# Nyquist Analog to Digital Converters <br> Tuesday, March 1st, 9:15-11:00 

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## February the 22th

- 3.1 Introduction
- 3.1.1 DAC applications
- 3.1.2 Voltage and current references
- 3.2 Types of converters
- 3.3 Resistor based architectures
- 3.3.1 Resistive divider
- 3.3.2 X-Y selection
- 3.3.3 Settling of the output voltage
- 3.3.4 Segmented architectures
- 3.3.5 Effects of mismatch
- 3.3.6 Trimming and calibration
- 3.3.7 Digital Potentiometer
- 3.3.8 R-2R Resistor Ladder DAC
- 3.3.9 Deglitching
3.4 Capacitor based architectures
3.4.1 Capacitive divider DAC
3.4.2 Capacitive MDAC
3.4.3 "Flip around" MDAC

3.4.4 Hybrid capacitive resistive DACs
3.5 Current source based
 architectures

Figure 1.1. Block diagram of the basic functions of an $\mathrm{A} / \mathrm{D}$ (a) and a D/A (b) converter.
3.5.1 Basic operation
3.5.2 Unity current generator
3.5.3 Random mismatch with
unary selection
3.5.4. Current sources
selection
3.5.5 Current switching and
segmentation
3.5.6 Switching of current sources
3.6 Other architectures
(The contents refer to "Maloberti")


1. mars 2011

## March the 1st

- Contents of Chapter 4:
- 4.1 Introduction
- 4.2 Timing accuracy

- 4.3 Full flash converters


Figure 1.1. Block diagram of the basic functions of an $\mathrm{A} / \mathrm{D}$ (a) and a D/A (b) converter.

- 4.4 Sub-ranging and two-step converters
- 4.5 Folding and interpolation
- 4.6 Time interleaved converters
- 4.7 Successive approximation converter
- 4.8 Pipeline converters
- 4.9 Other architectures


1. mars 2011

## ADCs and throughput (1 of 2)

- Depending on the bandwidth of the input signal, ADCs may use one or multiple clock cycles per conversion.


## Throughput Rate Comparison



## ADCs and throughput ( 2 of 2)

- Depending on the bandwidth of the input signal, ADCs may use one or multiple clock cycles per conversion.


Fig. 1: Resolution vs. clock cycles/sample for different ADC algorithms

## Full Flash Converters



Figure 4.4. Basic block diagram of the full-flash converter.

(b)

(c)

- Compare input with all transition points between adjacent quantization intervals - "brute force"
- Quick-1 CP, "flash".
- $n$ bit: $2^{n}-1$ reference voltages and comparators.
- "1" output up to a certain level, "0" over: thermometer code.
- ROM decoder.
- Quantization step $\Delta=\left(\mathrm{V}_{\text {ref }+}{ }^{-}\right.$ $\mathrm{V}_{\text {reft }+}$ )/ $2^{\mathrm{n}-1}$, with first and last step equal to $\Delta / 2$.


## Successive approx ADC algorithm



$$
\text { IS } X \geq(32+16) ?
$$



YES $\rightarrow$ RETAIN $8 \rightarrow 1$YES $\rightarrow$ RETAIN $4 \rightarrow 1$
IS $X \geq(32+8+4)$ ?
NO $\rightarrow$ REJECT $2 \rightarrow 0$$\square$ IS $X \geq(32+8+4+2+1) ? \quad$ YES $\rightarrow$ RETAIN $1 \rightarrow 1$ TOTALS: $X=32+8+4+1=45^{10}=101101^{2}$

Figure 4. Successive-approximation ADC algorithm using balance scale and binary weights.

Which ADC Architecture Is Right for Your Application?

By Walt Kester [walt.kester@analog.com]

- If we have weights of 1 kg, $2 \mathrm{~kg}, 4 \mathrm{~kg}, 8 \mathrm{~kg}, 16$ $\mathrm{kg}, 32 \mathrm{~kg}$ and will find the weight of an unknown X assumed to be 45 kg .
- $101101_{2}$
$=1 * 32+0 * 16+1^{*} 8+1^{*} 4+$
$0 * 2+1 * 1$
$=45_{10}$


## Successive approximation converter


(a)

(b)

Figure 4.28. Timing (a) and flow diagram (b) of the successive approximation technique.


Figure 4.29. Basic circuit diagram of the successive approximation algorithm.

- Multiple clock periods
- Exploits knowledge of previously determined bits to find next significant bit.
- Low complexity and low power consumption
- For a given dynamic range 0 - $V_{\text {FS }}$ the MSB distinguishes between input signals below or above $\mathrm{V}_{\mathrm{FS}}$ / 2. Comparing the input with $\mathrm{V}_{\mathrm{FS}} / 2$ obtains the first bit as seen in Fig. 4.28 a)

Fig. 4.29 shows a typical block diagram.

## Successive approximation converter


(a)

(b)

Figure 4.28. Timing (a) and flow diagram (b) of the successive approximation technique.


Figure 4.29. Basic circuit diagram of the successive approximation algorithm.

- For a given dynamic range gec $\mathrm{V}_{\text {FS }}$ the MSB distinguishes between input signals below or above $\mathrm{V}_{\mathrm{FS}}$ / 2 . Comparing the input with $\mathrm{V}_{\mathrm{FS}} / 2$ obtains the first bit as seen in Fig. 4.28 a)
- The knowledge of the MSB restricts the search for the next bit to either the upper or lower half of the 0 to $\mathrm{V}_{\mathrm{FS}}$ interval. Threshold for second bit is $\mathrm{V}_{\text {FS }} /$ 2 or $3 \mathrm{~V}_{\mathrm{FS}} / 4$ (the case here). After this the next bit is chosen and next bit can be estimated . The timing diagram (upper left) describes the case for three bits.
- Voltages for comparisons are generated by a DAC under control of the SAR register.


## Successive approximation converter



Figure 4.29. Basic circuit diagram of the successive approximation algorithm.

- Timing diagram: S/H samples the input during the 1st clock period and holds it for N successive clock intervals.
- The DAC is controlled by the SAR algorithm (Fig. 4.28 b))
- Initially the SAR sets MSB to 1 as a prediction, though this may be changed to 0 .
- The process continues until all $n$ bits have been determined.
- As the start of the next conversion (while the S\&H is sampling the next input, the SAR provides the $n$-bit output and resets the registers.
- The name of the algorithm comes from the fact that the voltage from the DAC is an improving approximation of the sampled input voltage.

Suck. Approx ADC, example 13.2 $\boldsymbol{2}$
Example 13.2



$$
\begin{aligned}
V_{\text {ret }} & =8 \mathrm{~V} \\
V_{\text {in }} & =2.831 \mathrm{~V}
\end{aligned}
$$

3-bit conversion
cycle 1: $B_{\text {out }}=100$, so that $V_{D / A}=4.0 \mathrm{~V}$. Since $V_{\text {in }}<V_{D / A}, b_{1} \rightarrow 0$
cycle 2: $B_{\text {out }}=010$, so that $V_{D / A}=2.0 \mathrm{~V}$. Since $V_{\text {in }}>V_{D / A}, b_{2} \rightarrow 1$
Eyck 3: Bout $=011$, - 1 . Since $V_{\text {in }}<V_{D / A}, D_{3} \rightarrow 0$
010

## Sub-ranging and Two-step convertersit


$\sqrt{V}(\mathrm{M}+\mathrm{N})$-bit Output


[^0]- Sub-ranging and two-step Ancer have better speed-accuracy tradeoff than full flash for $n>8$.
- 2 (or 3) clock periods per conversion, but smaller number of comparators and thus benefitting silicon area, power consumption and capacitive loading of the $\mathrm{S} / \mathrm{H}$.
- The DAC converts the M MSBs back to an analog signal that is subtracted from the held input that is converted to digital by the 2nd N bit flash that yields the LSBs.
- Digital Logic combine coarse and fine bits to obtain the $n=(M+N)$ bit output.
- Subranging ADCs does not have the amplification by K (two-step has).


## Sub-ranging and Two-step converter ${ }^{2}$ <br>  diagram.

DIGITALLOGIC
$\checkmark(\mathrm{M}+\mathrm{N})$-bit Output


Figure 4.10. Block diagram of sub-ranging $(\mathrm{K}=1)$ and two-step architectures $(\mathrm{K}>1)$.

- Four logic signals (below main clock signal) are derived from the main clock.
- Assuming half a clock period is used to provide each function or group of functions means that 2 clock periods are enough for 1 conversion.
- For an 8 bit conversion: $\mathrm{M}=\mathrm{N}=4$ $\rightarrow 2(16-1)=30$ comparators are needed, instead of 255 for an 8 -bit flash ADC.
- The spared area and power are much more than what is needed for the DAC and residue generator.
- $\mathrm{S} / \mathrm{H}$ is only loaded by $2^{\mathrm{M}}$ comp.


## Folding and interpolation



Figure 4.17. Non-linear blocks that obtain input folding.


Figure 4.18. Basic architecture of a folding converter.

- Splits the input range into a number of sectors
- Single folding bends the input around $1 / 2 \mathrm{~V}_{\mathrm{FS}}$ and gives rise to 2 sectors (1-bit) with peak amplitude $1 / 2 \mathrm{~V}_{\text {FS }}$.
- Folding 2 times leads to 4 sectors (2-bit) with peak amplitude $1 / 4 \mathrm{~V}_{\mathrm{FS}}$.
- $M$ bit folding needs $2^{n-M-1}$ comparators to complete an n bit conversion.
- Knowledge of which segment the input is in determines the MSBs, which are combined with LSBs for $\mathrm{M}+\mathrm{N}$-bit output.


## Folding and interpolation



Figure 4.18. Basic architecture of a folding converter.

- Fig. 4.18 is a conceptual block diagram: The M-bit folder produces the analog folded output and the M-bit code which identifies which segment the output is in. The gain stage augments the dynamic range to become $\mathrm{V}_{\text {Fs }}$. The N -bit ADC determines the LSBs that are combined with the MSBs to give the overall output of $n=(N+M)$ bits.
- The folding circuit is normsally used for high conversion rates and medium-high resolutions.


## Folding A/D Converters (13.7) fi

- The number ofanches is
 reduced compared to the interpolating ADC, and even more from FLASH
- The figure shows a 4 bit converter with folding rate of 4
- A group of LSBs are found separately from a group of MSBs.
- The MSB converter determines whether the input signal, $\mathrm{V}_{\text {in }}$, is in one of four voltage regions (between 0 and $1 / 4,1 / 4$ and $1 / 2,1 / 2$ and $3 / 4$, or $3 / 4$ and 1)
- $\mathrm{V}_{1}$ to $\mathrm{V}_{4}$ produce a thermometer code for each of the four MSB regions


## Similar to folding block responses on previoussi

- Bipolar folder outpûts

- Ex: Input 1.05:
- F1 > threshold=0 -> "1"
- F2 > threshold=0 -> "1"
- F3 > threshold=0 -> "1"
- F4 < threshold=0 -> "0"
- Thermometer code produced for each of the four MSB regions (between 0 and $1 / 4,1 / 4$ and $1 / 2,1 / 2$ and $3 / 4$, or $3 / 4$ and 1 for previous slide)
- (in certain respects related to intel
3.24)


## Folding - problems with unsharp edg



Figure 4.17. Non-linear blocks that obtain input folding.

(a)

(b)

Figure 4.19. (a) Real folding response and (b) its unfolded version.

- Sharp edges are desired, but hard to obtain.
- Linearity is good in the regions midway between the folding points and becomes bad as the input approaches the segment borders.
- This may give rise to an INL which sometimes can make the method impractical. There is a solution..


## 2 folders to avoid non-linear regions

- Bad regions may be discarded and only good ones used.
- 2 folders and transfer characteristics shifted by a quarter of the folding period.
- $\rightarrow$ One folder is always in the linear region.
- The combining logic (MSBs + LSBs) must take into account the sign of the slope in the used segment and decide which folder provides the best linear response.


## Interpolation



Figure 4.21. Simple circuits that achieve interpolation.
$\xi_{V_{2}} \quad V_{\text {inter }}=\frac{V_{1} R_{2}+V_{2} R_{1}}{R_{1}+R_{2}}$.

- Interpolation provides an electrical value that is intermediate between two other electrical quantities.
- Voltage inputs: resistive or capacitive dividers
- Current inputs: schemes based on current mirrors.


## Interpolation in Flash ADCs ${ }_{\text {(ch. } 4.54)}$


(a)

Figure 4.22. a) Use of interpolation in flash converters. b) Outputs and interpolated responses.

- Reduces the number of preamplifiers by generating the median of adjacent pre-amplifier outputs. This interpolated voltage is then used by intermediate latches.
- Equal slopes at the zero crossing equalize the speed and the metastabilityu error of the latches.
- The number of pre-amps ( and reference voltages diminish by a factor of 2, reducing the capacitive load on the $\mathrm{S} / \mathrm{H}$. (may be extended to 4 or 8 resistors between neighbouring pre-amplifiers.
- $\rightarrow$ less power consumption or higher speed.


## Time-Interleaved Converters $($ ch. 4.54$)$



Figure 4.27. Time-interleaved architecture.

- Converters working in parallel for simultaneous quantization of input samples.
- A suitable combination of the results makes the operation equivalent to a single converter whose speed has been increased by a factor equal to the number of parallel elements.
- An alternative solution that relaxes the demanding specification associated with one full speed $\mathrm{S} / \mathrm{H}$ employs one $\mathrm{S} / \mathrm{H}$ in each path.
- Problems: gain mismatch between channels transformed into dynamic errors.


## Time-Interleaved - best compromise betwgeq complexity and sampling rate - may be used (arm different architectures [Elbjornsson '05]



Figure 7 Comparison between ADC architectures. The time interleaved successive approximation ADC gives the best compromise between complexity and sampling rate.

# Pipeline Converters $($ (ch 4.8) 



Figure 4.36. Block diagram of a pipeline stage.
Two-step expanded to a multi-step algorithm and implemented as a pipeline architecture.

- May generate multiple bits / stage.
- Total resolution is given by the sum of the bits at each stage.
- Fig. 4.36: generic pipeline stage.


# Pipelined ADC -example 

## A Cost-Efficient High-Speed 12-bit Pipeline ADC in $0.18-\mu \mathrm{m}$ Digital CMOS

Terje Nortvedt Andersen, Bjørnar Hernes, Member, IEEE, Atle Briskemyr, Frode Telstø Johnny Bjørnsen, Member, IEEE, Thomas E. Bonnerud, and Øystein Moldsvor



Fig. 7. Power dissipation versus conversion rate. The input frequency and signal swing is 10 MHz and $2 \mathrm{~V}_{\mathrm{P}-\mathrm{P}}$, respectively.

| Nominal sampling rate | $110 \mathrm{MS} / \mathrm{s}$ |
| :---: | :---: |
| Technology | $0.18 \mu \mathrm{~m}$ digital CMOS |
| Nominal supply voltage | 1.8 V |
| Resolution | 12 bit |
| Full scale analog input | $2 \mathrm{~V}_{\mathrm{P}-\mathrm{P}}$ |
| Area | $0.86 \mathrm{~mm}^{2}$ |
| Power consumption | 97 mW |
| DNL | $\pm 1.2 \mathrm{LSB}$ |
| INL | $-1.5 /+1 \mathrm{LSB}$ |
| SNR $\left(f_{\text {in }}=10 \mathrm{MHz}\right)$ | 67.1 dB |
| SNDR $\left(f_{\text {in }}=10 \mathrm{MHz}\right)$ | 64.2 dB |
| SFDR $\left(f_{i n}=10 \mathrm{MHz}\right)$ | 69.4 dB |
| ENOB $\left(f_{\text {in }}=10 \mathrm{MHz}\right)$ | 10.4 bit |



Fig. 12. Figure of Merit $(F M)$ versus $1 / A$ for 12 -bit $\mathrm{ADCs} . f_{\mathrm{CR}}$ is given in $\mathrm{MS} / \mathrm{s}, A$ is given in $\mathrm{mm}^{2}$ and $P_{\text {SUP }}$ is given in mW .

## Integrating converters (ch. 49.2)



Figure 4.50. Waveforms of a two-slope ADC (a) and (b) offset cancelled scheme.

- Input signal determines the slope of the output from the integrator.
- When it's shifted, the slope becomes fixed, and the time it takes to return is measured by a counter, and translated to a digital output.


## Integrating Converters (13.1) ifj


(Vin is held constant during conversion.)

- $\mathrm{V}_{\mathrm{x}}(\mathrm{t})=\mathrm{V}_{\text {in }} \mathrm{t} / \mathrm{RC}\left(\mathrm{V}_{\mathrm{x}}\right.$ ramp derivative depending on $\left.\mathrm{V}_{\text {in }}\right)$
- High linearity and low offset/gain error
- Small amount of circuitry
- Low conversion speed
- $2^{\mathrm{N}+1}{ }^{*} 1 / \mathrm{T}_{\mathrm{ck}}$ (Worst case)


## Integrating Converters


(Constant slope)
$T_{2}$ (Three values for three inputs)

- The digital output is given by the count at the end of $T_{2}$
- The digital output value is independent of the time-constant RC


$$
V_{L S B}=\frac{V_{F S}}{2^{n}}
$$

Input offset voltage: voltage required to dive the output voltage to zero

Comp. offset is aced to the diff. input and modifies the threshold transition. offset in a flash architecture alter the isth quantization
 interval $\Delta$ :

$$
\Delta_{i}=V_{t h r, i}-V_{t h r, i-1}=\Delta-V_{0 s, i}+V_{0 s, i-1}
$$

To ensure no missing codes or monotonicity for a given yield, the max-value of the offset must be lower than $\frac{1}{2}$ LSB divided by the number of sigma required to obtain that yield.
$\varepsilon_{x}:$

COMPARATOR OFFSET
Example: 8-bit FLASH, 10 FS, $99.9 \%$ yodel :

$$
\begin{aligned}
& V_{L S B}=\frac{1 V}{2^{8}}=3.90625 \mathrm{mV} \\
& \frac{1}{2} V_{L S B}=0.001953125 \mathrm{~V}=1.953125 \mathrm{mV}
\end{aligned}
$$



To ensure $99.9 \%$ yield, the corresponding sigma with a normal distribution of errors is $\sigma=3.3$

$$
\frac{1}{2} \text { LSB }: 3.3>1.953125 \mathrm{mV} / 3.3=0.592 \mathrm{mV}
$$

The offset is mainly caused by the pre-amplifier of the comparator. $\Rightarrow$ Design the 1st stage and optimize the layout for minimum threshold, tran scond. param. $P$ lox and $W / L$ mismatches in the input diff. pair and active loads $\Delta V_{t_{n}}=\frac{A_{v_{T}}}{\sqrt{w_{L}}}$.

Metastability - probability of undefined comparator output

- A sampled-data comparator is typically realized using a pre-amplifier and a latch.
- $\Phi_{\text {amp }}, \Phi_{\text {latch }}$ (latch logic level)
- If $\mathrm{V}_{\mathrm{in}, \mathrm{d}}$ is too small, the comparator may be undefined at the end of the latch phase giving an error in the output code and possibly causing a code bubble error in the thermometric output of some converter architectures (or other circuits making use of comparators).


## Metastability - probability of undefined comparator output



The differential latch of Fig. 4.3 is the positive loop of two transconductors whose regenerative time constant, $\tau_{L}$, is

$$
\begin{equation*}
\tau_{L} \simeq \frac{C_{p}}{g_{m}} \tag{4.10}
\end{equation*}
$$

The metastability error probability can be approximated by

$$
\begin{equation*}
P_{E}=\frac{V_{0}}{V_{i n} A_{0}} e^{-t_{r} / \tau_{L}} \tag{4.11}
\end{equation*}
$$

where $V_{0}$ is the voltage swing required for valid logic levels and $t_{r}$ is the period of the latch phase.


- $\mathrm{V}_{0}$ : voltage swing for valid logic levels
- $t_{r}=\Phi_{\text {latch }} \rightarrow$ Probability of a metastability error increases with the sampling frequency and at high frequencies becomes equal to 1 (since more than 1 is not a valid result. If $P_{E}$ is according to eq. $4.11>1$, the result means $P_{E}$ is 1.)
- $P_{E}$ is inversely proportional to the input amplitude $\mathrm{V}_{\text {in,d }}$.


## Approximate Evaluation of max frequency of operation for ADCs (

- $f_{\text {Tech }}$ : Technology unity gain frequency
- $\mathrm{f}_{\mathrm{T}}$ : unity gain frequency of OTA or op-amp
- $f_{T}=f_{\text {Tech }} / \alpha$, where $\alpha$ is at least 2-4 (ultimately depending on accuracy).
- $f_{C K}=f_{T} / \gamma$, where $\gamma$ is a suitable margin between the op-amps $f_{T}$ and the clock frequency, $f_{\mathrm{CK}}$, as some time for settling is needed. $\left(\mathrm{f}_{\mathrm{CK}}<\mathrm{f}_{\mathrm{T}}\right)$
- In order to estimate $\gamma$, suppose that the input $\mathrm{V}_{\text {in }}$ is a step at $\mathrm{t}=0$.
- A single pole band-limitation gives rise to an output $\mathrm{V}_{\text {out }}(\mathrm{t})$ (approaching $V_{\text {in }}$ ) given by:
$V_{\text {out }}(t)=V_{\text {in }}\left(1-e^{-t / \tau}\right)$


The exponential function $y=e^{x}$

## Approximate Evaluation of max frequency of operation for ADCs (2)

- $\mathrm{f}_{\text {Tech }}$ : Technology unity gain frequency
- $f_{T}$ : unity gain frequency of OTA or op-amp
- $f_{T}=f_{\text {Tech }} / \alpha$, where $\alpha$ is at least 2-4 (ultimately depending on accuracy).
- $f_{C K}=f_{T} / \gamma$, where $\gamma$ is a suitable margin between the op-amps $f_{T}$ and the clock frequency, $f_{\mathrm{CK}}$, as some time for settling is needed. ( $\mathrm{f}_{\mathrm{CK}}<\mathrm{f}_{\mathrm{T}}$ )
- In order to estimate $\gamma$, suppose that the input $V_{\text {in }}$ is a step at $t=0$.
- A single pole band-limitation gives rise to an output $\mathrm{V}_{\text {out }}(\mathrm{t})$ (approaching $\mathrm{V}_{\text {in }}$ ) given by:

$$
\begin{align*}
& V_{\text {out }}(t)=V_{\text {in }}\left(1-e^{-t / \tau}\right)  \tag{4.1}\\
& \quad \tau=\frac{1}{2 \pi \beta f_{T}} \tag{4.2}
\end{align*}
$$ output is fed back to the negative input

- Since an n-bit ADC needs an accuracy better than 2-( $\mathrm{n}+1$ ), the settling time must be

$$
t_{s e t t}>\tau \cdot(n+1) \ln (2)
$$

$$
\begin{gather*}
f_{C K}<\frac{\pi \beta f_{T}}{(n+1) \ln (2)}  \tag{4.3}\\
\gamma=\frac{f_{T}}{f_{C K}}>\frac{(n+1) \ln (2)}{\pi \beta} . \tag{4.4}
\end{gather*}
$$

- Since the time allowed for settling is half clock frequency.

The foreseen order of the anti-aliasing filter sets a given margin $\lambda$, which is the ratio between sampling rate and signal band. Moreover, since the conversion algorithm can use multiple clock periods (say $k$ ), the conversion rate is therefore given by $f_{C K} /(\lambda k)$.
$\beta$ - feedback factor - how much of the

## Litterature

- Johns \& Martin: "Analog Integrated Circuit Design"
- Franco Maloberti: "Data Converters"
- http://inst.eecs.berkeley.edu/~ee247/fa04/fa04/lectures/L19_f04.pdf

```
SYSTEM ARCHITECTURE AND KEY COMPONENTS FOR AN 8 BIT/1 GH
GaAs MESFET ADC
```

J. Sauerer, R. Hagelauer, F. Oehler, G. Rohmer, U. Schlag, D. Seitzer + T. Grave, W. Kellner ++

+ FhG-IIS, Wetterkreuz 13, D-8520 Erlangen, Germany
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## Which ADC Architecture Is Right for Your Application?

By Walt Kester [walt.kester@analog.com]

## Next week, 01/03: ifj

- Next week: To be defined..
- Messages are given on the INF4420 homepage.
- Questions: sa@ifi.uio.no , 22852703 / 90013264


[^0]:    Figure 4.10. Block diagram of sub-ranging $(\mathrm{K}=1)$ and two-step architectures $(\mathrm{K}>1)$.

