

$$1. \left( \bigcup_{\alpha \in \Lambda} A_\alpha \right)^c = \bigcap_{\alpha \in \Lambda} A_\alpha^c.$$

$$2. \left( \bigcap_{\alpha \in \Lambda} A_\alpha \right)^c = \bigcup_{\alpha \in \Lambda} A_\alpha^c. \quad \square$$

### 2.4.10 DEFINITION

Let  $A$  and  $B$  be sets. The set

$$A \setminus B = \{x : x \in A \text{ but } x \notin B\}$$

is called the **set difference** of  $A$  and  $B$ . (Notice that if both  $A$  and  $B$  are subsets of a set  $U$ ,  $A \setminus B = A \cap B^c$ .)

Here are some (set) algebraic identities associated with set difference.

### 2.4.11 THEOREM

For any sets  $A$ ,  $B$ , and  $C$ :

$$1. C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B).$$

$$2. C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B).$$

$$3. B \setminus (B \setminus A) = A \cap B.$$

$$4. (A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B).$$

(These identities can be proved in at least two different ways:

- set theoretically by using element arguments, and
- algebraically by converting the set differences to intersections and using the various algebraic identities that have already been established.

Try your hand at both.) □

### 2.4.12 DEFINITION

The set described in part 4 of Theorem 2.4.11 is called the **symmetric difference** of  $A$  and  $B$ . It is often denoted by  $A \Delta B$ .

## 2.5 The Power Set

You will see that we often need to consider sets whose elements are themselves sets. Power sets are among the most important examples of such sets.

### 2.5.1 DEFINITION

If  $A$  is a set, then the **power set** of  $A$  is the set of all subsets of  $A$ . It is denoted by  $\mathcal{P}(A)$ .

### 2.5.2 EXERCISES

Find  $\mathcal{P}(\{1\})$ ,  $\mathcal{P}(\{1, 2\})$ .

### 2.5.3 EXERCISES

Let  $S$  be any set  $v$

1. Looking o  
will have.
2. Does your

### 2.5.4 THEOREM

Let  $A$  and  $B$  be se

### 2.5.5 THEOREM

Let  $A$  and  $B$  be se

1.  $\mathcal{P}(A \cap B)$
2.  $\mathcal{P}(A) \cup \mathcal{P}(B)$

### 2.5.6 EXERCISES

Let  $A$  and  $B$  be se

1. Provide a
2. Is it ever tr

That is, can  
 $\mathcal{P}(A) \cup \mathcal{P}(B)$

### 2.5.7 PROBLEM

The purpose of th  
double the size of  
the special cases y  
into what is going

Let  $S$  be any

1. Prove that

(That is, sh