(b) Does complementation distribute over set difference? In other words, is it true that

$$(A \setminus B)^{\mathsf{c}} = A^{\mathsf{c}} \setminus B^{\mathsf{c}}?$$

Give a proof or a counterexample.

7. Consider the symmetric difference of two sets A and B.

$$A \Delta B = (A \setminus B) \cup (B \setminus A).$$

(a) Draw a venn diagram that shows this set.

(b) We have discussed a variety of set operations. Formulate and either prove or disprove at least three distribution principles (as in the previous problem) involving symmetric difference.

8. Let A and B be subsets of a set U.

(a) Prove that $\mathcal{P}(B^{\mathsf{c}}_U) \neq \mathcal{P}(B)^{\mathsf{c}}_{\mathcal{P}(U)}$.

(b) Prove that $\mathcal{P}(A \setminus B) \neq \mathcal{P}(A) \setminus \mathcal{P}(B)$.

(c) Prove that $\mathcal{P}(B^{\mathsf{c}}_U) \setminus \{\emptyset\} \subseteq \mathcal{P}(B)^{\mathsf{c}}_{\mathcal{P}(U)}$.

(Hint: look at examples.)

QUESTIONS TO PONDER

1. In Problem 2.5.7, you needed to do some counting to come to the necessary conclusions. I used the word "argue" (rather than prove) in two parts of that problem because we do not, as yet, have any mathematical theory of counting. In other words, you have no definitions, axioms, theorems, or any other basis for proving your conclusions. What might such a theory of counting look like? If I say, "These two sets have the same number of elements," what does that mean mathematically? If I say, "This set has twice as many elements as that set," what does *that* mean? How might we define this rigorously?

2. Following up on this theme, suppose that two sets A and B have the same number of elements. Must their power sets have the same number of elements? Conversely, suppose $\mathcal{P}(A)$ has the same number of elements as $\mathcal{P}(B)$, does it follow that A and B have the same number of elements?

3. I did not define indexing set precisely. Can you figure out how to define indexing set?

4. Do you see a connection between Question 1 and Question 3? Does either question shed light on the other?

5. **The Berry Paradox**—*a paradox of naming*. Here is another paradox for you to ponder and try to resolve. Suppose we wish to consider "the least integer not nameable in fewer than 19 syllables." Paradox: This description has 18 syllables!

6. I recommend Appendices A and B as entire chapters full of set-theoretic questions to ponder. The Appendices provide an axiomatic treatment of set theory. The second appendix culminates in the construction of the real numbers from elementary sets. (By the middle of the second appendix, some of the gaps left to the reader are fairly big. However, following the general pattern of the ideas is not too hard and the treatment should give you a good idea of how it is all done.)