shift due to the on-resistance of $M_{1}$ and $C_{H}$. Assuming $V_{\text {out }} \approx V_{i n}$, we need not distinguish between the source and drain terminals, obtaining

$$
R_{o n 1}=\frac{1}{\mu_{n} C_{o x} \frac{W}{L}\left(V_{D D}-V_{i n}-V_{T H}\right)}
$$

(12.22)

Thus, $R_{\text {on } 1, \text { max }} \approx 1.11 \mathrm{k} \Omega$ and $R_{\text {on } 1, \text { min }} \approx 870 \Omega$. By contrast, if the maximum input level is raised to 1.5 V , then $R_{\text {on 1, max }}=2.5 \mathrm{k} \Omega$.

## Ex. 12.3 Sampling speed considerations



Figure 12.13 Sampling circuit using PMOS switch.



Figure 12.14 Definition of speed in a sampling circuit.

- Speed: Time from zero to maximum input level after the switch turns on, or more relevant: time to settle within a certain "error band", $\Delta \mathrm{V}$.
- Sampling speed is given by the on-resistance of the switch and the value of the sampling capacitor.
- $R_{\text {on }}$ depends on input level, giving a greater time constant for more positive inputs (in the case of NMOS switches)


# D/A (DAC) settling time and sampling rate 

- In a DAC the settling time is defined as the time it takes for the converter to settle within some specified amount of the final value (usually 0.5 LSB ).
- The sampling rate is the rate a which samples can be continously converted and is typically the inverse of the settling time.
- Different combinations of input vectors give different settling
 dissertation for the dr. scient. degree by Leif Hanssen, Ifi, UiO, 1990.


Fig 3.9 Risetime 6 bit DAC, input 0 to 128. (Xdivision $=50 \mathrm{~ns}$ )


## Ex. 12.3 Complimentary MOS switch


(a)

(b)

Figure 12.15 On-resistance of (a) NMOS and (b) PMOS devices as a function of input voltage.


Figure 12.16 (a) Complementary switch, (b) on-resistance of the complementary switch.

- NMOS on resistance increases as the input voltage becomes more positive (and vice versa)
- PMOS on-resistance has the opposite behaviour and decreases as the input voltage becomes more positive.
- Combine PMOS and NMOS for complementary switches and rail-to-rail swings (, when needed).


## Complementary switches need complementary clocks


(a)

(b)

Figure 12.16 (a) Complementary switch, (b) on-resistance of the complementary switch.


Figure 12.17 Distortion generated if complementary switches do not turn off simultaneously.


- The complementary switch revesals much less variation in on-resistance than that corresponding to each switch alone.
- For high-speed input signals the PMOS and NMOS switches must turn off simultaneously. (If for example the NMOS turns off $\Delta t$ seconds earlier than the PMOS, the output voltage tends to track the input for the remaining $\Delta t$ seconds, giving rise to distortion in the sampled value.)
- Fig. 12.18 shows a complementary clock generator for moderate precision.


# A BiCMOS Sample-and-Hold for satellite communications 

Snorre Aunet*, Leif Hanssen**

*Department of Informatics, University of Oslo, Norway
**Norwegian Telecom Research, Kjeller, Norway

## Summary

70 MHz is a very commonly used intermediate frequency (IF) in satellite communication systems. Direct conversion of this high frequency signal requires a very expensive ADC capable of operation at a clock rate twice this frequency. However, a narrow band 68-72 MHz can be folded down to $4-8 \mathrm{MHz}$ with a sample-and-hold clocked at 16 MHz . Such undersampling allows conversion using a simple ADC with an accuracy of $6-8$ bit. This ADC can be realized in CMOS with low area- and power consumption.

This configuration puts heavy requirements on the sample-and-hold. Two factors needing special attention are analog input bandwidth and aperture uncertainty (jitter). Other important specifications include total harmonic distortion (THD), signal-to-noise-plus-distortion ratio $[\mathrm{S} /(\mathrm{N}+\mathrm{D})]$ and intermodulation distortion (IMD).

A traditional CMOS sample-and-hold contains an input MOSFET switch, a hold capacitor and an unity-gain output buffer. The high analog input frequency makes this an inadequate solution. The ON-resistance in the switch varies with the input level, resulting in variation in magnitude and phase, hence distortion. Limited slew rate on the switch gate voltage combined with a high $\mathrm{dV} / \mathrm{dt}$ for the input signal also introduce aperture uncertainty (jitter).
BiCMOS offers the possibility of designing integrated circuits with speed-power-density performance previously unattainable with either technology individually. A BiCMOS sample-and-hold circuit has been made with a special attempt to overcome problems concerning the high analog input frequency. The input switch is implemented as a diode bridge using bipolar transistors. In sample mode this switch works as an input buffer.In hold mode the capacitor is buffered and held by a feedback loop containing two of the transistors in the diode bridge. The output buffer and the bias circuits have mainly been implemented with MOSFETs. A 1 bit current-steering DAC switches between the sample and hold modes. Turning current sinks on and off is then avoided, resulting in increased switching speed. With this configuration an input bandwidth of 100 MHz with a 2.5 pF hold capacitor has been obtained. Input voltage swing is limited to about 1.2 V p-p. The circuit has been integrated in a 2 micron BiCMOS technology offered by Euroch-p. The active area is $0.16 \mathrm{~mm}^{2}$ and the circuit has a single +5 V power supply. Power consump-
tion has been calculated to less than 20 mW


- $2 \mu \mathrm{~m}$ BiCMOS, 1993
- "...A traditional CMOS Sample-and-Hold contains an input MOSFET switch, a hold capacitor and an unity-gain buffer. The high analog input frequency makes this an inadequate solution. The ONresistance of the switch varies with the input level, resulting in variation in magnitude and phase, hence distortion. Limited slew rate on the switch gate voltage combined with a high $d V / d t$ for the input signal also introduce aperture uncertainty (jitter)..."

8. februar 2011

## Charge injection due to channel capacitance

- When the clock signal goes low, the charge is distributed equally between the drain and source of $\mathrm{M}_{1}$.

$$
\begin{align*}
Q_{c h} & =W L C_{o x}\left(V_{D D}-V_{i n}-V_{T H}\right),  \tag{12.28}\\
\Delta V & =\frac{W L C_{o x}\left(V_{D D}-V_{i n}-V_{T H}\right)}{2 C_{H}} .
\end{align*}
$$

- is linearly related to $\mathrm{V}_{\text {in }}$, resulting in a gain error for

$$
\begin{equation*}
V_{o u t} \approx V_{i n}-\frac{W L C_{o x}\left(V_{D D}-V_{i n}-V_{T H}\right)}{C_{H}}, \tag{12.30}
\end{equation*}
$$ the $\mathrm{S} / \mathrm{H}$.

There is also a linear relationship to $\mathrm{V}_{\mathrm{TH}}$, which is nonlinearly related to Vin ( through Vsb ) resulting in distortion for the overall S/H.

- Equal distribution (S/D) imprecise; worst-case: all charge to one node.


Figure 12.20 Effect of charge injection.

Charge injection leads to gain error, dc offsets and nonlinearity


Figure 12.20 Effect of charge injection.


Figure 12.21 Input/output characteristic of sampling circuit in the presence of charge injection.

- In reality the fraction of charge that exits through the source and drain terminals is a relatively complex function of various parameters such as impedance seen at each terminal to ground and the transition time of the clock. No good rule of thumb.
- Most circuit simulators model charge injection quite innacurately.
- The assumed linear function of the input voltage, leading to gain error and dc offset (only) is imprecise, due to nonlinear behaviour of $\mathrm{V}_{\mathrm{TH}}$ upon $\mathrm{V}_{\text {in }}$.


## Clock feedthrough



Figure 12.22 Clock feedthrough in a sampling circuit.


$$
\Delta V=V_{C K} \frac{W C_{o v}}{W C_{o v}+C_{H}},
$$

- Clock transitions are couplea through gate-drain and gatesource overlap capacitances.
- The error, $\Delta \mathrm{V}$, is independent of the input level, manifesting itself as a constant offset in the input/output characteristic.


## kT/C noise



Figure 12.23 Thermal noise in a sampling circuit.
value of the input voltage. It can be proved that the rms voltage of the sampled noise in this case is still approximately equal to $\sqrt{k T / C}[3,4]$.

- A resistor charging a capacitor gives rise to a total rms noise voltage of SQRT ( $\mathrm{kT} / \mathrm{C}$ ).
- The noise gets stored on the capacitor along with the instantaneous value of the input voltage.
- In order to achieve low noise the sampling capacitor must be sufficiently large, thus loading other circuits and degrading the speed.


## Charge injection cancellation



Figure 12.24 Addition of dummy device to reduce charge injection and clock feedthrough.


Figure 12.26 Use of complementary switches to reduce charge injection.

Fig. 12.24: The deposited chanell charge is absorbed by the latter.
Dimension W2 $=0.5 \mathrm{~W} 1$ and $\mathrm{L} 1=$ L2. But: The approach is not very attractive as the underlying assumption of equal splitting between $S$ and $D$ is generally invalid.

- Fig. 12.26: Attempts to match dimensions leads to cancellation for only one input level.
- Better: differential operation (Fig. 12.27) Charge inj. Is common mode disturbance. Nonlinearity of body effect leads to odd order distortion.
- Charge injection limits the speed precision envelope in sampled-data systems



## Some systems exploiting data converters, "Allen \& Hentorg


(a)


Figure 10.1-1 (a) A/D converters and (b) D/A converters in signal processing systems.

## Different ADCs depending on needs



Figure 1. ADC architectures, applications, resolution, and sampling rates.


Fig. 2. ADC sample rate vs. ENOB from 1987 to 2005.

## Which ADC Architecture Is Right for Your Application?

By Walt Kester [walt.kester@analog.com]

IEEE 2005 CUSTOM INTEGRATED CIRCUITS CONFERENCE
Scaling of Analog-to-Digital Converters into Ultra-Deep-Submicron CMOS
Y. Chiu ${ }^{1}$, B. Nikolici ${ }^{2}$, and P. R. Gray ${ }^{2}$

Electrical and Computer Engineering, University of Illinois at Urbana-Champaign Electrical Engineering and Computer Sciences, University of California at Berkeley

## Main data converter types: iff

- Nyquist-rate converters:
- Each value has a one-to-one correspondencewith a single input
- The sample-rate must be at least equal to twice the signal frequency (Typically somewhat higher)
- Oversampled converters:
- The sample-rate is much higher than the signal frequency, typically $20-512$ times.
- The extra samples are used to increase the SNR
- Often combined with noise shaping


## Nyquist Sampling, Oversampling, Noise Shaping <br> - Figurenfrome

 [Kest05]

- Straight oversampling gives an SNR improvement of $3 \mathrm{~dB} /$ octave
- $\mathrm{fs}>2 \mathrm{f}_{0}\left(2 \mathrm{f}_{0}=\right.$ Nyquist Rate
- $\operatorname{OSR}=\mathrm{f}_{\mathrm{s}} / 2 \mathrm{f}_{0}$
- SNRmax = 6.02N+1.76+ 10log (OSR)


## Flash ADC from 1926 (Analog Digitad Conversion handbook, Analog Devices

The first documented flash converter was part of Paul M. Rainey's electro-mechanical PCM facsimile system described in a relatively ignored patent filed in 1921 (Reference 5-see further discussions in Chapter 1 of this book). In the ADC, a current proportional to the intensity of light drives a galvanometer which in turn moves another beam of light which activates one of 32 individual photocells, depending upon the amount of galvanometer deflection (see Figure 3.49). Each individual photocell output activates part of a relay network which generates the 5 -bit binary code.


Figure 3.49: A 5-Bit Flash ADC Proposed by Paul Rainey Adapted from Paul M. Rainey, "Facsimile Telegraph System," U.S. Patent

1,608,527, Filed July 20, 1921, Issued November 30, 1926

## Ideal D/A converter



$$
\begin{gathered}
\mathrm{B}_{\text {in }}=\mathrm{b}_{1} 2^{-1}+\mathrm{b}_{2} 2^{-2}+\ldots+\mathrm{b}_{\mathrm{N}} 2^{-\mathrm{N}} \\
\mathrm{~V}_{\text {out }}=\mathrm{V}_{\text {ref }}\left(\mathrm{b}_{1} 2^{-1}+\mathrm{b}_{2} 2^{2}+\ldots+\mathrm{b}_{\mathrm{N}} 2^{-\mathrm{N}}\right)
\end{gathered}
$$

## Example : 8-bit D/A converter

An ideal D/A converter has

$$
\mathrm{V}_{\mathrm{ref}}=5 \mathrm{~V}
$$

Find Vout when

$$
\begin{aligned}
& \quad \mathrm{B}_{\text {in }}=10110100 \\
& \qquad \mathrm{~B}_{\text {in }}=2^{-1}+2^{-3}+2^{-4}+2^{-6}=0,703125 \\
& \mathrm{~V}_{\text {out }}=\mathrm{V}_{\text {re }} \mathrm{B}_{\text {in }}=3,516 \mathrm{~V} \\
& \text { Find } \\
& \qquad V_{\text {LSB }} \\
& \quad \mathrm{V}_{\text {LSB }}=5 / 256=19,5 \mathrm{mV}
\end{aligned}
$$



## Ideal A/D converter ( Fig. 11.3 )



$$
\mathrm{V}_{\mathrm{ref}}\left(\mathrm{~b}_{1} 2^{-1}+\mathrm{b}_{2} 2^{-2}+\ldots+\mathrm{b}_{\mathrm{N}} 2^{-\mathrm{N}}\right)=\mathrm{V}_{\mathrm{in}} \pm \mathrm{V}_{\mathrm{x}}
$$

where

$$
-\frac{1}{2} \mathrm{~V}_{\mathrm{LSB}} \leq \mathrm{V}_{\mathrm{x}}<\frac{1}{2} \mathrm{~V}_{\mathrm{LSB}}
$$

## Ideal transfer curve for a 2-bit A/D converter (Fig. 2.2,


-A range of input values produce the same output value (QA range of input values produce the same output value (Quantization error)
-Different from the D/A case

## Quantization noise ("J\&M" + "M")



$$
\mathrm{V}_{\mathrm{Q}}=\mathrm{V}_{1}-\mathrm{V}_{\mathrm{in}}
$$



## Quantization noise model



Quantizer

$$
\mathrm{V}_{1}=\mathrm{V}_{\mathrm{in}}+\mathrm{V}_{\mathrm{Q}}
$$

Model
-TThe model is exact as long as Vq is properly defined
$\cdot \mathrm{Vq}$ is most often assumed to be white and uniformely distributed between +/VIsb/2

## Quantization noise

-The rms-value of the quantization noise can be shown to be:

$$
\mathrm{V}_{\mathrm{Q}(\mathrm{rms})}=\frac{\mathrm{V}_{\mathrm{LSB}}}{\sqrt{12}}
$$

-Total noise power is independent of sampling frequency
-In the case of a sinusoidal input signal with p-p amplitude of $\quad \mathrm{V}_{\text {ref }} / 2$

$$
\begin{aligned}
& \mathrm{SNR}=20 \log \left(\frac{\mathrm{~V}_{\mathrm{in}(\mathrm{rms})}}{\mathrm{V}_{\mathrm{Q}(\mathrm{rms})}}\right)=20 \log \left(\frac{\mathrm{~V}_{\mathrm{ref}} /(2 \sqrt{2})}{\mathrm{V}_{\mathrm{LSB}} /(\sqrt{12})}\right) \\
& \mathrm{SNR}=6,02 \mathrm{~N}+1,76 \mathrm{~dB}
\end{aligned}
$$

## Quantization noise


-Signal-to Noise ratio is highest for maximum input signal amplitude

## Signed codes

- Unipolar / bipolar
- Common signed digitalepr sign magnitude, 1's complement, 2's compl.

| Table 11.1 | Some 4-bit signed digital representations |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Normalized <br> number | Sign <br> magnitude | 1's <br> complement | Offset <br> binary | 2's <br> complement |
| +7 | $+7 / 8$ | 0111 | 0111 | 1111 | 0111 |
| +6 | $+6 / 8$ | 0110 | 0110 | 1110 | 0110 |
| +5 | $+5 / 8$ | 0101 | 0101 | 1101 | 0101 |
| +4 | $+4 / 8$ | 0100 | 0100 | 1100 | 0100 |
| +3 | $+3 / 8$ | 0011 | 0011 | 1011 | 0011 |
| +2 | $+2 / 8$ | 0010 | 0010 | 1010 | 0010 |
| +1 | $+1 / 8$ | 0001 | 0001 | 1001 | 0001 |
| +0 | +0 | 0000 | 0000 | 1000 | 0000 |
| $(-0)$ | $(-0)$ | $(1000)$ | $(1111)$ |  |  |
| -1 | $-1 / 8$ | 1001 | 1110 | 0111 | 1111 |
| -2 | $-2 / 8$ | 1010 | 1101 | 0110 | 1110 |
| -3 | $-3 / 8$ | 1011 | 1100 | 0101 | 1101 |
| -4 | $-4 / 8$ | 1100 | 1011 | 0100 | 1100 |
| -5 | $-5 / 8$ | 1101 | 1010 | 0011 | 1011 |
| -6 | $-6 / 8$ | 1110 | 1001 | 0010 | 1010 |
| -7 | $-7 / 8$ | 1111 | 1000 | 0001 | 1001 |
| -8 | $-8 / 8$ |  |  | 0000 | 1000 |

- Sign. M.: 5:0101, -5:1101, two repr. Of 0, $2^{\mathrm{N}}-1$ numb.
- 1's compl.: Neg. Numbers are complement of all bits for equiv. Pos. Number: 5:0101, -5:1010
- Offset bin: 0000 to the most neg., and then counting up..
+: closely related to unipolar through simple offset


## 2's complement

| A3a2a1a0 | Sign <br> magnitude | 2s complement |
| :--- | :--- | :--- |
| 0111 | +7 | +7 |
| 0110 | +6 | +6 |
| 0101 | +5 | +5 |
| 0100 | +4 | +4 |
| 0011 | +3 | +3 |
| 0010 | +2 | +2 |
| 0001 | +1 | +1 |
| 0000 | +0 | +0 |
| 1000 | -0 | -8 |
| 1001 | -1 | -7 |
| 1010 | -2 | -6 |
| 1011 | -3 | -5 |
| 1100 | -4 | -4 |
| 1101 | -5 | -3 |
| 1110 | -6 | -2 |
| 111 | -7 | -1 |

- $5_{10}: 0101=2^{2}+2^{0}$
- $-5_{10}:(0101)^{\prime}+1=1010+1=$ 1011
- Addition of positive and negative numbers is straightforward, using addition, and requires little hardware
- 2's complement is most popular representation for signed numbers when arithmetic operations have to be performed
$7_{10}-6_{10}$ via addition using two's complement of -6
- $0000000000000000000000000000000000111_{2}=7_{10}$
- $0000000000000000000000000000000000110_{2}=6_{10}$
- Subtraction uses addition: The appropriate operand is negated before being added
- Negating a two's complement number: Simply invert every 0 and 1 and add one to the result. Example:
- $000000000000000000000000000000000110_{2}$ becomes
- $111111111111111111111111111111111001_{2}$

$$
+
$$$1_{2}$

= $111111111111111111111111111111111010^{2}$ $000000000000000000000000000000000111_{2}=7_{10}$

+ $11111111111111111111111111111111{11010_{2}}^{2}=-6_{10}$
$=000000000000000000000000000000000001_{2}=1_{10}$


## performance limitations

- Resolution
- Offset and gain error
- Accuracy
- Integral nonlinearity (INL) error
- Differential nonlinearity (DNL) error
- Monotonicity
- Missing codes
- A/D conversion time and sampling rate
- D/A settling time and sampling rate
- Sampling time uncertainty
- Dynamic range
- NB!! Different meanings and definitions of performance parameters sometimes exist. $\rightarrow$ Be sure what's meant in a particular specification or scientific paper.. There are also more than those mentioned here.


## Resolution

- Resolution usually refers to the number of bits in the input (D/A) or output (ADC) word, and is often different from the accuracy.
- Analog-Digital Conversion Handbook, Analog Devices, 3rd Edition, 1986: An n-bit binary converter should be able to provide $2 n$ distinct and different analog output values corresponding to the set of $n$ binary words. A converter that satisfies this criterion is said to have a resolution of $n$ bits.
in $0.18-\mu \mathrm{m}$ Digital CMOS

[^0]| TABLE I |  |
| :---: | :---: |
| KEY DATA For the ADC |  |



Fig. 8. SFDR, SNR, and SNDR versus conversion rate. The input frequency and signal swing is 10 MHz and $2 \mathrm{~V}_{\mathrm{P}-\mathrm{P}}$, respectively

## Litterature

- Johns \& Martin: "Analog Integrated Circuit Design"
- Franco Maloberti: "Data Converters"


## Next week:

- Data converter fundamentals, among them some principles especially relevant for your project.
- Messages are given on the INF4420 homepage.
- Questions: sa@ifi.uio.no , 22852703 / 90013264


## Transistor stuff.

Writing

$$
\begin{equation*}
I_{D}=\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left(V_{G S}-V_{T H}\right)^{2}\left(1+\lambda V_{D S}\right) \tag{2.29}
\end{equation*}
$$

and $\lambda \propto 1 / L$, we note that if the length is doubled, the slope of $I_{D}$ vs. $V_{D S}$ is divided by four because $\partial I_{D} / \partial V_{D S} \propto \lambda / L \propto 1 / L^{2}$ (Fig. 2.26). For a given gate-source overdrive, a larger $L$ gives a more



Figure 2.15 Saturation of drain current.

Figure 2.12 Linear operation in deep triode region.

current. With the condition $V_{D S} \ll 2\left(V_{G S}-V_{T H}\right)$, we say the device operates in deep triode region.


Figure 2.11 Drain current versus
drain-source voltage in the triod region.
Fig. 2.11 plots the parabolas given by (2.8) for different values of $V_{G S}$, indicating that can show that the peak of each parabola occurs at $V_{D S}=V_{G S}-V_{T H}$ and the peak current is

$$
I_{D, \text { max }}=\frac{1}{2} \mu_{n} C_{o x} \frac{W}{L}\left(V_{G S}-V_{T H}\right)^{2} .
$$

We call $V_{G S}-V_{T H}$ the "overdrive voltage" 4 and $W / L$ the "aspect ratio." If $V_{D S} \leq V_{G S}$ $V_{T H}$, we say the device operates in the "triode region."

If in (2.8), $V_{D S} \ll 2\left(V_{G S}-V_{T H}\right)$, we have

$$
\begin{equation*}
I_{D} \approx \mu_{n} C_{o x} \frac{W}{L}\left(V_{G S}-V_{T H}\right) V_{D S} \tag{2.10}
\end{equation*}
$$

that is, the drain current is a linear function of $V_{D S}$. This is also evident from the characteristics of Fig. 2.11 for small $V_{D S}$ : as shown in Fig. 2.12, each parabola can be approximated by a straight line. The linear relationship implies that the path from the source to the drain can be represented by a linear resistor equal to

$$
\begin{equation*}
R_{o n}=\frac{1}{\mu_{n} C_{o x} \frac{W}{L}\left(V_{G S}-V_{T H}\right)} \tag{2.11}
\end{equation*}
$$

- Saturation. Vgs $\geq$ Vth. Vds sufficiently high so that Vgd < Vth; Vds $\geq(\mathrm{Vgs}-\mathrm{Vth})$

Triode: Vgs $\geq$ Vth, Vds sufficiently high so Vgd < Vth

- Cutoff / subthreshold: Vgs < Vth


[^0]:    Terje Nortvedt Andersen. Bjerrar Hernes, Member, IEEE, Atl Briskemyr, Frode Testor,
    Johnny Bjornsen, Member IEEE. Thomas E. Bonnenud, and ©ystein Moldsvor

