2.3.13 DEFINITION (Unions and intersections, revisited)

Suppose we have a collection of sets $\{B_\alpha\}_{\alpha\in\Lambda}$.

1. The union of all the sets is denoted by $\bigcup_{\alpha\in\Lambda} B_\alpha$, which we read as "the union over alpha in lambda of the B-alpha's." An element x is in $\bigcup_{\alpha\in\Lambda} B_\alpha$ if $x \in B_\alpha$ for some $\alpha \in \Lambda$. That is,
$$\bigcup_{\alpha\in\Lambda} B_\alpha = \{x : x \in B_\alpha \text{ for some } \alpha \in \Lambda\}.$$
2. The intersection of all the sets is $\bigcap_{\alpha\in\Lambda} B_\alpha$, which we read as "the intersection over alpha in lambda of the B-alpha's." An element x is in $\bigcap_{\alpha\in\Lambda} B_\alpha$ if $x \in B_\alpha$ for all $\alpha \in \Lambda$. That is,
$$\bigcap_{\alpha\in\Lambda} B_\alpha = \{x : x \in B_\alpha \text{ for all } \alpha \in \Lambda\}.$$

2.3.14 EXERCISE

Let $\Lambda = \{1, 2, 3\}$. Do the definitions of union and intersection given in Definition 2.3.13 correspond to the one that you gave in Exercise 2.3.10? What if $\Lambda = \{1, 2, 3, 4\}$? □

2.3.15 EXERCISE

1. Let $\{I_n\}_{n\in\mathbb{N}}$ be the collection of intervals described in Example 2.3.11.

   (a) Find $\bigcup_{n\in\mathbb{N}} I_n$. (b) Find $\bigcap_{n\in\mathbb{N}} I_n$.

   How would your answer be different if the intervals were open intervals instead of closed intervals?
2. Let $\{C_t\}_{t\in\mathbb{R}}$ be the collection of circles described in Example 2.3.11.

   (a) Find $\bigcup_{t\in\mathbb{R}} C_t$. (b) Find $\bigcap_{t\in\mathbb{R}} C_t$. □

2.4 The Algebra of Sets

In this section you will be asked to prove some important set-theoretic identities. To prove each identity you will need to prove the equality of two sets. Recall that two sets are equal if each is a subset of the other. Suppose that we wish to prove that $X = Y$; then we must prove that $X \subseteq Y$ and that $Y \subseteq X$. Each of these will require an *element argument* (as described on page 42).

2.4.1 EXERCISE

Soon you will be asked to show that union distributes over intersection, and that intersection distributes over union. Drawing Venn diagrams can help you to understand what