

$$1. \left(\bigcup_{\alpha \in \Lambda} A_\alpha \right)^c = \bigcap_{\alpha \in \Lambda} A_\alpha^c.$$

$$2. \left(\bigcap_{\alpha \in \Lambda} A_\alpha \right)^c = \bigcup_{\alpha \in \Lambda} A_\alpha^c. \quad \square$$

2.4.10 DEFINITION

Let A and B be sets. The set

$$A \setminus B = \{x : x \in A \text{ but } x \notin B\}$$

is called the **set difference** of A and B . (Notice that if both A and B are subsets of a set U , $A \setminus B = A \cap B^c$.)

Here are some (set) algebraic identities associated with set difference.

2.4.11 THEOREM

For any sets A , B , and C :

$$1. C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B).$$

$$2. C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B).$$

$$3. B \setminus (B \setminus A) = A \cap B.$$

$$4. (A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B).$$

(These identities can be proved in at least two different ways:

- set theoretically by using element arguments, and
- algebraically by converting the set differences to intersections and using the various algebraic identities that have already been established.

Try your hand at both.) \square

2.4.12 DEFINITION

The set described in part 4 of Theorem 2.4.11 is called the **symmetric difference** of A and B . It is often denoted by $A \Delta B$.

2.5 The Power Set

You will see that we often need to consider sets whose elements are themselves sets. Power sets are among the most important examples of such sets.

2.5.1 DEFINITION

If A is a set, then the **power set** of A is the set of all subsets of A . It is denoted by $\mathcal{P}(A)$.

2.5.2 EXERCISES

Find $\mathcal{P}(\{1\})$, $\mathcal{P}(\{1, 2\})$.

2.5.3 EXERCISES

Let S be any set w

1. Looking o
will have.
2. Does your

2.5.4 THEOREM

Let A and B be se

2.5.5 THEOREM

Let A and B be se

1. $\mathcal{P}(A \cap B)$
2. $\mathcal{P}(A) \cup \mathcal{P}(B)$

2.5.6 EXERCISES

Let A and B be se

1. Provide a
2. Is it ever tr

That is, can
 $\mathcal{P}(A) \cup \mathcal{P}(B)$

2.5.7 PROBLEM

The purpose of th
double the size of
the special cases y
into what is going

Let S be any

1. Prove that

(That is, sh