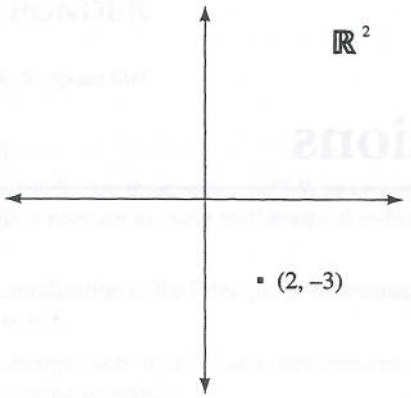

Figure 4.1 The Cartesian plane

4.1.2 EXAMPLE

The Cartesian product that you are most familiar with is probably $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$, the Cartesian plane (see Figure 4.1). ■

4.1.3 EXERCISE

Let $A = \{1, 2, 3\}$, $B = \{a, b\}$.

1. Find $A \times A$ and $B \times B$.
2. Find $A \times B$. □

4.1.4 DEFINITION

If A and B are sets, then any subset of $A \times B$ is called a **relation** between A and B. A subset of $A \times A$ is called a relation on A.

4.1.5 EXAMPLE

On the set $\mathbb{R}$, we have the relation $\leq$. The pair $(2, 3)$ is in the relation because $2 \leq 3$. The pair $(4, -\frac{1}{3})$ is not in the relation because $4 \not\leq -\frac{1}{3}$. ■

Remark. The notion of a relation is a very general one. Different notation schemes are used in different contexts, and I will indicate notational conventions as we go. When talking about relations in the abstract, we will indicate that a particular pair is in the relation by some notation like $a \sim b$. (We would read this as "a is related to b.") In concrete cases, other symbols are associated with particular relations. For instance, in the case of Example 4.1.5 we write $a \leq b$ instead of $a \sim b$.

4.1.6 EXERCISE

Let S be the set of students in your class. Let B be the collection of books in your library. Define a relation between S and B. □