MAT2410 - MANDATORY ASSIGNMENT #1, FALL 2010

REMINDER: The assignment must be handed in before 14:30 on Thursday September 23, 2010, at the reception of the Department of Mathematics, in the 7th floor of Niels Henrik Abels hus, Blindern. Be careful to give reasons for your answers. To have a passing grade you must have correct answers to at least 50% of the questions and moreover have attempted to solve all of them.

Exercise 1.

a. Determine

$$\mathrm{Arg}(-6-6i), \quad \mathrm{Arg}(-\pi), \quad \mathrm{Arg}(3i), \quad \mathrm{Arg}(\sqrt{3}-i).$$

b. Express the complex numbers

$$z_1 = -6-6i, \quad z_2 = -\pi, \quad z_3 = 3i, \quad z_4 = \sqrt{3}-i$$

in polar form, and compute

$$z_1 z_2 z_3 z_4.$$

c. Describe the set of points in the complex plane that satisfy

$$z\bar{z} = \frac{1}{2}(z+\bar{z})^2 + 1.$$

d. Write e^{e^i} in standard form, i.e., as $a+bi$ for some real numbers a, b.

Exercise 2.

a. Suppose z is a complex number different from 1 and $n \geq 1$ is an integer. Show that

$$1 + z + z^2 + \cdots + z^n = \frac{z^{n+1}-1}{z-1}. \tag{1}$$

b. Utilize (1) and De Moivre's formula to show that

$$\sin(\theta) + \sin(2\theta) + \cdots + \sin(n\theta) = \frac{\sin(n\theta/2)\sin((n+1)\theta/2)}{\sin(\theta/2)}, \qquad \theta \in [0, 2\pi].$$

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c. Similarly, show that

$$1 + \cos(\theta) + \cos(2\theta) + \cdots + \cos(n\theta) = \frac{1}{2} + \frac{\sin((n+1/2)\theta)}{2\sin(\theta/2)}.$$

d. Compute all the values of

$$(1-i)^{5/2}.$$

Plot these values in the complex plane.

Exercise 3.

a. Consider the Riemann sphere

$$\Sigma = \{(x_1, x_2, x_3) \in \mathbf{R}^3 : x_1^2 + x_2^2 + x_3^2 = 1\},$$

and the subset

$$S = \{(x_1, x_2, x_3) \in \Sigma : x_3 = 1/2\}.$$

Describe the set in the complex plane that under stereographic projection maps to the set S on the Riemann sphere.

b. Let z be a complex number with $|z| < 1$. For $n = 1, 2, \ldots$, set

$$S_n = 1 + z + z^2 + \cdots + z^n.$$

Show that

$$\lim_{n\to\infty} S_n = \frac{1}{1-z}.$$