collections, and the collection of natural numbers were proposed as likely "safe" sets—and some, such as the collection of all sets, were not to be considered sets. How then was the mathematical community to tell which were safe collections and which would lead to contradiction? The answer lay in carefully considered axioms that would give criteria for identifying collections that are sets.

In addition to the empty set, finite collections, and the collection of natural numbers, the axioms tell us that any collections that can be built from sets by taking unions, intersections, subsets, and power sets (provided that these operations are handled a bit carefully) are to be considered sets. For more details see Appendix A.

PROBLEMS

1. Is there a distinction between $\emptyset$ and $\{\emptyset\}$? Explain.

2. Consider the following pairs of sets.

$$A = \{x \in \mathbb{N} : x \geq 12\} \quad \text{and} \quad B = \{x \in \mathbb{N} : x \leq 14\}$$
$$A = \{x \in \mathbb{N} : x \geq 32\} \quad \text{and} \quad B = \{x \in \mathbb{N} : x < 32\}$$
$$A = \{x \in \mathbb{R} : -3 < x \leq 3\} \quad \text{and} \quad B = \{x \in \mathbb{R} : x < 10\}$$

For each pair find:

(a) $A \cap B$. (b) $A \cup B$.

3. **Playing with Venn diagrams.** Figure 2.2 shows A, B, and C in "general position." That means that we are assuming maximum possible overlap between the sets. (Specifically, A and B intersect, A and C intersect, and B and C intersect. Furthermore, there is a place where all three intersect.) In the absence of special information about the composition of the sets, we always draw Venn diagrams in general position, but if we know more about the situation, we incorporate that information into the diagram. The Venn diagram in Figure 2.3 shows three sets A, B, and C in which A and B intersect, A and C intersect, and B and C intersect, but

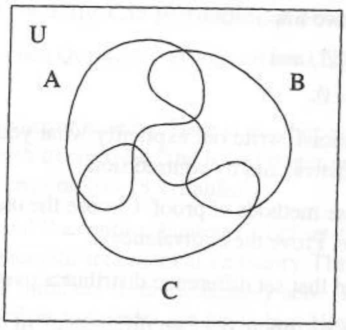

Figure 2.3