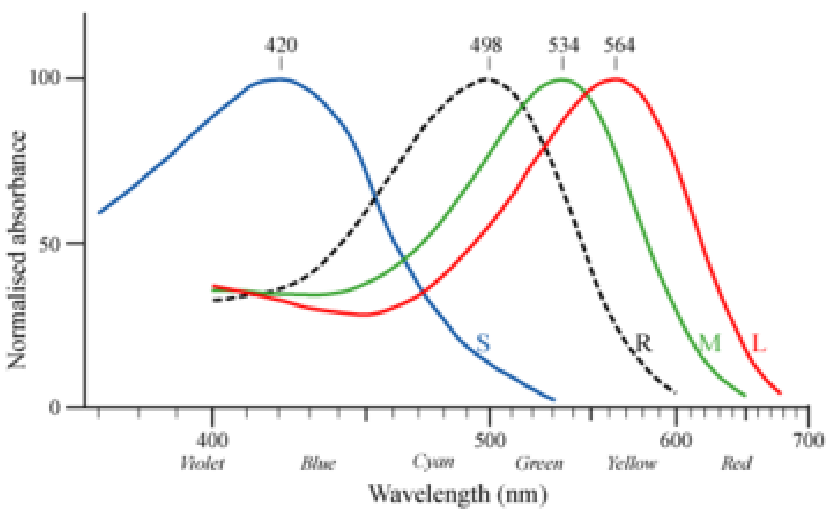
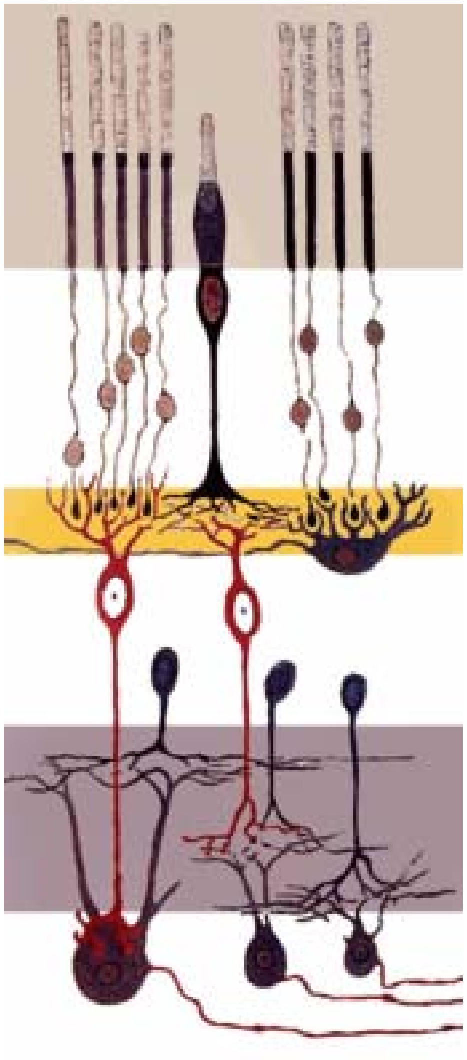
COLOUR THEORY

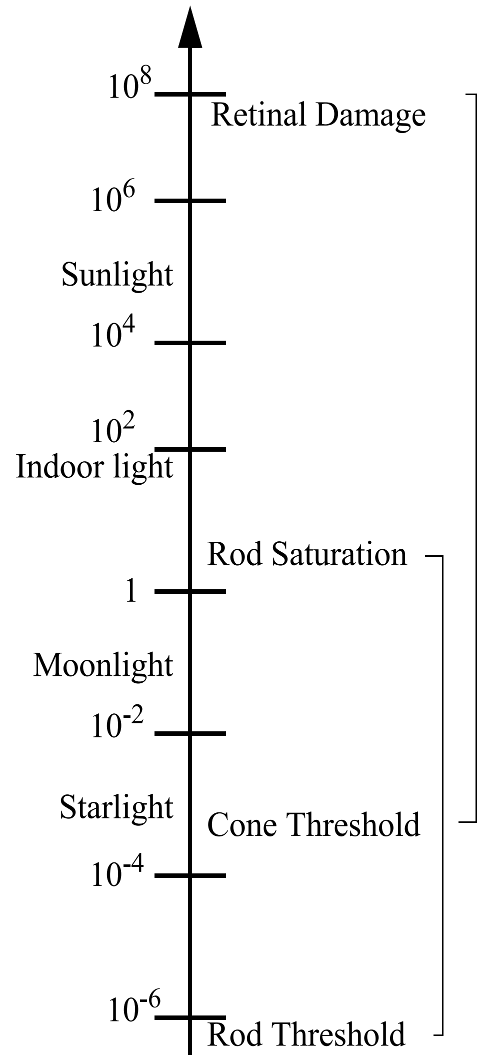
Main ref.: Nakamura

Response of the eye

- The human eye has three kinds of cones S, M, and L which is sensitive to different wavelength ranges: Blue, green, red. Colour view
- Rods has higher sensitivity but for one range only, slightly below the green range. Night vision without colour.







Ref.: www.wikipedia.org

Adaptive and Subtractive Colour Mixing

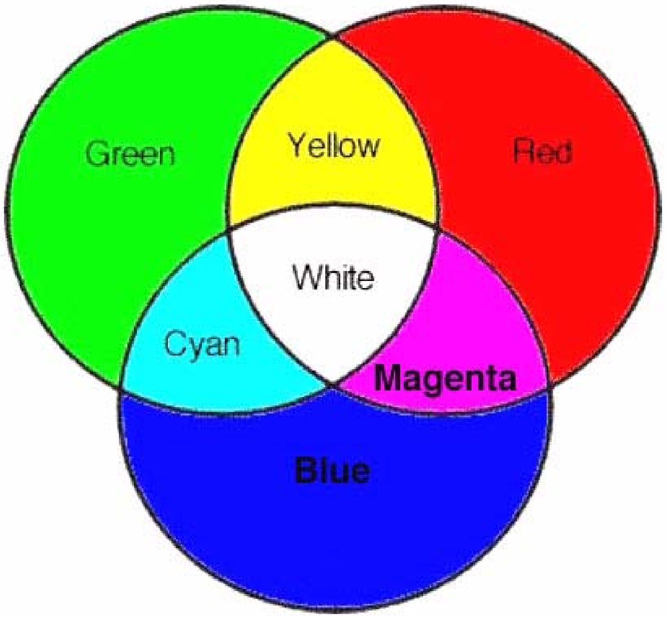


Figure 3. Additive colour mixtures of blue, green and red to produce cyan, magenta, yellow and white.

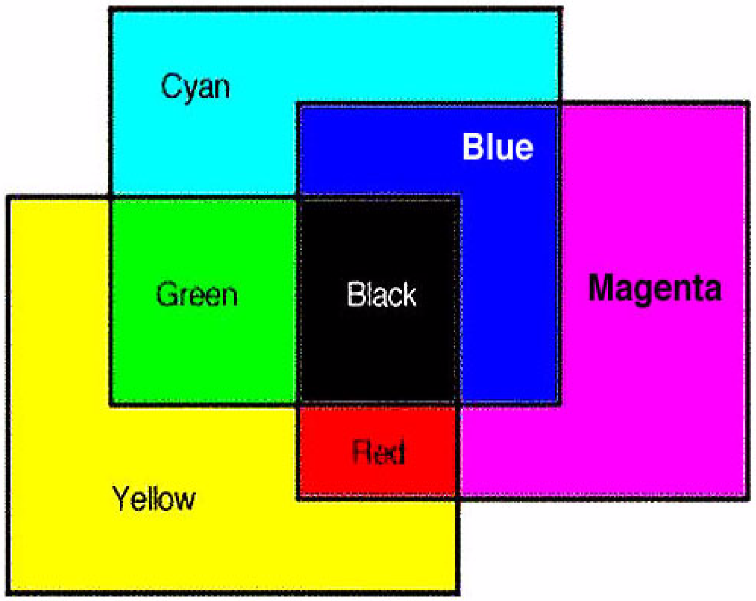


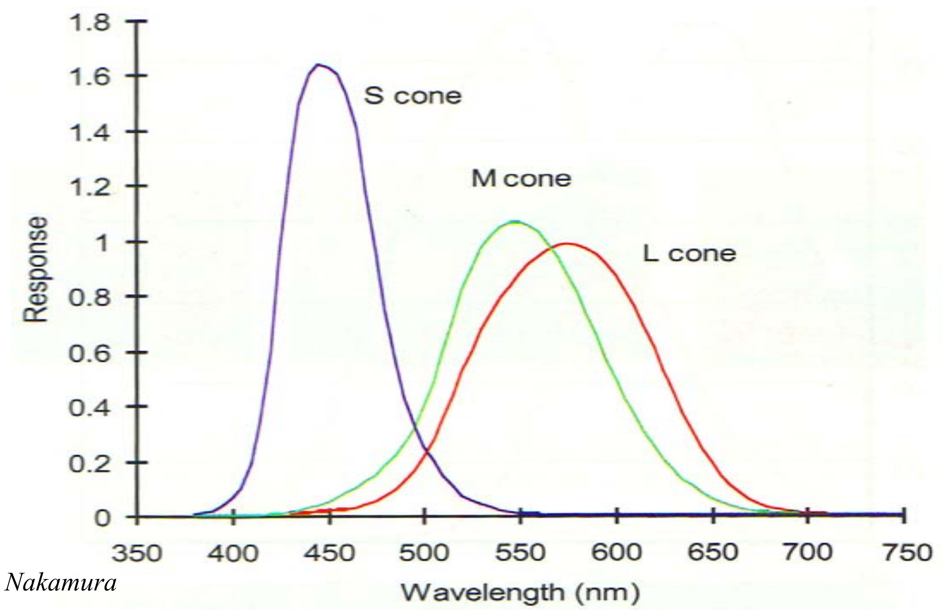
Figure 4. Subtractive colour mixtures of cyan, magenta and yellow to produce blue, green and red.

Ref.: http://webvision.med.utah.edu/KallColor.html

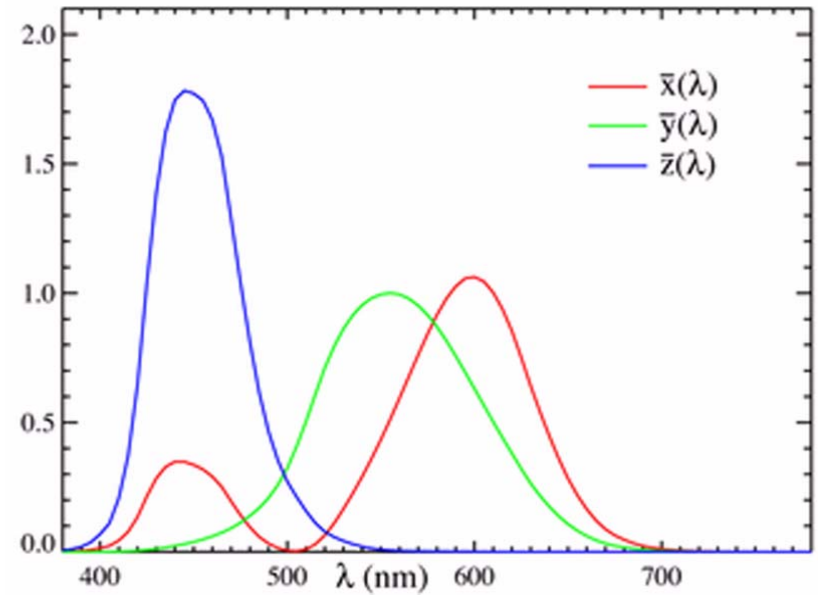
Colour Matching Functions

How to quantify colours

- X(λ), Y(λ), and Z(λ)
- CIE 1931 standard
  (Commission Internationale de l'Eclairage)
  International Commission on Illumination
- The Colour space is associated with the colour sensitivity of the human eye.



Ref. Nakamura



Ref. Wikipedia

Colour Matching Functions (cont.)

Tristimulus values: X, Y, Z is obtained by weighting the source intensity L(λ) and the illuminated object's reflectance R(λ) with the colour matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$.

$$X = \int L(\lambda)R(\lambda)\bar{x}(\lambda)d\lambda$$

$$Y = \int L(\lambda)R(\lambda)\bar{y}(\lambda)d\lambda \qquad \textbf{(7.1)}$$

$$Z = \int L(\lambda)R(\lambda)\bar{z}(\lambda)d\lambda$$

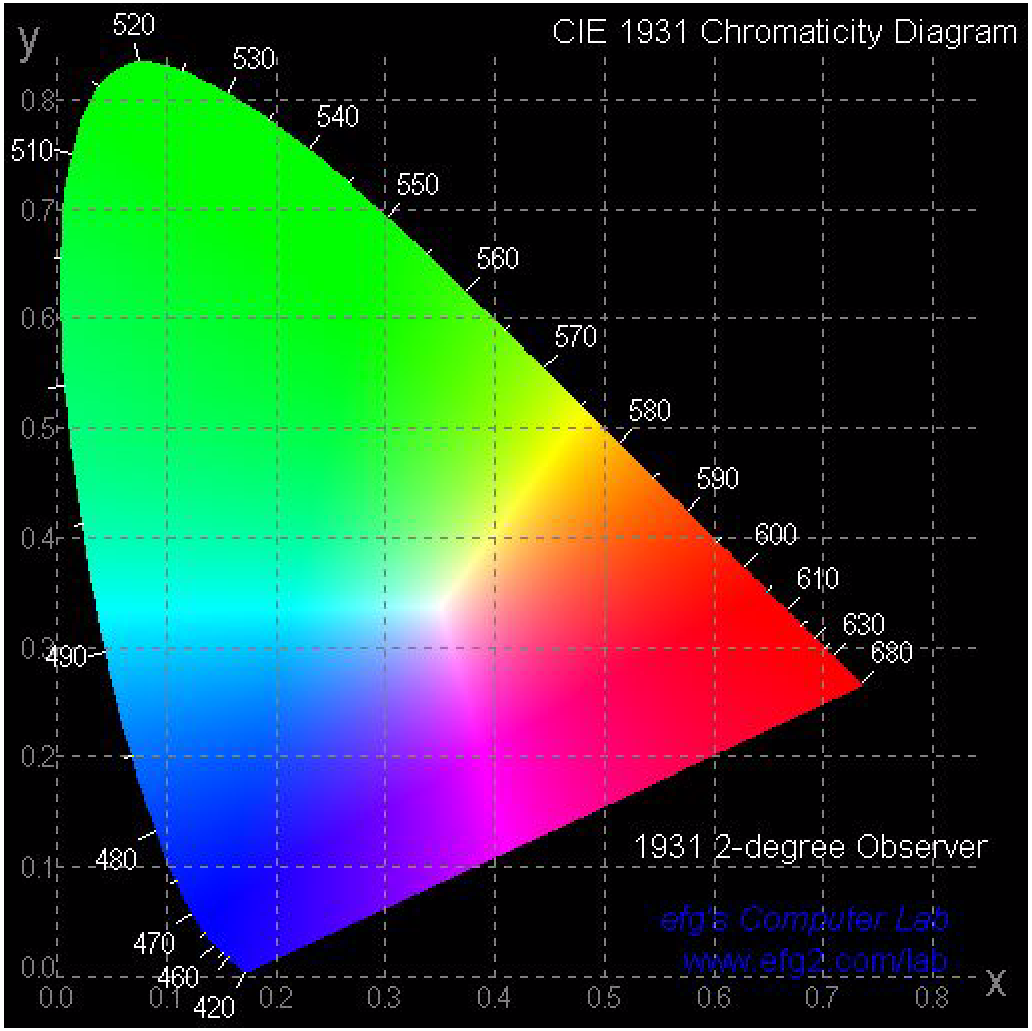
Luminance: Y

Chrominance: x, y
values is represented in a 2 dimensional space.

$$x = \frac{X}{X+Y+Z} \qquad y = \frac{Y}{X+Y+Z} \qquad \textbf{(7.2)}$$

Converting back to X, Z

$$X = \frac{Y}{y}x \qquad Z = \frac{Y}{y}(1-x-y) \qquad \textbf{(7.3)}$$



Ref.: efg'a Computer Lab

Colour Matching Functions (cont.)

Improved geometric uniformity is achieved with the standard CIE 1976: L*u*v* and L*a*b*.
Developed for gray background (L*=50)

Common L*:

$$L^* = \begin{cases} 116\left(\frac{Y}{Y_0}\right)^{1/3} - 16 & \frac{Y}{Y_0} > 0.008856 \\ 903.29\left(\frac{Y}{Y_0}\right) & \frac{Y}{Y_0} \le 0.008856 \end{cases} \quad \textbf{(7.4)}$$

$$u^* = 13L^*(u' - u'_0) \qquad v^* = 13L^*(v' - v'_0) \quad \textbf{(7.5)}$$

where

$$u' = \frac{4X}{X + 15Y + 3Z} \qquad v' = \frac{9Y}{X + 15Y + 3Z} \quad \textbf{(7.6)}$$

Index 0 represents tristimulus values for white

$$a^* = 500(X_n - Y_n) \qquad b^* = 200(Y_n - Z_n) \quad \textbf{(7.7)}$$

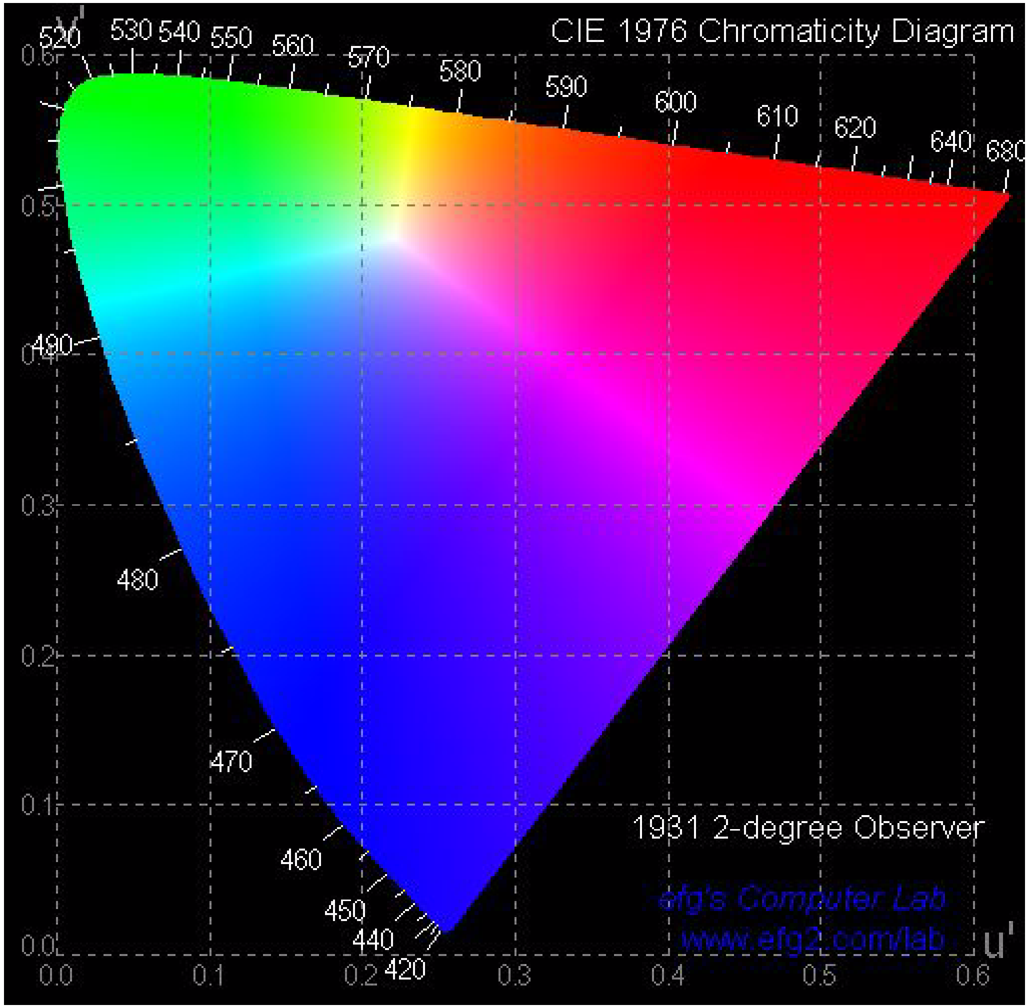
where

$$X_n = \begin{cases} \left(\frac{X}{X_0}\right)^{1/3} & \frac{X}{X_0} > 0.008856 \\ 7.787\left(\frac{X}{X_0}\right) + \frac{16}{116} & \frac{X}{X_0} \le 0.008856 \end{cases} \quad \textbf{(7.8)}$$

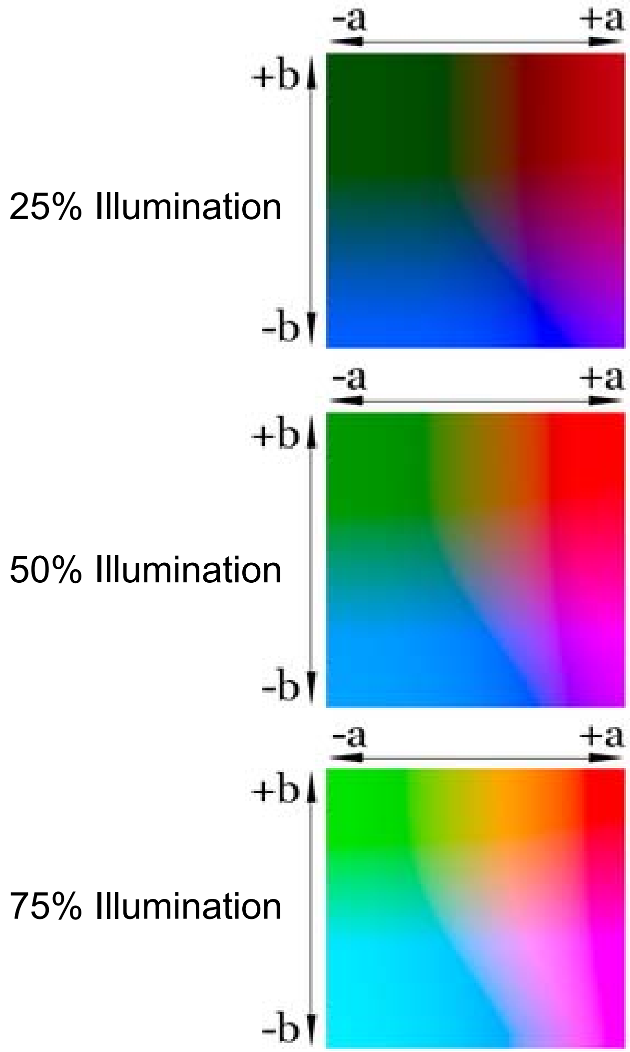
$$Y_n = \begin{cases} \left(\frac{Y}{Y_0}\right)^{1/3} & \frac{Y}{Y_0} > 0.008856 \\ 7.787\left(\frac{Y}{Y_0}\right) + \frac{16}{116} & \frac{Y}{Y_0} \le 0.008856 \end{cases} \quad \textbf{(7.9)}$$

$$Z_n = \begin{cases} \left(\frac{Z}{Z_0}\right)^{1/3} & \frac{Z}{Z_0} > 0.008856 \\ 7.787\left(\frac{Z}{Z_0}\right) + \frac{16}{116} & \frac{Z}{Z_0} \le 0.008856 \end{cases} \quad \textbf{(7.10)}$$

L*a*b* (CIE1976)



Ref.: efg'a Computer Lab



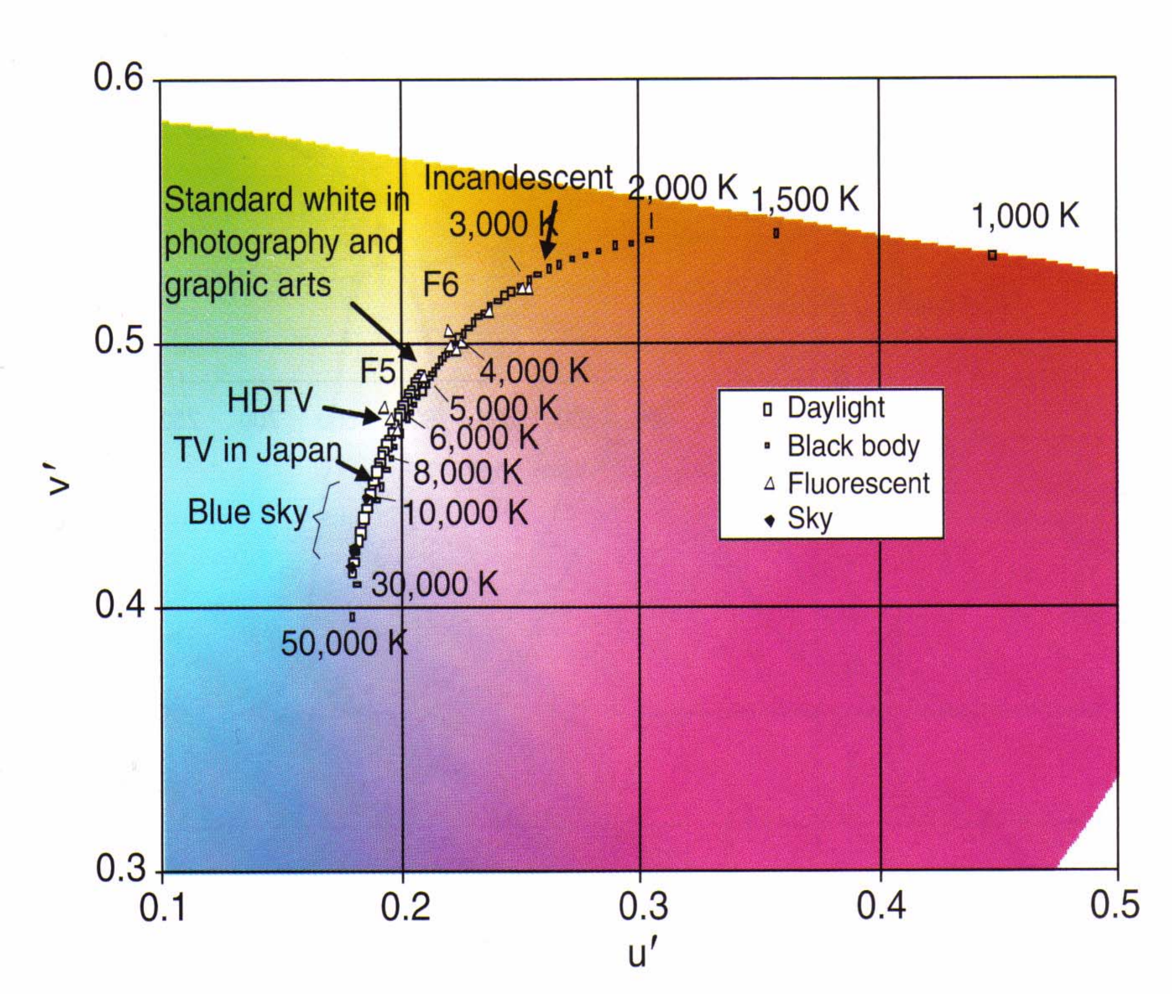
L*C*H* (CIECAM02)

Ref.: Wikipedia

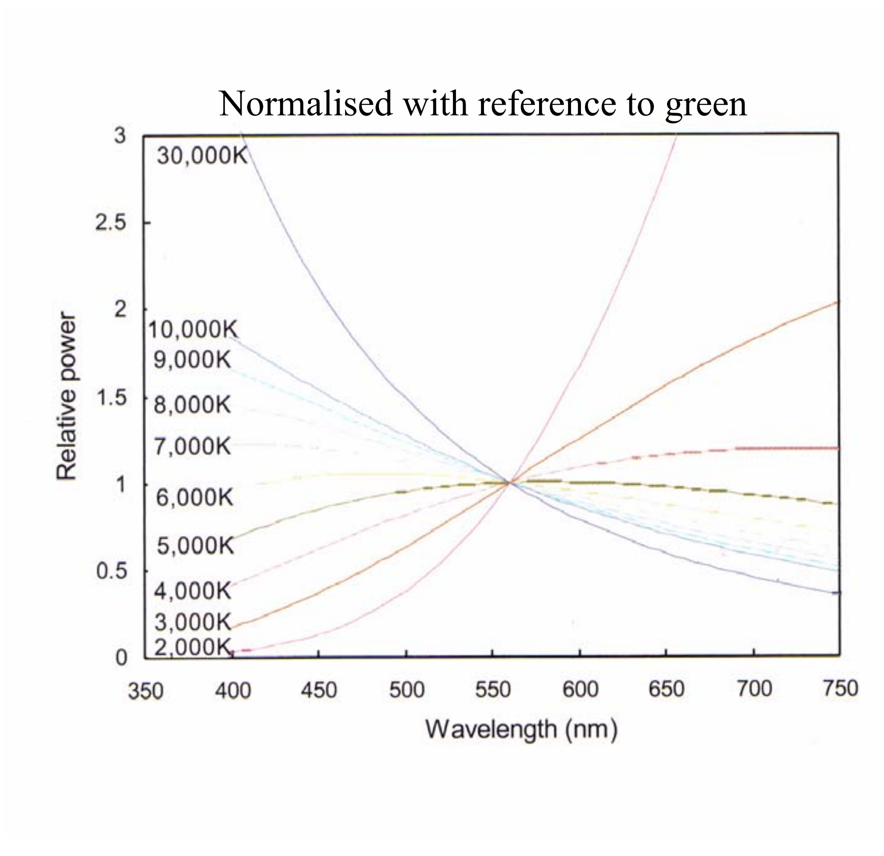
L* as in (7.4).

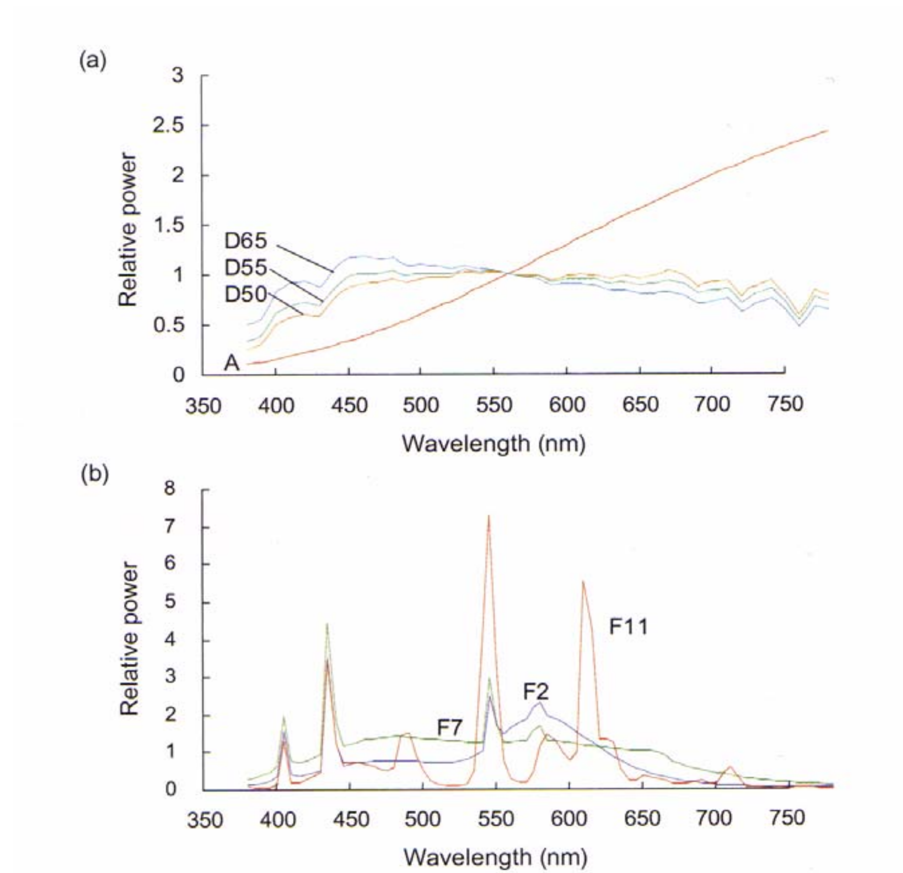
$$C_{ab}^* = \sqrt{(a^{*2} + b^{*2})} \qquad h_{ab} = \frac{180^o}{\pi}\,\mathbf{atan}\left(\frac{b^*}{a^*}\right) \qquad \mathbf{(7.11)}$$

Colour temperature of light sources



Colour temperature of light sources (cont.)





Equal colour temperature is not the same as equal spectrum

Colour Rendering index (CRI)

A measure on a light source capability to reproduce colour.

- 100 is best, 0 is worst
- The light source is measured against a reference, a filament lamp (blackbody radiation) with CRI=100.
- A standardized set of colours are illuminated and compared.
- The reference is daylight for colour temperatures above 5000 °K.

Average colour rendering index R_a is calculated according to formula

$$R_a = \frac{1}{8}\sum_{i=1}^{8}(100 - 4,6\Delta E_i)$$

Where ΔE_i is the colour deviation in a given colour space, e.g. 1964 L*U*V*

$$\Delta E_i = \sqrt{(L_i - L_{i0})^2 + (U_i - U_{i0})^2 + (V_i - V_{i0})^2}$$

Index i gives the colour in the standard set, index 0 represents the reference source.

Ref.: Wikipedia

Colour Rendering index (forts.)

TABLE 7.1
Color Rendering Index for Some Light Sources

Sample light source			F2	F7	F11	A	D65	D50	D55
Chromaticity	x		0.3721	0.3129	0.3805	0.4476	0.3127	0.3457	0.3324
	y		0.3751	0.3292	0.3769	0.4074	0.3290	0.3585	0.3474
Reference light source (P: black body; D: daylight)			P	D	P	P	D	D	D
Correlated color temperature			4200	6500	4000	2856	6500	5000	5500
Average color rendering index	Ra		64	90	83	100	100	100	100
Special color rendering index	R1	7.5 R 6/4	56	89	98	100	100	100	100
	R2	5 Y 6/4	77	92	93	100	100	100	100
	R3	5 GY 6/8	90	91	50	100	100	100	100
	R4	2.5 G 6/6	57	91	88	100	100	100	100
	R5	10 BG 6/4	59	90	87	100	100	100	100
	R6	5 PB 6/8	67	89	77	100	100	100	100
	R7	2.5 P 6/8	74	93	89	100	100	100	100
	R8	10 P 6/8	33	87	79	100	100	100	100
	R9	4.5 R 4/13	−84	61	25	100	100	100	100
	R10	5 Y 8/10	45	78	47	100	100	100	100
	R11	4.5 G 5/8	46	89	72	100	100	100	100
	R12	3 PB 3/11	54	87	53	100	100	100	100
	R13	5 YR 8/4	60	90	97	100	100	100	100
	R14	5 GY 4/4	94	94	67	100	100	100	100
	R15	1 YR 6/4	47	88	96	100	100	100	100

Camera Colour Properties

To generate tristimulus values unlikely, these have to be a linear combination of the camera's sensitivity functions:

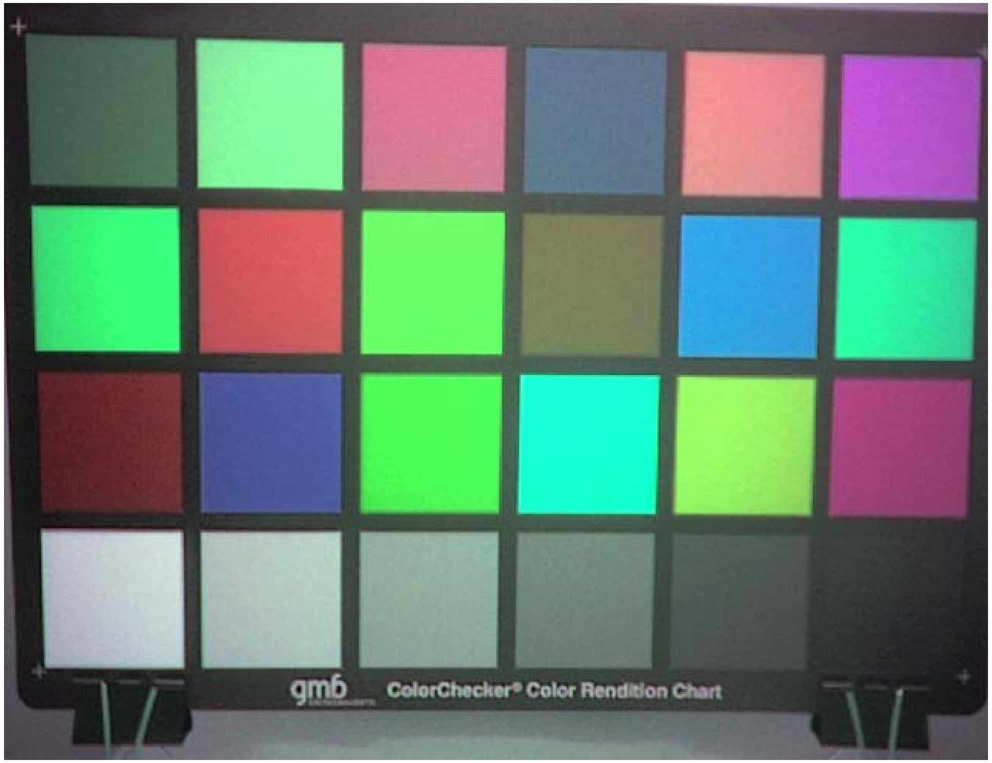
$X=c_{11}S_x + c_{12}S_y + c_{13}S_z$
$Y=c_{21}S_x + c_{22}S_y + c_{23}S_z$
$Z=c_{31}S_x + c_{32}S_y + c_{33}S_z$

Called the Luther condition.

Provided the reflectance has a continuous transitions in the spectrum such that the reflected spectrum can be decomposed into three basic components, an object's tristimulus values can be calculated using any set of sensitivity functions.

Colour chart for test is made to simulate the reflectance of real objects.

The camera colour functions can be generated from the chart. Either an analytical or recursive method.



Gretag Macbeth colour checker

Camera Colour Properties (cont.)

The target:

$$T = \begin{bmatrix} X_1 & \cdots & X_i & \cdots & X_n \\ Y_1 & \cdots & Y_i & \cdots & Y_n \\ Z_1 & \cdots & Z_i & \cdots & Z_n \end{bmatrix}$$

Estimated value: Compensation Camera response **(7.12)**

$$\hat{T} = \begin{bmatrix} \hat{X}_1 & \cdots & \hat{X}_i & \cdots & \hat{X}_n \\ \hat{Y}_1 & \cdots & \hat{Y}_i & \cdots & \hat{Y}_n \\ \hat{Z}_1 & \cdots & \hat{Z}_i & \cdots & \hat{Z}_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} r_1 & \cdots & r_i & \cdots & r_n \\ g_1 & \cdots & g_i & \cdots & g_n \\ b_1 & \cdots & b_i & \cdots & b_n \end{bmatrix} = A \cdot S$$

Solving for the compensation matrix:

$$A = T \cdot S^T \cdot (S \cdot S^T)^{-1}$$

Camera Colour Properties (cont.)

Recursive method to minimize colour differences:

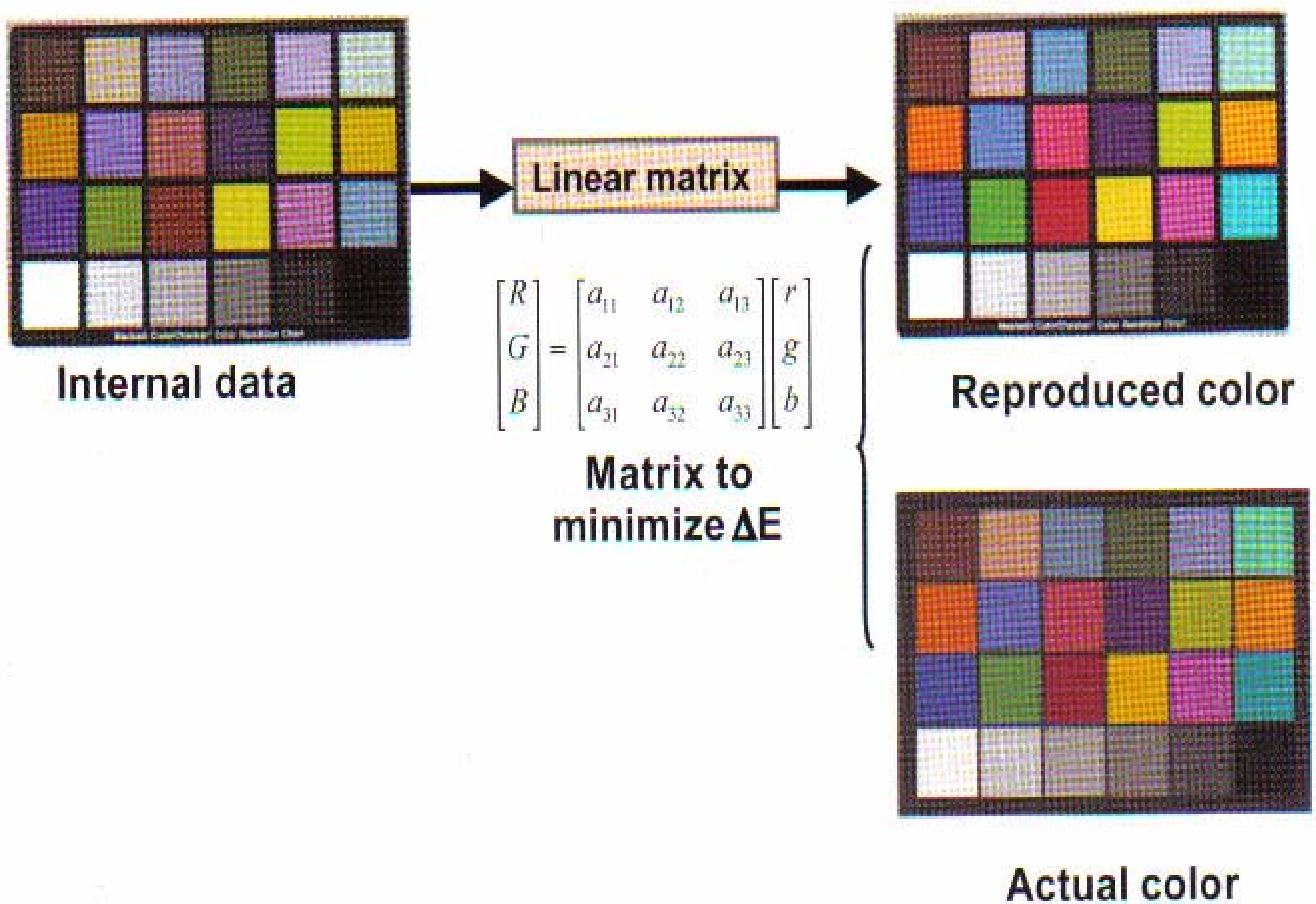
$$J = \sum_{i=1}^{n} w_i \Delta E(X_i, Y_i, Z_i, \hat{X}_i, \hat{Y}_i, \hat{Z}_i) \quad \textbf{(7.13)}$$

Where ΔE is the direct colour difference (e.g. L*a*b)

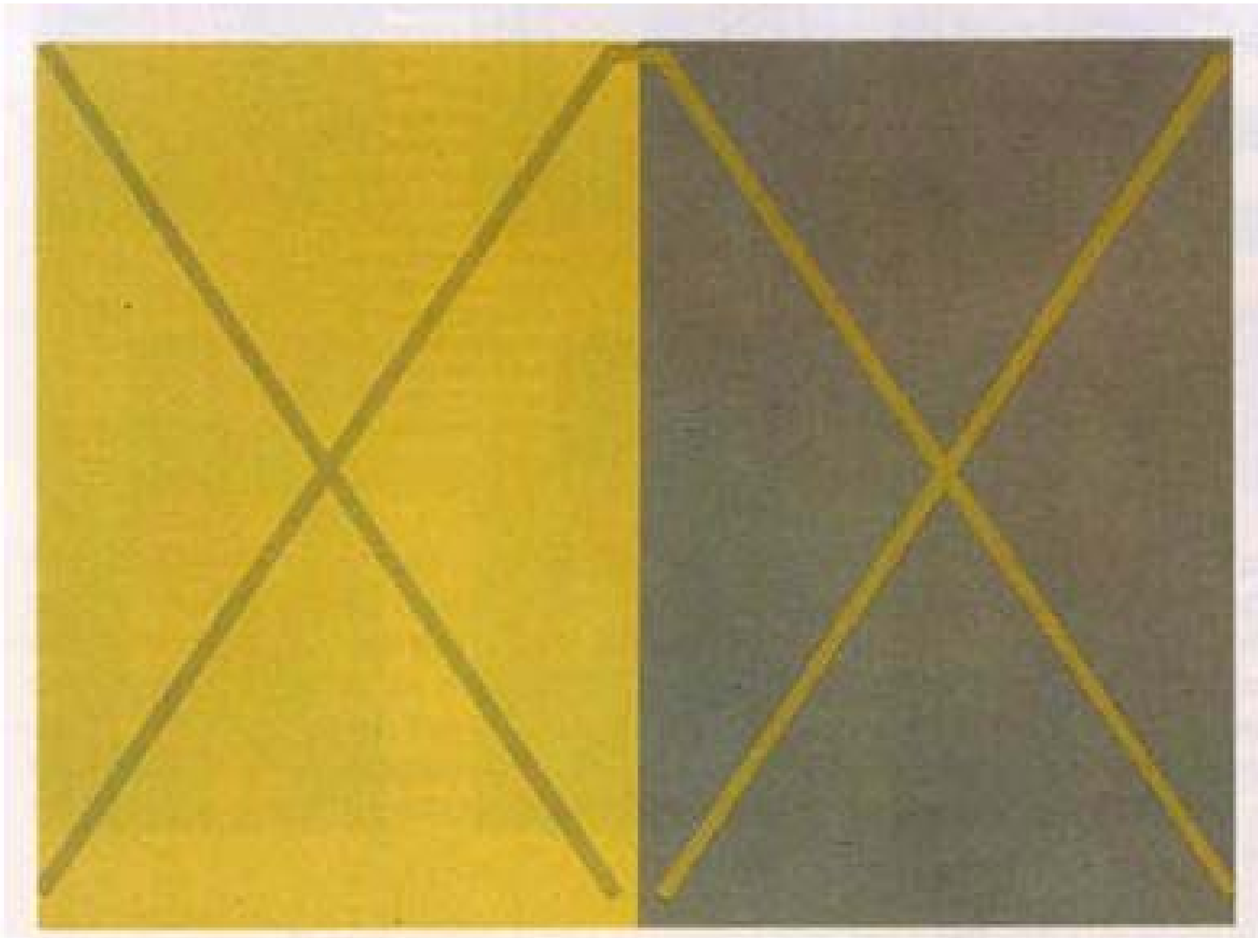
$$\Delta E_{ab}{}^{*} = \sqrt{(L_1{}^{*} - L_2{}^{*})^2 + (a_1{}^{*} - a_2{}^{*})^2 + (b_1{}^{*} - b_2{}^{*})^2} \quad \textbf{(7.14)}$$

J: Visible colour difference (deviation form the object’s colour)

w_i: Coefficient associated to each colour on the palette.



$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$



http://neuroanthropology.net/2008/09/04/colour-is-it-in-the-brain/

White Balance

White act as the reference for the colours, i.e. the colour of the light source. The eye adjusts the sensitivity of cones to adapt to the lightning conditions.

The question is: What is white in the image? There are several methods.

- Scene average.
  Assuming the “world is grey” and that the average colour is the colour of the light source.
- The brightest spot is white.
  Assuming the light that is reflected by the brightest elements represents the light source.
- Gamut of the scene (colour space).
  Assuming the total spectrum has a expected distribution, analyse the spectrum from all part of the scene and move the colours towards the highest correlation between expected and measured distribution.

White Balance - Chromatic adaption

von Kries model:

Images are viewed in a background illumination different form the recording background light.
Cone response at background illumination 1, is tristimulus values L_1,M_1,S_1 at the state white = L_{w1},M_{w1},S_{w1}
Cone response at background illumination 2, is tristimulus values L_2,M_2,S_2 at the state white = L_{w2},M_{w2},S_{w2}

$$L_2 = \frac{L_{w2}}{L_{w1}} L_1 \qquad M_2 = \frac{M_{w2}}{M_{w1}} M_1 \qquad S_2 = \frac{S_{w2}}{S_{w1}} S_1$$

New r’ b’ g’ values is calculated from r g b by:

$$\begin{bmatrix} r' \\ g' \\ b' \end{bmatrix} = A^{-1}B^{-1}\begin{bmatrix} \frac{L'_w}{L_w} & 0 & 0 \\ 0 & \frac{M'_w}{M_w} & 0 \\ 0 & 0 & \frac{S'_w}{S_w} \end{bmatrix} BA\begin{bmatrix} r \\ b \\ g \end{bmatrix}, \qquad \begin{bmatrix} L \\ M \\ S \end{bmatrix} = BA\begin{bmatrix} r \\ g \\ b \end{bmatrix} \tag{7.15}$$

A: Sensor RGB -> Tristimulus values XYZ
B: Tristimulus values XYZ -> cone response LMS.

White Balance - Colour constancy

Subjective properties of the eye ensures that an object's colour stays nearly unchanged at varying light conditions (needed for recognition of objects). It is however required that the spectrum of the light has a certain width, that Ra (average colour rendering index) is high.

A simple method is to use (7.15) with the adaption of B to a standard light source.

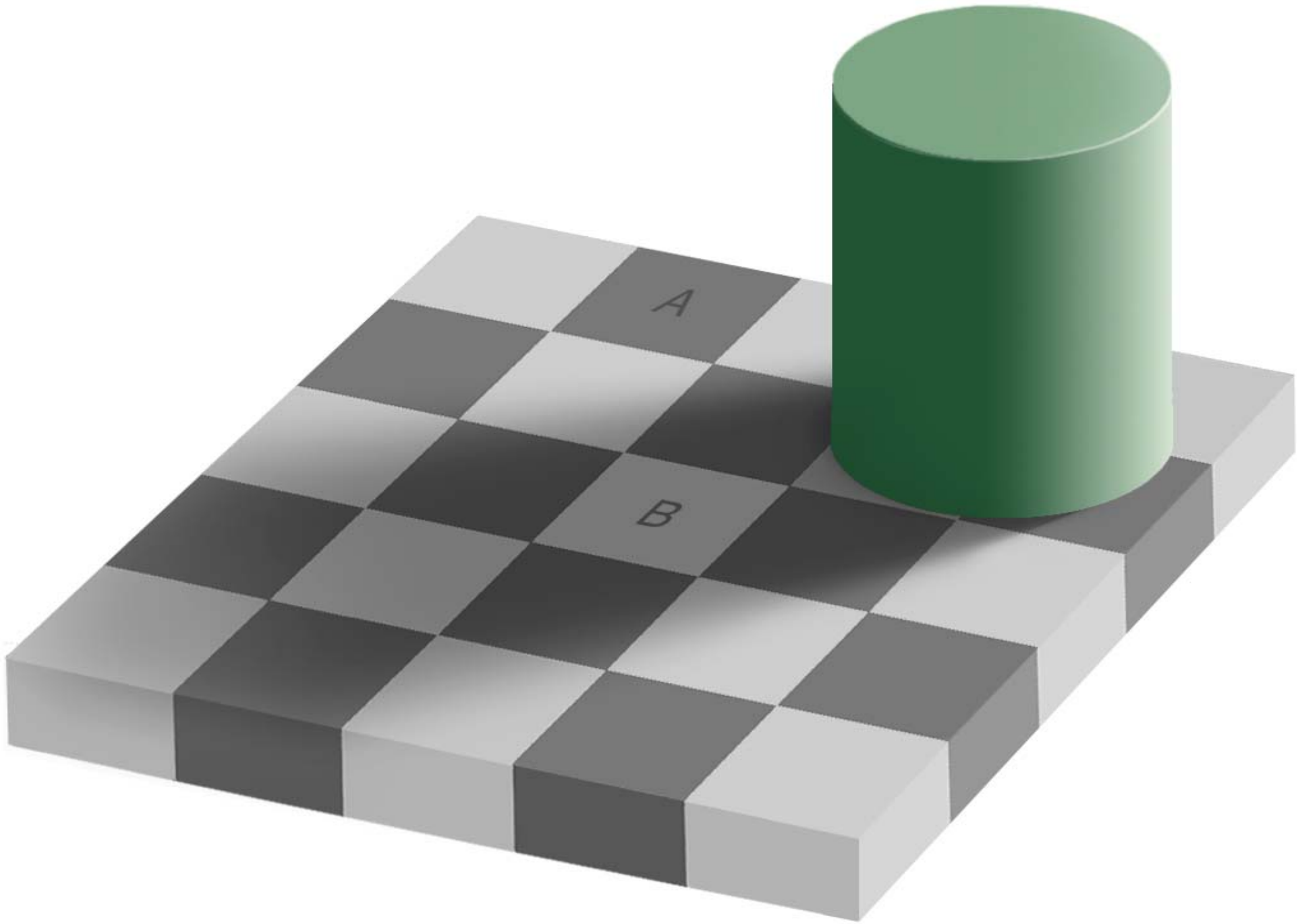
http://en.wikipedia.org/wiki/colour_constancy

Perception of colour depend on the colour of the background

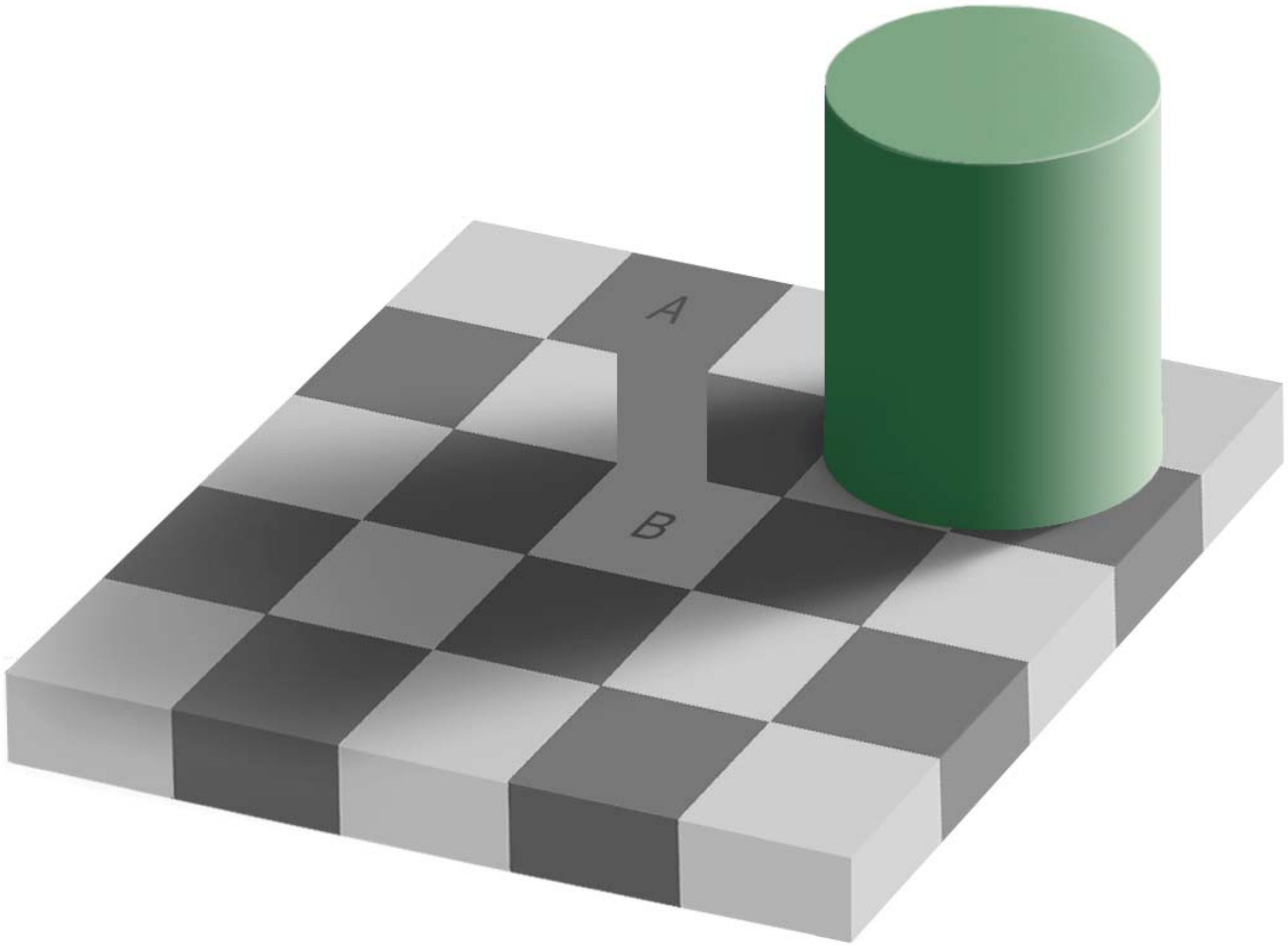
The pink sheet in top image looks more like white in the bottom image.

http://en.wikipedia.org/wiki/colour_constancy

A is at the same grey level as B?



http://en.wikipedia.org/wiki/colour_constancy

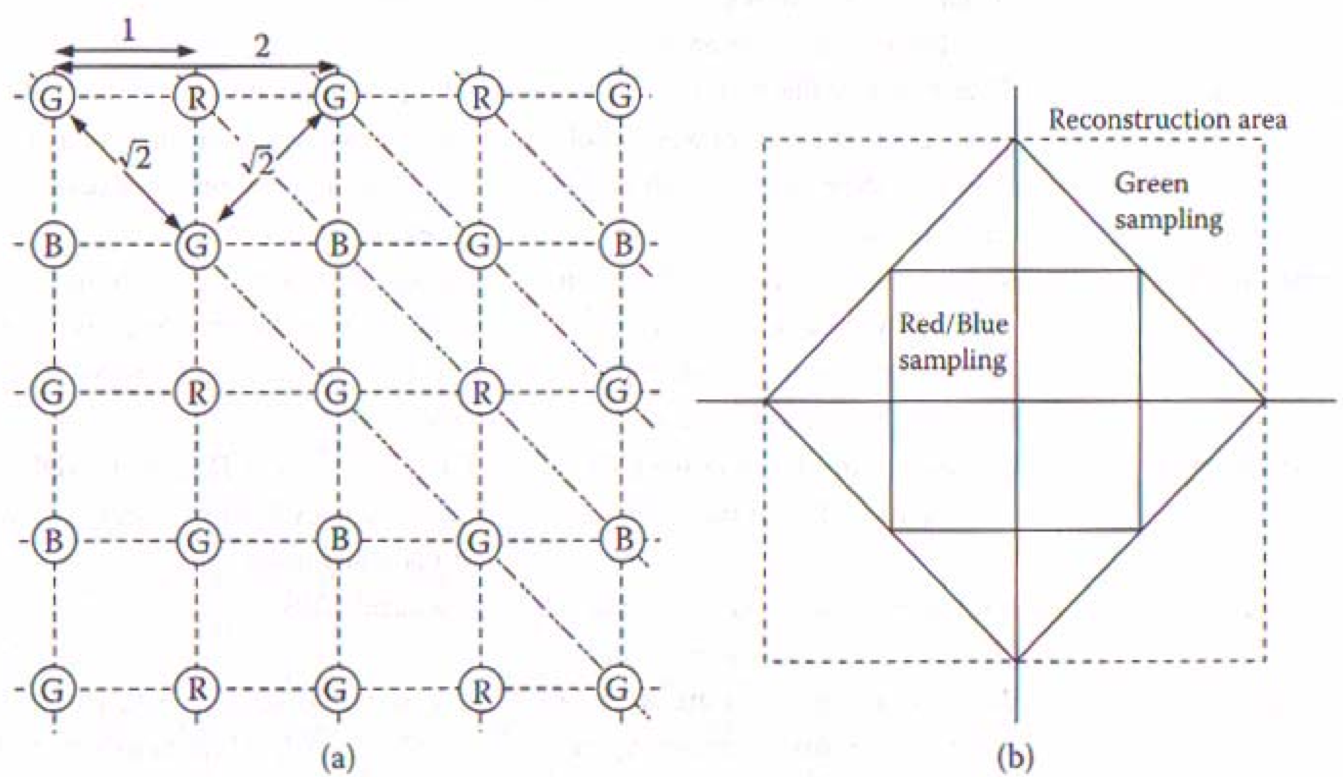


http://en.wikipedia.org/wiki/colour_constancy

Colour Interpolation

<u>Sampling of Bayer pattern CFA (Colour Filter Array)</u>

Red and blue distance = 2. Green distance = $\sqrt{2}$. Nyquist frequency is 1.4x higher for green, in the directions 45° and 135°



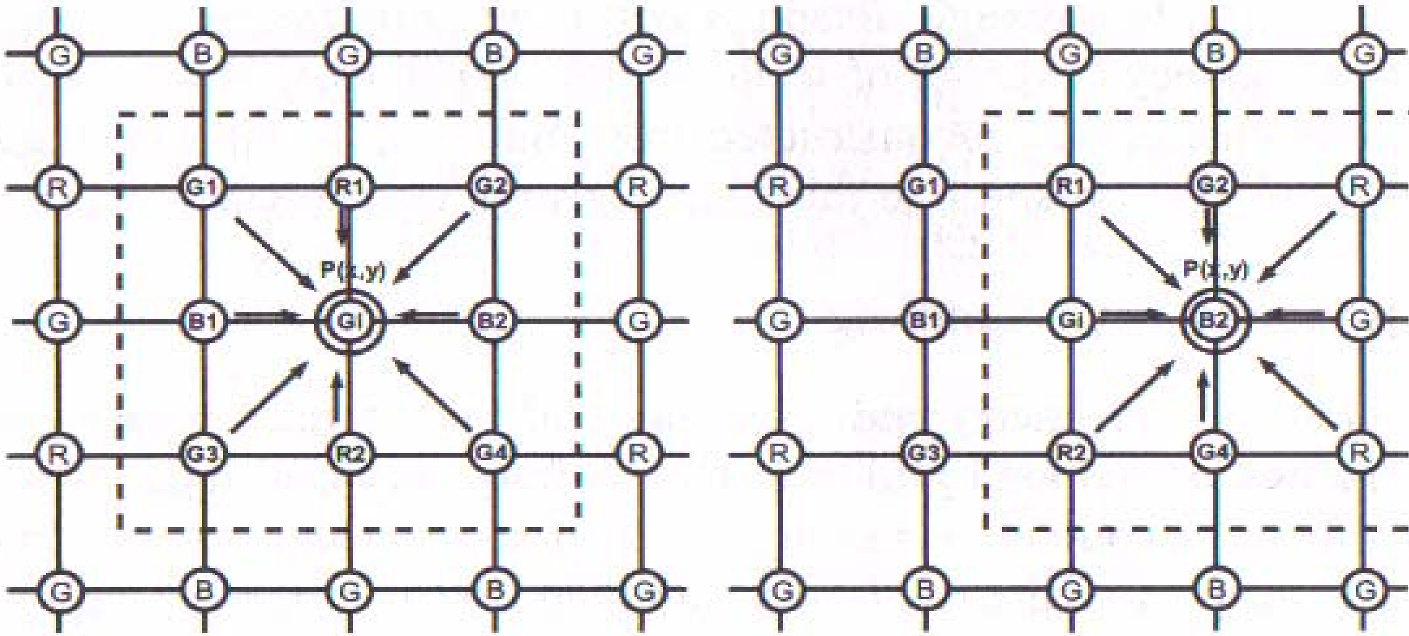
Bilinear Interpolation

Interpolation: Insert a new point with values based on averaging of the neighbouring points, i.e. spatial low pass filtering of the data.

Position 1: $G(x_1,y_1)|_R = [R(x_1,y_1+1) + R(x_1,y_1-1)]/2$
$G(x_1,y_1)|_B = [B(x_1+1,y_1) + B(x_1-1,y_1)]/2$

Position 2: $B(x_2,y_2)|_G = [G(x_2,y_2+1) + G(x_2,y_2-1) + G(x_2+1,y_2) + G(x_2-1,y_2)]/4$
$B(x_2,y_2)|_R = [G(x_2+1,y_2+1) + G(x_2+1,y_2-1) + G(x_2-1,y_2+1) + G(x_2-1,y_2-1)]/4$

Corresponding for red pixel.



(a) Interpolation at point Gi (b) Interpolation at point B2

Colour correction

$$\begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad \textbf{(7.16)}$$

The coefficients at the diagonal adjust the relation between R,G and B, white balance, directly.
The matrix elements outside the diagonal can be used to compensate for colour crosstalk.

As the eye has lower spatial response for chrominance than for luminance, separation of these is advantageous.
This can be done by converting RGB to YC_bC_r.
This is especially useful in data compression.

The ITU standard:

$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 0.2988 & 0.5869 & 0.1143 \\ -0.1689 & -0.3311 & 0.5000 \\ 0.5000 & -0.4189 & -0.0811 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}, \qquad \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1.0000 & 0.00000 & 1.40200 \\ 1.0000 & -0.34410 & -0.71410 \\ 1.0000 & 1.77200 & -0.00015 \end{bmatrix} \begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} \quad \textbf{(7.17)}$$

Fixed point representation:

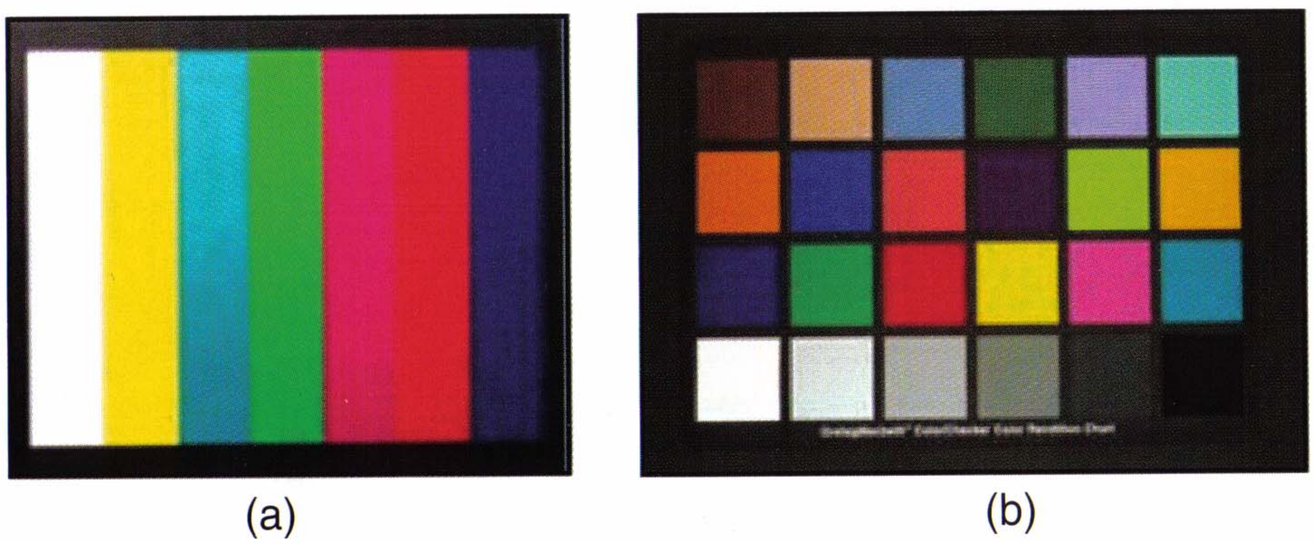
$$\begin{bmatrix} Y \\ C_b \\ C_r \end{bmatrix} = \begin{bmatrix} 306 & 601 & 117 \\ -173 & -339 & 512 \\ 512 & -429 & -83 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad \textbf{(7.18)}$$

Colour Reproduction

Method: Recording of Macbeth colour checker with a well defined reflectance for each colour, converting to Chrominance (and luminance using tristimulus functions. Quantifying deviation from expected Chrominance values for each colour.

Our perception of colour depends on the illuminance, background light, and some properties of our vision system that are not caught by the tristimulus functions.

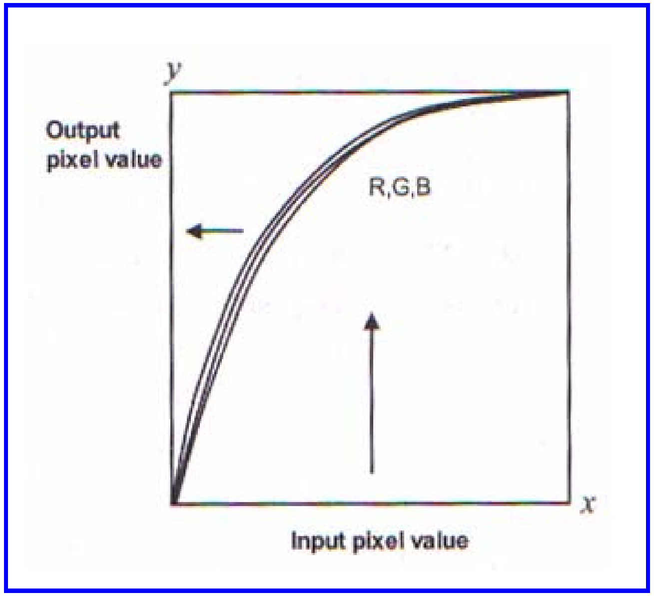
Therefore the latest adjustment is done by observing the chart plus some typical objects.



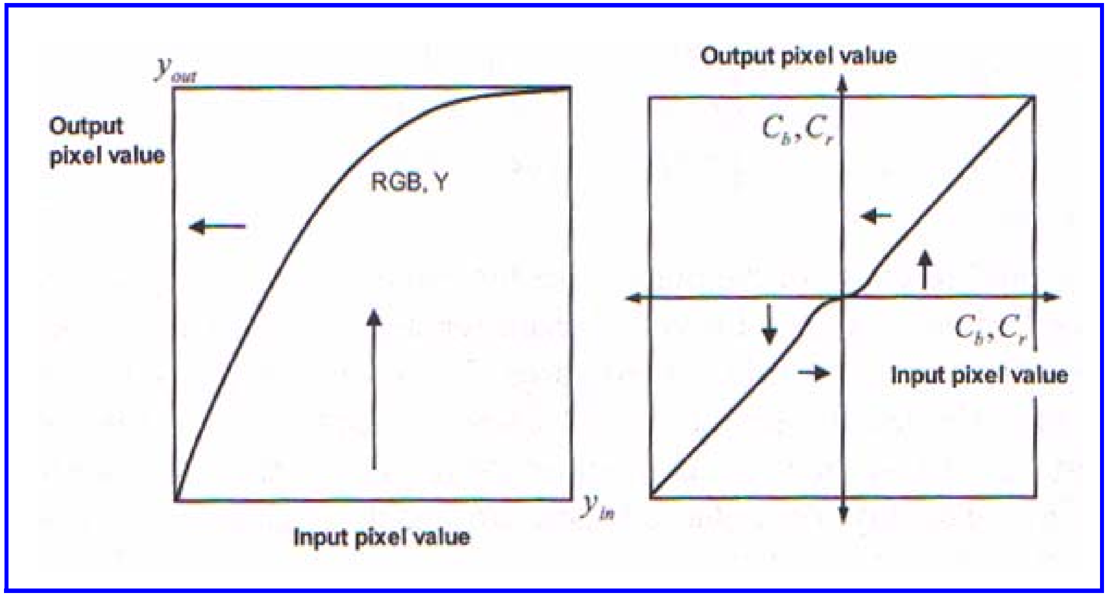
COLOR FIGURE 10.8 Test chart for measuring color reproduction: (a) color bar chart; (b) Macbeth's color chart.

Gamma Correction and Hue Correction

Must take into account the non-linear response, the form of $y=x^{\gamma}$. For example, the Cathode Ray Tube (CRT) has a typical gamma of 0.45. The transform from one response to another is the gamma correction. Can be done in both the RGB space or in the YC_bC_r space.



RGB



YC_bC_r

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http://en.wikipedia.org/wiki/CIE_1931_color_space
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http://en.wikipedia.org/wiki/colour_space
http://en.wikipedia.org/wiki/colour_constancy

efg's Computer Lab

www.efg2.com/Lab