We often consider relations that have convenient special properties. Reflexivity, symmetry, antisymmetry, and transitivity are some very useful ones.

4.1.8 DEFINITION

Let $\sim$ be a relation on a set A.

1. $\sim$ is said to be **reflexive** if for all $x \in A$, $x \sim x$. (That is, a relation on A is reflexive if every element of A is related to itself.)
2. $\sim$ is said to be **symmetric** if for all $x, y \in A$, $x \sim y$ implies that $y \sim x$. (A relation on A is symmetric if y is related to x whenever x is related to y.)
3. $\sim$ is said to be **antisymmetric** if for all $x, y \in A$, $x \sim y$ and $y \sim x$ imply that $x = y$. (Say in words what this means.)
4. $\sim$ is said to be **transitive** if for all x, y, and $z \in A$, $x \sim y$ and $y \sim z$ imply that $x \sim z$. (How would we say this in words?)

4.1.9 EXERCISE

Recall the relation "knows the name of" on the set of people at a party. (In this instance, instead of saying "Mark $\sim$ Karen," we would say "Mark knows Karen's name.")

Translate the definitions above into "knows the name of" language. For instance, saying that "knows the name of" is symmetric is to say that if Mark knows Karen's name, then Karen must know Mark's name. Is this true? Say in words what it would mean to say that the relation "knows the name of" is reflexive, antisymmetric, and transitive. Which of these properties holds? □

> Note that just because we refer to things by different names does not necessarily mean that they are different; when we say $x \sim y$, it may be that $x = y$. In the case of the relations "knows the name of," any person at the party that knows his or her own name is related to him or herself.

4.1.10 PROBLEM

Indicate whether the following relations on the given sets are reflexive, symmetric, antisymmetric, or transitive. (These are not mutually exclusive conditions, so the relations may satisfy more than one.) Justify your answer.

1. $A = \{p : p \text{ is a person in Alaska}\}$. $x \sim y$ if x is at least as tall as y.
2. $A = \mathbb{N}$. $x \sim y$ if $x + y$ is even.
3. $A = \mathbb{N}$. $x \sim y$ if $x + y$ is odd.
4. $A = \mathcal{P}(\mathbb{N})$. $x \sim y$ if $x \subseteq y$.
5. $A = \mathbb{R}$. $x \sim y$ if $x = 2y$.