$A \cap B \cap C = \emptyset$. Draw a Venn diagram that depicts sets A, B, and C in the following special configurations.

(a) A and B are disjoint, but each has nonempty intersection with C.

(b) A is disjont from B and C, but $B \cap C \neq \emptyset$.

(c) No two of the sets intersect. (Such a collection of sets is said to be **pairwise disjoint.**)

4. (a) For each $n \in \mathbb{N}$ let

$$A_n = \left(\frac{1}{2}, \frac{1}{2} + \frac{1}{n}\right).$$

(i) Find $\bigcup_{n\in\mathbb{N}} A_n$. (ii) Find $\bigcap_{n\in\mathbb{N}} A_n$.

How would your answer change if the intervals were closed instead of open?

(b) For each $r \in \mathbb{Q}$, let

$$D_r = \left(\frac{1}{2}, \frac{1}{2} + r\right).$$

(i) Find $\bigcup_{r\in\mathbb{Q}} D_r$. (ii) Find $\bigcap_{r\in\mathbb{Q}} D_r$.

(c) For each $r \in \mathbb{Q}$, let

$$D_r = \left(\frac{1}{2} - r, \frac{1}{2} + r\right).$$

(i) Find $\bigcup_{r\in\mathbb{Q}} D_r$. (ii) Find $\bigcap_{r\in\mathbb{Q}} D_r$.

(d) For each $r \in \mathbb{Q}$, let K_r be the set containing all real numbers *except* r. That is, $K_r = \{r\}^{\mathrm{c}}_{\mathbb{R}}$.

(i) Find $\bigcup_{r\in\mathbb{Q}} K_r$. (ii) Find $\bigcap_{r\in\mathbb{Q}} K_r$.

5. Suppose A and B are subsets of some set U. In this problem you will prove the following statement:

$$A \cap B^{\mathrm{c}} = \emptyset \text{ if and only if } A \subseteq B.$$

It is a proposition that lends itself very well to a good review of logical principles. Note that to prove it you have to prove two implications:

$\Longrightarrow$ If $A \cap B^{\mathrm{c}} = \emptyset$, then $A \subseteq B$, and

$\Longleftarrow$ If $A \subseteq B$, then $A \cap B^{\mathrm{c}} = \emptyset$.

(a) For each of these implications, write out explicitly what you would have to do to prove them directly, by contrapostive, and by contradition.

(b) Now consider each of these methods of proof. Choose the one that you think makes each implication most tractable. Prove the equivalence.

6. In Theorem 2.4.11 you proved that set difference distributes over union and intersection.

(a) Do union and intersection distribute over set difference? In other words, is it true that

$$A \cup (B \setminus C) = (A \cup B) \setminus (A \cup C) \quad \text{and} \quad A \cap (B \setminus C) = (A \cap B) \setminus (A \cap C)?$$

Give a proof or a counterexample for each.

(b) Does comple

Give a proof c

7. Consider the symı

(a) Draw a venn c

(b) We have discu
at least three
difference.

8. Let A and B be su

(a) Prove that $\mathcal{P}(B$

(b) Prove that $\mathcal{P}(A$

(c) Prove that $\mathcal{P}(B$

(Hint: look at exam

QUESTIONS

1. In Problem 2.5.7, y
used the word "arg
as yet, have any ma
axioms, theorems, c
of counting look like
that mean mathema
does *that* mean? Ho

2. Following up on this
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elements?

3. I did not define inde

4. Do you see a connec
on the other?

5. The Berry Paradox-
try to resolve. Suppo
syllables." Paradox: '

6. I recommend Appenc
The Appendices prov
in the construction o
appendix, some of th
pattern of the ideas is
all done.)