INF5410: Signal Processing in Space and Time

Mandatory Exercise 1

Part 1: From Johnson & Dudgeon

1) Exercise 3.7.
   In order to make the answer more straightforward: Assume circular elements instead of hexagonal elements. See Fig. 1.

2) Exercise 3.11.

3) Exercise 4.5. *Hint: See Fig. 1*

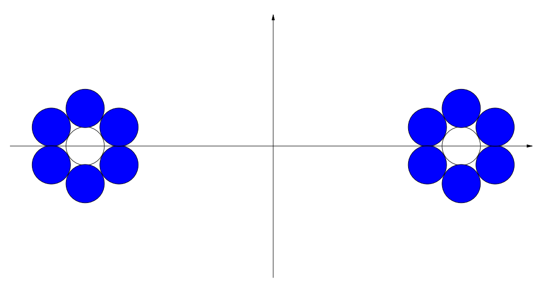

Figure 1: Hints regarding the geometry to be applied in Exercise 3.7 and Exercise 4.5. Each sub-aperture (blue circle) is of radius r, and the distance between the two big sub-apertures is $2 \cdot D$. In Exercise 4.5 b): Assume $r = 6$ cm and $D = 3$ m.

Part 2: Matlab coding project

Array Pattern

Write a Matlab program for computing the array pattern (aperture smoothing function) of an M-element array with the elements symmetrically located on the x-axis. Assume initially that each element is omni-directional. Plot the result in dB and give the result both as a function of wavenumber (*e.g.* as a function of sin(angle)) and angle. Plot only the visible region.

Grating lobes

1) Assume a linear array with $M = 10$ elements with unity weight. Let the elements be uniformly spaced and try four different element spacings: $d = \lambda/4, \lambda/2, \lambda$, and 2λ. Plot the results and discuss the differences as the inter-element distance varies.

Element weighting and element spacing

2) Consider the array with spacing $d = \lambda/2$ and $M = 10$, and use the Remez program in MATLAB's Signal Processing Toolbox to find the symmetric set of element weights that produces a uniform sidelobe level of -20 dB.
   [MATLAB 7 command: `firpm`, MATLAB 6 command: `remez`. See Mitra, Digital Signal Processing, chapter 7.7.1, Oppenheim & Schafer, Discrete-Time Signal Processing, 1999, chapter 7.6 or Proakis & Manolakis, 3rd edition, chapter 8.2.4 for more on the Remez method]

3) Let the array have $M = 10$ elements with unity weights and let the element spacings be given by $\{0.21464, 0.38517, 0.46147, 0.52586\}$ where the first number is the spacing from $x = 0$ to the first element, the second number is the distance from element 1 to element 2 and so on in units of λ. The last element is placed so that the aperture is equal to the equi-spaced array with $\lambda/2$ spacing, and the array is symmetric about $x = 0$.
   [The element locations have been taken from H. Schjær-Jacobsen and K. Madsen, "Synthesis of nonuniformly spaced arrays using a general nonlinear minimax optimization method," IEEE Trans. Antennas and Propagation, vol. AP–24, pp. 501–506, July 1976.]
   Hint: The solution should resemble that of the previous paragraph.

   Compare the solutions from 1), 2) and 3) with respect to beamwidth, aperture, sidelobes, grating lobes, etc.

Thinning

4) Assume an array as in 1) with $d = \lambda/2$. Make a thinned, symmetric array with 6 elements with the same aperture as the 10-element array by discarding 4 elements. By trying various combinations of thinning, find the thinning pattern which gives the smallest maximum sidelobe.

   Compare with the full array.

Element directivity

5) Consider again the arrays in 1) with $d = \lambda$ and $d = 2\lambda$. Assume that each element is of size d and include the element factor in the computation of the array pattern. Compare with the results in 1) and discuss the differences.

6) Apply steering to the array with the element responses in the previous paragraph and see what happens as the steering angle is increased.

7) Linear arrays with element spacing larger than $\lambda/2$ are regularly used in medical ultrasound imaging systems. Assume 10 unity weighted, regularly spaced elements and an element of size d equal to the element spacing. Design a linear array with as large aperture as possible that makes it possible to steer $\pm 15°$ with acceptable sidelobe and grating lobe levels.

Write a report with all plots, results and discussions. Include your MATLAB code as an appendix.