2.2.7 DEFINITION

Suppose that A and B are sets. Then $A = B$ if $A \subseteq B$ and $B \subseteq A$.

A word about definitions: Although definitions are written in the same form as theorems, they are fundamentally different. Suppose we have the following definition: "If X is a clacking waggler and all subsets of X are marint, then X is a **supreme clacking waggler.**" Because we are giving a definition, we are saying *exactly* what we mean by supreme clacking waggler. We are not just describing some possibility for supreme clacking waggler. The statements

> *"X is a supreme clacking waggler,"* and
> *"X is a clacking waggler and all subsets of X are marint"*

mean *exactly the same thing.* Definitions are always equivalences. By convention, the if and only if is understood and never said explicitly.

2.3 Set Operations

The axioms of set theory allow us to "build new sets from old ones." They tell us that any subset of a set that we have at our disposal is also a set. They also allow us to take unions, intersections, and complements of sets.

2.3.1 DEFINITION

Let U be a set. Let $S \subseteq U$. Define

$$S_U^{\mathrm{c}} = \{x \in U : x \notin S\}.$$

The set S_U^{c} is called the **complement** of S in U. If the set U is understood, we may just call S_U^{c} "the complement of S" and denote it by S^{c}.

(For technical reasons having to do with set-theoretic paradoxes, complements must always be taken relative to a larger set—see Section 2.6. In the absence of the set U, S^{c} makes no sense.)

2.3.2 EXERCISE

Consider the intervals $U = [-5, 5]$ and $S = [-5, 2]$. Find S_U^{c}. □

2.3.3 EXERCISE

As in Exercise 2.3.2, let $S = [-5, 2]$. What is $S_{\mathbb{R}}^{\mathrm{c}}$? □