Norges teknisk–
naturvitenskapelige universitet
Institutt for matematiske fag

Contact during exam:
Carl Fredrik Berg   Telephone: 975 05 585

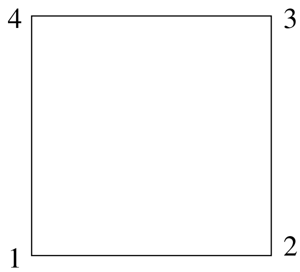
EXAM IN TMA4150 ALGEBRA AND NUMBER THEORY

English
Friday 28. may 2004
Time: 09.00 - 14.00
Gradings: 21. june 2004

Permitted aids: HP30S

Problem 1

Let $G = \mathcal{D}_4$ be the symmetry group of the square



a) Write the 8 elements of G as permutations of $\{1, 2, 3, 4\}$.

b) In how many ways can the 4 corners of the square above be painted when the colors yellow, blue and red are available? (Two colorings are viewed as equal if they can be carried over to each other by one of the symmetries of the square.)

Problem 2

Let G be the group of invertible 2×2-matrices over the real numbers $\mathbb{R}$. Let H be the subset of G consisting of matrices with determinant equal to 1.

a) Show that H is a subgroup of G, and show that this subgroup is normal.

b) Show that the factor group G/H is isomorphic to the multiplicative group $\mathbb{R}^*$, i.e. $\mathbb{R} \setminus \{0\}$ where the group operation is the usual multiplication.

Problem 3

Let G be a group with 143 elements, and $H \subset G$ a subgroup with $H \neq G$. Explain why H is a cyclic group.

Problem 4

a) Find all abelian groups of order 8 up to isomorphism.

b) Let G be the group of units in the commutative ring $\mathbb{Z}_{10} \times \mathbb{Z}_3$. Find all elements in G, and decide which of the groups in (a) G is isomorphic to.

Problem 5

Let $\sigma = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8 \\ 3\,4\,5\,2\,1\,6\,7\,8 \end{pmatrix}$ be an element in S_8.

a) Write σ as a product of disjoint cycles and as a product of transpositions (i.e. cycles of length 2).

b) Find the order of σ. Find an element in S_8 of order 12.

Decide whether there are elements of order 27 and 30 in S_8.

Problem 6

Let R be a commutative ring with unity (1). Show that R is a field if and only if (0) and R are the only ideals in R.