MAT2410: Mandatory assignment #1, autumn 2013

To be handed in by September 19, 14:30

Exercise 1.

(**a**) Solve the equation $1/z + z = i$.
(**b**) Write the complex number i^i in Cartesian form.
(**c**) Use DeMoivre's formula to find an expression for $\cos(5\theta)$ in terms of $\cos\theta$ and $\sin\theta$.
(**d**) Find all solutions of $z^4 = 2\sqrt{2} + 2\sqrt{2}i$.

Exercise 2.

Find the limits

(**a**)

$$\lim_{n\to\infty}\left(\frac{i}{2}\right)^n,$$

(**b**)

$$\lim_{z\to -i}\frac{z^2+1}{z+i},$$

(**c**)

$$\lim_{n\to\infty}\left(1+\frac{i}{n}\right)^{n\pi}.$$

Exercise 3.

(**a**) Find the real and imaginary parts of the function $f(z) = z^2$.
(**b**) Let $P : \mathbb{C} \cup \{\infty\} \to S^2$ be the stereographic projection of the extended complex plane to the Riemann sphere, see Section 1.7 in Saff & Snider for an explanation. The complex function $f(z) = z^2$ induces a map $g : S^2 \to S^2$ by

$$g(x_1, x_2, x_3) = P\left(\left(P^{-1}(x_1, x_2, x_3)\right)^2\right).$$

Show that

$$g(x_1, x_2, x_3) = \frac{1}{x_3^2 + 1}\left(x_1^2 - x_2^2, 2x_1x_2, 2x_3\right).$$

(**c**) Let $l(t) = (0, \sin(t), \cos(t)) \subset S^2$, $t \in \mathbb{R}$. Describe the set $\{g(l(t)) \,|\, t \in \mathbb{R}\}$.

Exercise 4.

Let $f(z)$ be defined as

$$f(z) = \begin{cases} \frac{x^3-y^3}{x^2+y^2} + i\frac{x^3+y^3}{x^2+y^2} & z \neq 0, \\ 0 & z = 0. \end{cases}$$

(**a**) Show that f is continuous at $z = 0$. Hint: You can use that $|x^3 \pm y^3| \leq 2(|x| + |y|)(x^2 + y^2)$.

(**b**) Is f analytic?