INTEGER DIVISJON — SUBTRAKTIVE ALGORITMER

- Innledning

  - Divisjon kategoriseres etter hvilke verdier kvotient bitene kan anta:

    - Restoring divisjon: $\{0, 1, 2, \ldots, r-1\}$

    - Nonrestoring divisjon: $\{-(r-1), -(r-2), \ldots, -1, +1, \ldots, r-2, r-1\}$

    - SRT divisjon: $\{-1, 0, 1\}$

    - Generalisert SRT divisjon: $\{-m, \ldots, -1, 0, 1, \ldots, m\}$

$$\frac{r-1}{2} \leq m \leq r-1.$$

- VI HAR: DIVIDENDEN 'X',
  DIVISOR 'D',
  RESULTAT / KVOTIENT 'Q',
  REST 'R'

SOM GIR:

$$X = Q \cdot D + R \text{ med } R < D.$$

- HVIS 'X' HAR DOBBELT SÅ MANGE BIT SOM 'D' OG RESULTATET 'Q' SKAL HA LIKE MANGE BIT SOM 'D' MÅ:

$$X < 2^{n-1} \cdot D$$

ELLERS OPPSTÅR OVERFLOW.
'n' ER ANTALL BIT I ~~[illegible]~~ DIVISOR 'D' OG RESULTAT 'Q'.

- I TILLEGG MÅ 'D' $\neq 0$ FOR Å UNNGÅ DIVISJON MED NULL EXCEPTION.

- SUBTRAKTIV DIVISJON BASERES PÅ FORMLEN :

$$r_i = 2 \cdot r_{i-1} - q_i \cdot D .$$

HVOR

$$i = 1, 2, \ldots m = n-1 .$$

OG HVOR

$$r_0 = 'X' ,$$

$$R = r_m \cdot 2^{-m} .$$

RESTORING DIVISION

Example 3.4

Let $X = (0.100000)_2 = 1/2$ and $D = (0.110)_2 = 3/4$. The dividend occupies a double-length register. The condition $X < D$ is clearly satisfied.

$r_0 = X$			0	.1	0	0	0	0	0	
$2r_0$		0	1	.0	0	0	0	0		set $q_1 = 1$
Add $-D$	+	1	1	.0	1	0				
$r_1 = 2r_0 - D$		0	0	.0	1	0	0	0		
$2r_1$			0	.1	0	0	0			set $q_2 = 0$
$r_2 = 2r_1$			0	.1	0	0	0			
$2r_2$		0	1	.0	0	0				set $q_3 = 1$
Add $-D$	+	1	1	.0	1	0				
$r_3 = 2r_2 - D$		0	0	.0	1	0				

som brukes for å regne ut endelig rest: R.

Note that the generation of $2r_0$ should not result in an overflow indication (multiplying a positive number by 2 should result in a positive number), since the quotient and remainder are within the proper range for the given dividend and divisor. Hence, an extra bit position in the arithmetic unit is needed.

The final results are $Q = (0.101)_2 = 5/8$ and $R = r_m 2^{-m} = r_3 2^{-3} = 1/4 \cdot 2^{-3} = 1/32$. (The precise quotient is the infinite binary fraction $2/3 = 0.1010101\cdots$.) The quotient and final remainder satisfy the equation $X = Q \cdot D + R = 5/8 \cdot 3/4 + 1/32 = 16/32 = 1/2$. □

- FOR INTEGER GJELDER SAMME FREMGANGSMÅTE HVOR

$$2^{2n-2} \cdot X_F = 2^{n-1} \cdot Q_F \cdot 2^{n-1} \cdot D_F + 2^{n-1} \cdot R_F$$

HVOR ALLE TALLENE REPRESENTERES SOM FRACTIONS.

DELE PÅ 2^{2n-2} GIR:

$$\underline{X_F = Q_F \cdot D_F + 2^{-(n-1)} \cdot R_F}$$

Example 3.5

We repeat the previous example with all operands and results being integers. In this case the double-length dividend is $X = 0100000_2 = 32$, and the divisor is $D = 0110_2 = 6$. The overflow condition $X < 2^{n-1}D$ is tested by comparing the most significant half of X, 0100, to D, 0110. The results of the division are $Q = 0101_2 = 5$ and $R = 0010_2 = 2$. Observe that in the final step of the process the true remainder R is generated and, as can be verified from equation (3.14), there is no need to further multiply it by $2^{-(n-1)}$. □

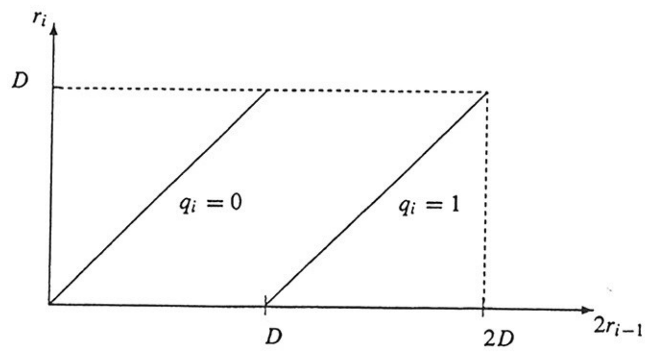

Figure 3.1 Restoring division.

NON-RESTORING DIVISION

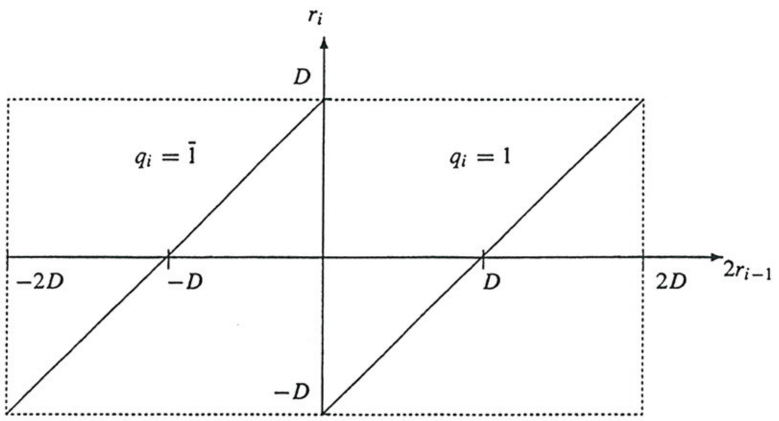

Figure 3.2 Nonrestoring division.

Example 3.6

Let $X = (0.100)_2 = 1/2$, and $D = (0.110)_2 = 3/4$, as in Example 3.4.

$r_0 = X$			0	.1	0	0	
$2r_0$		0	1	.0	0	0	set $q_1 = 1$
Add $-D$	+	1	1	.0	1	0	
r_1		0	0	.0	1	0	
$2r_1$			0	.1	0	0	set $q_2 = 1$
Add $-D$	+		1	.0	1	0	
r_2			1	.1	1	0	
$2r_2$			1	.1	0	0	set $q_3 = \bar{1}$
Add D	+		0	.1	1	0	
r_3			0	.0	1	0	

The final remainder is the same as before, and the quotient is $Q = 0.11\bar{1} = 0.101_2 = 5/8$. □

Example 3.7

Let $X = (0.100)_2 = 1/2$ and $D = (1.010)_2 = -3/4$.

$r_0 = X$			0	.1	0	0	
$2r_0$		0	1	.0	0	0	set $q_1 = \bar{1}$
Add D		1	1	.0	1	0	
r_1		0	0	.0	1	0	
$2r_1$			0	.1	0	0	set $q_2 = \bar{1}$
Add D	+		1	.0	1	0	
r_2			1	.1	1	0	
$2r_2$			1	.1	0	0	set $q_3 = 1$
Add $-D$	+		0	.1	1	0	
r_3			0	.0	1	0	

Finally, $Q = 0.\bar{1}\bar{1}1 = 0.\bar{1}0\bar{1} = -(0.101)_2 = -5/8$, or in two's complement, 1.011. Note that the final remainder is $1/32$ and has the same sign as the dividend X.

□

Example 3.8

Let $X = (0.101)_2 = 5/8$, and $D = (0.110)_2 = 3/4$. Then

$$
\begin{array}{llcccccl}
r_0 = X & & & 0 & .1 & 0 & 1 & \\
2r_0 & & 0 & 1 & .0 & 1 & 0 & \text{set } q_1 = 1 \\
\text{Add } -D & + & 1 & 1 & .0 & 1 & 0 & \\
\hline
r_1 & & & 0 & .1 & 0 & 0 & \\
2r_1 & & 0 & 1 & .0 & 0 & 0 & \text{set } q_2 = 1 \\
\text{Add } -D & + & 1 & 1 & .0 & 1 & 0 & \\
\hline
r_2 & & & 0 & .0 & 1 & 0 & \\
2r_2 & & & 0 & .1 & 0 & 0 & \text{set } q_3 = 1 \\
\text{Add } -D & + & & 1 & .0 & 1 & 0 & \\
\hline
r_3 & & & 1 & .1 & 1 & 0 &
\end{array}
$$

The final remainder is negative, while the dividend is positive. We must correct the final remainder by adding D to r_3, yielding $1.110 + 0.110 = 0.100$, and then correct the quotient:

$$\boxed{Q_{corrected} = Q - ulp}$$

where $Q = 0.111$ and therefore $Q_{corrected} = 0.110_2 = 3/4$. □

Example 3.9

Let $X = (1.101)_2 = -3/8$ and $D = (0.110)_2 = 3/4$. The correct result of this division is $Q = -1/2$ with a zero remainder.

$r_0 = X$			1	.1	0	1	
$2r_0$			1	.0	1	0	set $q_1 = \bar{1}$
Add D	+		0	.1	1	0	
r_1			0	.0	0	0	zero remainder
$2r_1$			0	.0	0	0	set $q_2 = 1$
Add $-D$	+		1	.0	1	0	
r_2			1	.0	1	0	
$2r_2$		1	0	.1	0	0	set $q_3 = \bar{1}$
Add D	+	0	0	.1	1	0	
r_3			1	.0	1	0	

Note that although the final remainder r_3 and the dividend X have the same sign, a correction step is needed, since the quotient we get is $Q = 0.\bar{1}1\bar{1} = 0.\bar{1}01_2 = -3/8$ instead of $-1/2$. We must therefore detect the occurrence of a zero intermediate remainder and correct the final remainder (to obtain a zero remainder):

$$r_3(corrected) = r_3 + D = 1.010 + 0.110 = 0.000$$

We have to then correct the quotient $Q = 0.\bar{1}1\bar{1} = 0.\bar{1}01$ by subtracting ulp, yielding $Q_{corrected} = 0.\bar{1}00_2 = -1/2$. □

SRT DIVISJON

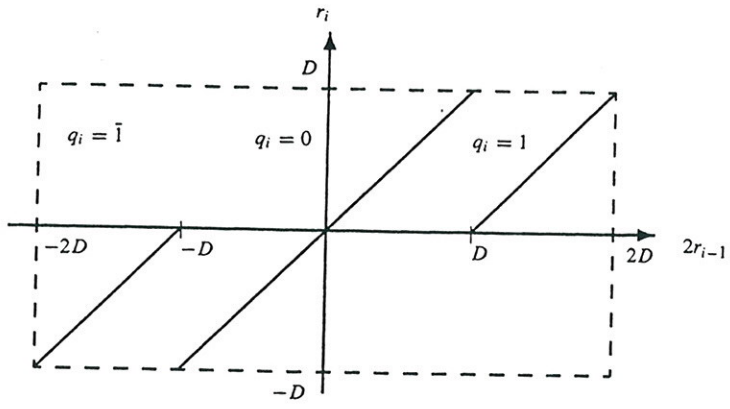

Figure 7.1 Nonrestoring division with $q_i = 0$.

$$q_i = \begin{cases} 1 & \text{if} \quad 2r_{i-1} \ge D \\ 0 & \text{if} \quad -D \le 2r_{i-1} < D \\ \bar{1} & \text{if} \quad 2r_{i-1} < -D \end{cases}$$

- HVIS VI NORMALISERER 'D' TIL Å VÆRE MELLOM $\frac{1}{2} \leq D < 1$ VIL $q_i = 0$ VELGES NÅR $-\frac{1}{2} \leq 2 \cdot r_{i-1} < \frac{1}{2}$ SOM FORENKLER VALGET AV q.

$$q_i = \begin{cases} 1 & \text{if} \quad 2r_{i-1} \geq 1/2 \\ 0 & \text{if} \quad -1/2 \leq 2r_{i-1} < 1/2 \\ \bar{1} & \text{if} \quad 2r_{i-1} < -1/2 \end{cases}$$

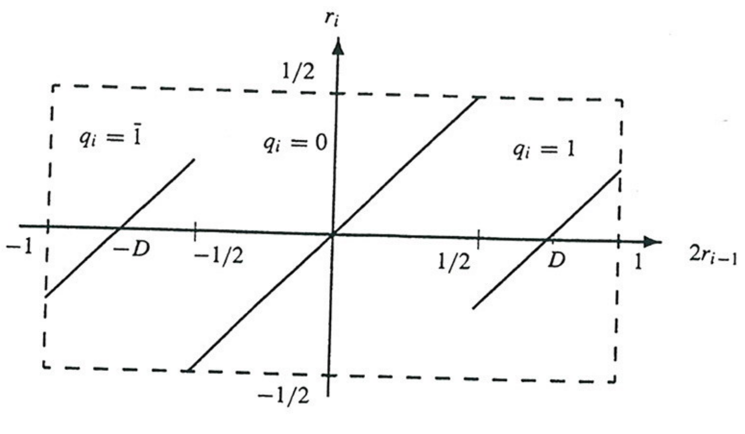

Figure 7.2 SRT division.

- SRT KAN ENKELT UTVIDES TIL Å INKLUDERE NEGATIV DIVISOR I 2'ER COMPLEMENT:

$$q_i = \begin{cases} 0 & \text{if} \quad |2r_{i-1}| < 1/2 \\ 1 & \text{if} \quad |2r_{i-1}| \geq 1/2 \quad \& \quad r_{i-1} \text{ and } D \text{ have the same sign} \\ \bar{1} & \text{if} \quad |2r_{i-1}| \geq 1/2 \quad \& \quad r_{i-1} \text{ and } D \text{ have opposite signs} \end{cases}$$

Example 7.1

Let the dividend X be equal to $(0.0101)_2 = 5/16$ and the divisor D be $(0.1100)_2 = 3/4$. Applying the SRT algorithm yields

$r_0 = X$		0	.0	1	0	1	
$2r_0$		0	.1	0	1	0	$\geq 1/2$ set $q_1 = 1$
Add $-D$	+	1	.0	1	0	0	
r_1		1	.1	1	1	0	
$2r_1 = r_2$		1	.1	1	0	0	$\geq -1/2$ set $q_2 = 0$
$2r_2 = r_3$		1	.1	0	0	0	$\geq -1/2$ set $q_3 = 0$
$2r_3$		1	.0	0	0	0	$< -1/2$ set $q_4 = \bar{1}$
Add D	+	0	.1	1	0	0	
r_4		1	.1	1	0	0	negative remainder & positive X
Add D	+	0	.1	1	0	0	correction
r_4		0	.1	0	0	0	corrected final remainder

The quotient generated before the correction is $Q = 0.100\bar{1}$. This is a minimal representation of $Q = 0.0111$ in *SD* form. In other words, a minimal number of add/subtract operations is performed. After correction, Q becomes $0.0111 - ulp = 0.0110_2 = 3/8$, and the final remainder is $1/2 \cdot 2^{-4} = 1/32$. □