Answers to Odd-Numbered Problems

CHAPTER 1

Exercises 1.1, Page 4

5. **(a)** $0 + (-3/2)i = -3i/2$ **(b)** $3 + 0i = 3$ **(c)** $0 + (-2)i = -2i$

7. **(a)** $8 + i$ **(b)** $1 + i$ **(c)** $0 + (-8/3)i = -8i/3$

9. $\dfrac{61}{185} - \dfrac{107}{185}i$

11. $2 + 0i = 2$

13. $6 + 5i$

17. $8 - 10i$

19. $a^3 - 3ab^2 + 5a^2 - 5b^2 = a,\ 3a^2b - b^3 + 10ab = b + 3$, where $z = a + bi$

21. $z_1 = 1 + i,\ z_2 = -i$

Exercises 1.2, Page 12

3. -3

7. **(a)** horizontal line $y = -2$
(b) circle, center $= 1 - i$, rad $= 3$
(c) circle, center $= i/2$, rad $= 2$
(d) perpendicular bisector of segment joining $z = 1$ and $z = -i$
(e) parabola $y^2 = 4(x + 1)$ with vertex -1, focus 0
(f) ellipse with foci at ± 1
(g) circle, center $= \frac{9}{8}$, rad $= \frac{3}{8}$
(h) half-plane consisting of all points on or to the right of the vertical line $x = 4$
(i) all points inside the circle with center i and rad $= 2$ (open disk)
(j) all points outside the circle $|z| = 6$

Exercises 1.3, Page 22

3. $|z_1+z_2|^2+|z_1-z_2|^2=2\,|z_1|^2+2\,|z_2|^2$

5. **(a)** 1 **(b)** $5\sqrt{26}$ **(c)** $5\sqrt{5}/2$ **(d)** 1

7. **(a)** $\arg=\pi+2k\pi$, polar form: $\frac{1}{2}\text{cis}(\pi)$
(b) $\arg=3\pi/4+2k\pi$, polar form: $3\sqrt{2}\,\text{cis}(3\pi/4)$
(c) $\arg=-\dfrac{\pi}{2}+2k\pi$, polar form: $\pi\,\text{cis}(-\pi/2)$
(d) $\arg=7\pi/6+2k\pi$, polar form: $4\text{cis}(7\pi/6)$
(e) $\arg=7\pi/12+2k\pi$, polar form: $2\sqrt{2}\,\text{cis}(7\pi/12)$
(f) $\arg=5\pi/3+2k\pi$, polar form: $4\text{cis}(5\pi/3)$
(g) $\arg=5\pi/12+2k\pi$, polar form: $\frac{\sqrt{2}}{2}\,\text{cis}(5\pi/12)$
(h) $\arg=13\pi/12+2k\pi$, polar form: $\frac{\sqrt{14}}{2}\,\text{cis}(13\pi/12)$

9. rotation of vector z about origin through an angle ϕ in the counterclockwise direction

13. (b), (d)

21. $r=\sqrt{r_1^2+r_2^2+2r_1r_2\cos(\theta_1-\theta_2)}$, and θ is determined by the pair of equations: $\cos\theta=(r_1\cos\theta_1+r_2\cos\theta_2)/r$, $\sin\theta=(r_1\sin\theta_1+r_2\sin\theta_2)/r$

23. The center of mass of three particles that lie inside or on the unit circle also lies inside or on the circle.

29. $l=.0732$ m, $dl/dt=-.1155$ m/sec

Exercises 1.4, Page 31

1. **(a)** $\dfrac{\sqrt{2}}{2}-\dfrac{\sqrt{2}}{2}i$ **(b)** $e^2 i$ **(c)** $e^{\cos 1}\cos(\sin 1)+i\,e^{\cos 1}\sin(\sin 1)$

3. **(a)** $\dfrac{\sqrt{2}}{3}e^{-i\pi/4}$ **(b)** $16\pi e^{i4\pi/3}$ **(c)** $8e^{i3\pi/2}$

11. (a), (c), (d)

17. **(a)** circle $|z|=3$ traversed counterclockwise
(b) circle $|z-i|=2$ traversed counterclockwise
(c) upper half of circle $|z|=2$ traversed counterclockwise
(d) circle $|z-(2-i)|=3$ traversed clockwise

21. $\left|\dfrac{1-z^n}{1-z}\right|=|1+z+\cdots+z^{n-1}|\le 1+1+\cdots+1=n$ for $z=e^{i\theta}\neq 1$

23. **(a)** $\frac{35\pi}{64}$ **(b)** $\frac{5\pi}{8}$

Exercises 1.5, Page 37

5. **(a)** $2(\cos\pi/4+i\sin\pi/4)$, $2(\cos 3\pi/4+i\sin 3\pi/4)$, $2(\cos 5\pi/4+i\sin 5\pi/4)$, $2(\cos 7\pi/4+i\sin 7\pi/4)$

(b) 1, $\cos 2\pi/5+i\sin 2\pi/5$, $\cos 4\pi/5+i\sin 4\pi/5$, $\cos 6\pi/5+i\sin 6\pi/5$, $\cos 8\pi/5+i\sin 8\pi/5$

(c) $\cos\pi/8+i\sin\pi/8$, $\cos 5\pi/8+i\sin 5\pi/8$, $\cos 9\pi/8+i\sin 9\pi/8$, $\cos 13\pi/8+i\sin 13\pi/8$

(d) $\sqrt[3]{2}[\cos(-\pi/9)+i\sin(-\pi/9)]$, $\sqrt[3]{2}(\cos 5\pi/9+i\sin 5\pi/9)$, $\sqrt[3]{2}(\cos 11\pi/9+i\sin 11\pi/9)$

(e) $\sqrt[4]{2}(\cos 3\pi/8+i\sin 3\pi/8)$, $\sqrt[4]{2}(\cos 11\pi/8+i\sin 11\pi/8)$

(f) $\sqrt[12]{2}\exp[i\pi(1+8k)/24]$, $k=0,1,2,3,4,5$

7. **(a)** $-\frac{1}{4}\pm i\frac{\sqrt{23}}{4}$ **(b)** $2-i, 1-i$ **(c)** $1\pm\sqrt[4]{2}e^{-i\pi/8}$

9. $1, 1+i\sqrt{3}, 1-i\sqrt{3}$

11. $z=1/(w-1)$, $w=e^{i2k\pi/5}$, $k=1,2,3,4$

15. $\sqrt[4]{8}(\cos 5\pi/8+i\sin 5\pi/8)$, $\sqrt[4]{8}(\cos 13\pi/8+i\sin 13\pi/8)$

19. **(b)** $\frac{\pm\sqrt{2}+2i}{3}$

21. **(a)** $\pm(3+i)$ **(b)** $\pm(3+2i)$ **(c)** $\pm(5+i)$
(d) $\pm(2-i)$ **(e)** $\pm(1+3i)$ **(f)** $\pm(3-i)$

Exercises 1.6, Page 42

3. (b), (c), (f)

5. (a), (c)

7. (a), (b), (c), (d), (e)

11. all points of S and 0

17. No, $S\cap T$ might not be connected.

19. D is not connected.

21. $\ln(x^2+y^2)+C$

23. (a) and (d)

Exercises 1.3, Page 22

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21. $\left|\dfrac{1 - z^n}{1 - z}\right| = |1 + z + \cdots + z^{n-1}| \le 1 + 1 + \cdots + 1 = n \quad \text{for } z = e^{i\theta} \ne 1$

23. **(a)** $\frac{35\pi}{64}$ **(b)** $\frac{5\pi}{8}$

Exercises 1.7, Page 50

1. **(a)** $(0, 1, 0)$ **(b)** $(\frac{12}{101}, \frac{-16}{101}, \frac{99}{101})$ **(c)** $(\frac{-12}{25}, \frac{16}{25}, \frac{-3}{5})$

5. **(a)** The hemisphere $x_1 > 0$ **(b)** The bowl $x_3 < -\frac{3}{5}$
(c) The zone $0 < x_3 < \frac{3}{5}$ **(d)** The dome $x_3 > 0.8$
(e) The great circle $x_1 = x_2, -1 \le x_3 \le 1$ (45 and 225 degrees longitude)

CHAPTER 2

Exercises 2.1, Page 56

1. **(a)** $3x^2 - 3y^2 + 5x + 1 + i(6xy + 5y + 1)$ **(b)** $x/(x^2 + y^2) - iy/(x^2 + y^2)$
(c) $x/[x^2 + (y-1)^2] + i(1-y)/[x^2 + (y-1)^2]$
(d) $(2x^2 - 2y^2 + 3)/\sqrt{(x-1)^2 + y^2} + i4xy/\sqrt{(x-1)^2 + y^2}$
(e) $e^{3x}\cos 3y + ie^{3x}\sin 3y$ **(f)** $2\cos y \cosh x + i2\sin y \sinh x$

3. **(a)** half-plane $\text{Re}\, w > 5$ **(b)** upper half-plane $\text{Im}\, w \ge 0$ **(c)** $|w| \ge 1$
(d) circular sector $|w| < 2, -\pi < \text{Arg}\, w < \pi/2$

5. **(a)** domain of definition $= \mathbf{C}$, range $= \mathbf{C} \setminus \{0\}$ **(c)** circle $|w| = e$
(d) ray (half-line) $\text{Arg}\, w = \pi/4$ **(e)** infinite sector $0 \le \text{Arg}\, w \le \pi/4$

Exercises 2.2, Page 63

1. spirals to 0

7. **(a)** converges to 0 **(b)** does not converge **(c)** converges to π
(d) converges to $2 + i$ **(e)** converges to 0 **(f)** does not converge

11. **(a)** $-8i$ **(b)** $-7i/2$ **(c)** $6i$ **(d)** $-\frac{1}{2}$ **(e)** $2z_0$ **(f)** $4\sqrt{2}$

13. limits exist except at $z = -1$; continuous except at $z = -1$ and $z = 0$; removable discontinuity at $z = 0$

17. no

19. $-\frac{1}{2} - i$

21. **(a)** 1 **(b)** 0 **(c)** $-\frac{\pi}{2} + i$ **(d)** 1

Exercises 2.3, Page 70

7. **(a)** $18z^2 + 16z + i$
(b) $-12z\left(z^2 - 3i\right)^{-7}$
(c) $\dfrac{\left(iz^3 + 2z + \pi\right)2z - \left(z^2 - 9\right)\left(3iz^2 + 2\right)}{\left(iz^3 + 2z + \pi\right)^2}$
(d) $(z+2)^2[-5z^2 + (-16 - i)z + 3 - 8i]/(z^2 + iz + 1)^5$
(e) $24i\left(z^3 - 1\right)^3\left(z^2 + iz\right)^{99}\left(53z^4 + 28iz^3 - 50z - 25i\right)$

9. **(a)** $2 - 3i$ **(b)** $\pm i$ **(c)** $\frac{1}{2}(-1 \pm i\sqrt{15})$ **(d)** $\dfrac{1}{2}, 1$

11. **(a)** nowhere analytic **(b)** nowhere analytic **(c)** analytic except at $z = 5$ **(d)** everywhere analytic **(e)** nowhere analytic **(f)** analytic except at $z = 0$ **(g)** nowhere analytic **(h)** nowhere analytic

13. (a), (b), (d), (f), (g)

15. $\dfrac{3}{5}$

Exercises 2.4, Page 77

3. $g(z) = 3z^2 + 2z - 1$

5. $f'(z) = 2e^{x^2-y^2}(x + iy)[\cos(2xy) + i\sin(2xy)]$

7. Hint: If $f' = g'$ in D, consider $h := f - g$.

9. If F were analytic, then $\operatorname{Im} F(z) \equiv 0$ implies $F(z)$ is constant, which is a contradiction.

11. Hint: Consider $f(z) + \overline{f(z)}$.

13. Hint: $|f(z)| \equiv$ constant by condition (8). Now use the result of Prob. 12.

Exercises 2.5, Page 84

3. **(a)** $v = -x + a$ **(b)** $v = -e^x \cos y + a$ **(c)** $v = y^2/2 - y - x^2/2 - x + a$
(d) $v = \cos x \sinh y + a$ **(e)** $v = \operatorname{Tan}^{-1}(y/x) + a = \operatorname{Arg} z + a$
(f) $v = -\operatorname{Re}(e^{z^2}) + a = -e^{x^2-y^2}\cos(2xy) + a$

7. $\phi(x, y) = x + 1$

9. $\phi(x, y) = xy - 1$

13. (one example) $\phi(r, \theta) = r^4 \sin 4\theta = 4x^3y - 4xy^3$

15. $\text{Re}\,\frac{40}{63}(z^3 - z^{-3})$

17. **(a)** $\phi(x, y) = \text{Re}\left(z^2 + 5z + 1\right)$ **(b)** $\phi(x, y) = 2\,\text{Re}\left(\dfrac{z^2}{z + 2i}\right)$

19. $\text{Re}\left(\frac{1}{2z^2}\right) + \frac{1}{2}$

Exercises 2.6, Page 90

3. See Fig. 2.6(a).

Exercises 2.7, Page 95

1. $\zeta_{\pm} = \frac{1 \pm \sqrt{1-4c}}{2}$ are fixed, ζ_- attracts for $-\frac{3}{4} < c < \frac{1}{4}$

3. **(a)** i and $-i$ are fixed and repellors
(b) $\frac{1}{2}$ and -1 are fixed and repellors, $-\frac{1}{2}$ is fixed and an attractor

5. $(-1 + \sqrt{5})/2$ is fixed and attractor, $(-1 - \sqrt{5})/2$ is fixed and repellor

9. If $|\alpha| \le 1$ the whole complex plane is the filled Julia set; if $|\alpha| > 1$ the origin is the filled Julia set.

CHAPTER 3

Exercises 3.1, Page 108

1. $2(z + 1)^2(z^2 + 9)$

3. **(a)** $z^3(z + 1 + i)^2$ **(b)** $(z - 2)(z + 2)(z + 2i)(z - 2i)$
(c) $(z - \omega_7)(z - \omega_7^2) \cdots (z - \omega_7^6)$, where $\omega_7 = e^{i2\pi/7}$

5. **(a)** $42 + 83(z - 2) + 80(z - 2)^2 + 40(z - 2)^3 + 10(z - 2)^4 + (z - 2)^5$
(b) $\sum_{k=0}^{10} \binom{10}{k} 2^{10-k}(z - 2)^k$ (binomial expansion of $[(z - 2) + 2]^{10} = z^{10}$)
(c) $(z - 2)^3 + (z - 2)^4$

11. **(a)** Pole at 0 of order 3, at $(-1 - \sqrt{2})i$ of order 1, and at $(-1 + \sqrt{2})i$ of order 1
(b) Pole at 2 of order 1 and at 3 of order 2
(c) Pole at -2 of order 6
(d) Pole at -2 of order 1

13. **(a)** $\frac{(3+i)/2}{z} - \frac{3+i}{z+1} + \frac{(3+i)/2}{z+2}$
(b) $\frac{i}{z} + \frac{i/2}{z+i} - \frac{3i/2}{z-i}$
(c) $\frac{1/6 + i\sqrt{3}/6}{(z+1/2+i\sqrt{3}/2)^2} - \frac{i\sqrt{3}/9}{z+1/2+i\sqrt{3}/2} + \frac{1/6 - i\sqrt{3}/6}{(z+1/2-i\sqrt{3}/2)^2} + \frac{i\sqrt{3}/9}{z+1/2-i\sqrt{3}/2}$
(d) $\frac{5}{2}z^2 - \frac{15}{4}z + \frac{47}{8} + \frac{33/16}{z+1/2} - \frac{9}{z+1}$

15. **(a)** $\frac{3-2i}{4}$ **(b)** $-\frac{1}{512}$ **(c)** 6 **(d)** 0 **(e)** 3

Exercises 3.2, Page 115

3. sum $= \dfrac{1-e^{101z}}{1-e^z}$ for $z \neq 2k\pi i$; sum $= 101$ for $z = 2k\pi i$

5. **(a)** $e^2\dfrac{\sqrt{2}}{2} + ie^2\dfrac{\sqrt{2}}{2}$ **(b)** ie^2 **(c)** $i \sinh 2$
(d) $\cos(1)\cosh(1) + i\sin(1)\sinh(1)$
(e) $-\sinh(1)$ **(f)** 0

9. **(a)** $2\pi z \exp\left(\pi z^2\right)$ **(b)** $-2\sin(2z) - \left(i/z^2\right)\cos(1/z)$
(c) $2\cos(2z)\exp[\sin(2z)]$ **(d)** $3\tan^2 z \sec^2 z$
(e) $2(\sinh z + 1)\cosh z$ **(f)** $1 - \tanh^2 z = \operatorname{sech}^2 z$

17. **(a)** $z = ik\pi/2, k = 0, \pm1, \pm2, \ldots$ **(b)** $z = 2k\pi - i\ln 3, k = 0, \pm1, \pm2, \ldots$
(c) no solution

19. Hint: If $z_1 \neq z_2$ and $e^{z_1} = e^{z_2}$, then $|z_1 - z_2| = |2k\pi i| \geq 2\pi$

21. **(a)** The plane cut along the negative imaginary axis and along the interval $[-1, 1]$.
(b) Upper half plane

25. $e^{1/z} = w$ when $z = \frac{1}{\operatorname{Log} w + i2\pi k}$, so it is easy to choose an integer k that makes $|z| \leq 0.001$ for any of these w.

Exercises 3.3, Page 123

1. **(a)** $i(\pi/2 + 2k\pi), k = 0, \pm1, \pm2, \ldots$
(b) $\frac{1}{2}\operatorname{Log} 2 + i(7\pi/4 + 2k\pi), k = 0, \pm1, \pm2, \ldots$
(c) $-i\pi/2$ **(d)** $\operatorname{Log} 2 + i(\pi/6)$

5. **(a)** $\operatorname{Log} 2 + i(\pi/2 + 2k\pi), k = 0, \pm1, \pm2, \ldots$ **(b)** $\pm\sqrt[4]{2}\exp(i\pi/8)$
(c) $i(2\pi/3 + 2k\pi), i(4\pi/3 + 2k\pi), k = 0, \pm1, \pm2, \ldots$

9. cut plane: $\mathbf{C} \setminus \{z = x + i : x \geq 4\}$, $f'(z) = -1/(4 + i - z)$

11. (one example) $f(z) = \operatorname{Log}\left(z^2 + 2z + 3\right), f'(-1) = 0$

13. **(a)** $\operatorname{Log}(2z - 1)$
(b) $\mathcal{L}_0(2z - 1)$, where $\mathcal{L}_0\left(re^{i\theta}\right) = \operatorname{Log} r + i\theta, 0 < \theta < 2\pi$
(c) $\mathcal{L}_{\pi/2}(2z - 1)$, where $\mathcal{L}_{\pi/2}\left(re^{i\theta}\right) = \operatorname{Log} r + i\theta, \pi/2 < \theta < 5\pi/2$

15. $w = (1/\pi)\operatorname{Log} z$

19. Choose a branch of $\arg z$ that is continuous on the complement of the half-parabola; say, $\arg(re^{i\theta}) = \theta$, where $g(r) < \theta < g(r) + 2\pi$,
$g(r) = \operatorname{Tan}^{-1}\sqrt{2/(\sqrt{1 + 4r^2} - 1)}$.

Exercises 3.4, Page 129

1. 0.5

3. 0

5. $\frac{B-A}{\text{Log}\, r_1 - \text{Log}\, r_2} \text{Log}\,|z| + \frac{A\, \text{Log}\, r_1 - B\, \text{Log}\, r_2}{\text{Log}\, r_1 - \text{Log}\, r_2}$

7. $\text{Im}[(\log z)^2/2]$

Exercises 3.5, Page 136

1. **(a)** $\exp(-\pi/2 - 2k\pi),\ k = 0, \pm 1, \pm 2, \ldots$
(b) $+1, -\frac{1}{2} + i\frac{\sqrt{3}}{2}, -\frac{1}{2} - i\frac{\sqrt{3}}{2}$
(c) $\exp[-2k\pi^2 + \pi i\, \text{Log}\, 2],\ k = 0, \pm 1, \pm 2, \ldots$
(d) $(1+i)\exp[2k\pi + \pi/4 - (i/2)\text{Log}\, 2],\ k = 0, \pm 1, \pm 2, \ldots$
(e) $-2 + 2i$

3. **(a)** 2 **(b)** $e^{-\pi}$ **(c)** $(1+i)\exp[(i/2)\text{Log}\, 2 - \pi/4]$

5. Take, for example, $z_1 = -1 + i$, $z_2 = i$, $\alpha = 1/2$.

7. $(1+i)\exp(-\pi/2)$

11. $z = \pi/4 + k\pi,\ k = 0, \pm 1, \pm 2, \ldots$

15. **(a)** $i \exp\left[\frac{1}{2}\text{Log}\,(1 - z^2)\right]$ **(b)** $z \exp\left[\frac{1}{2}\text{Log}\,(4/z^2 + 1)\right]$
(c) $z^2 \exp\left[\frac{1}{2}\text{Log}\,(1 - 1/z^4)\right]$ **(d)** $z \exp\left[\frac{1}{3}\text{Log}\,(1 - 1/z^3)\right]$

17. $0 < \text{Sec}^{-1} x < \pi/2$ for $x > 1$; $\pi/2 < \text{Sec}^{-1} x < \pi$ for $x < -1$

19. $z = \log(-1 \pm \sqrt{1+w})$; $z = \text{Log}\, 3 + i\pi(2k+1)$ or $z = i2\pi k$, k any integer

Exercises 3.6, Page 143

1. $I_s = \frac{\sin(\omega t - \phi_0)}{R}$

5. $I_s \to \frac{\cos(\omega t)}{R}$

7. **(b)** Any number of symmetrically placed inductors will work, other than for two. (Two will work if they are placed at right angles.)

CHAPTER 4

Exercises 4.1, Page 159

1. **(a)** $z(t) = (1+i) + t(-3-4i), 0 \le t \le 1$
(b) $z(t) = 2i + 4e^{-it}, 0 \le t \le 2\pi$
(c) $z(t) = Re^{it}, \pi/2 \le t \le \pi$
(d) $z(t) = t + it^2, 1 \le t \le 3$

3. $z(t) = a\cos t + ib\sin t, 0 \le t \le 2\pi$

5. yes

7. $$z(t) = \begin{cases} -1 - i + 8t & 0 \le t \le \frac{1}{4} \\ 1 - i + 8i\left(t - \frac{1}{4}\right) & \frac{1}{4} \le t \le \frac{1}{2} \\ 1 + i - 8\left(t - \frac{1}{2}\right) & \frac{1}{2} \le t \le \frac{3}{4} \\ -1 + i - 8i\left(t - \frac{3}{4}\right) & \frac{3}{4} \le t \le 1 \end{cases}$$
length $= 8$

9. $$z(t) = \begin{cases} -2 + \exp(-6\pi i t) & 0 \le t \le \frac{1}{3} \\ -1 + 6\left(t - \frac{1}{3}\right) & \frac{1}{3} \le t \le \frac{2}{3} \\ 2 - \exp\left[6\pi i\left(t - \frac{2}{3}\right)\right] & \frac{2}{3} \le t \le 1 \end{cases}$$

11. 15π

13. **(a)** instantaneous velocity at time t
(b) instantaneous speed at time t
(c) infinitesimal (differential) distance traveled during time interval dt
(d) distance traveled from time $t = a$ to time $t = b$

Exercises 4.2, Page 170

1. yes

3. **(a)** $1 + i/3$ **(b)** $(1+i)\sinh 2$
(c) $\frac{i}{12}\left[1 - (1+2i)^6\right] = \frac{11}{3} - \frac{29i}{3}$ **(d)** $\frac{1}{2i} - \frac{1}{8+2i} = -\frac{2}{17} - \frac{8i}{17}$

5. $4\pi i$

7. $\frac{1}{2} + i$

9. $\frac{13}{10} + \frac{i}{6}$

11. **(a)** $3 + i$ **(b)** $3 + i$ **(c)** $3 + i$

13. $-2i$

Exercises 4.3, Page 178

1. **(a)** $-3+2i$ **(b)** $-2\sinh 1$ **(c)** $i\pi$ **(d)** 0 **(e)** $-\frac{i}{3}\sinh^3 1$
   (f) $\frac{e^\pi}{2}+\frac{e^i}{2}(\cosh 1 + i\sinh 1)$ **(g)** $-\frac{\sqrt{2}}{3}-\frac{2}{3}(\sqrt{\pi})^3+\frac{i\sqrt{2}}{3}$
   (h) $\pi - 2 + i\left(2-\frac{\pi^2}{4}\right)$ **(i)** $\frac{\pi}{4}-\frac{1}{2}\arctan 2+\frac{i}{4}\operatorname{Log} 5$
5. Hint: Consider Theorem 7.
7. Hint: Consider a branch of $\log(z-z_0)$ whose branch cut does not intersect C.

Exercises 4.4, Page 199

1. (a), (c)
3. (a), (b), (d), (e)
5. $z(s,t)=(2-s)\cos 2\pi t + i(3-2s)\sin 2\pi t,\ 0\le s, t\le 1$
9. (a), (c), (d), (f)
11. Since the whole plane **C** is simply connected, Theorem 10 (or Theorem 13) applies.
13. **(a)** π **(b)** 0 **(c)** $-\pi$
15. $-4\pi i$
17. 0

Exercises 4.5, Page 212

1. 0
3. **(a)** $-2\pi i$ **(b)** $\frac{3\pi i e^{3/2}}{2}$ **(c)** $\frac{2\pi i}{9}$ **(d)** $10\pi i$ **(e)** $-2e\pi i$ **(f)** $-i\pi/2$
5. $G(1)=4\pi i,\ G'(i)=-2\pi(2+i),\ G''(-i)=4\pi i$
7. $\frac{-2\pi i}{9}$
9. $\left|f^{(n)}(0)\right|\le Mn!$
11. $\frac{\partial^2 u}{\partial x^2}=\operatorname{Re} f''$
13. $g(z)$ is not analytic inside Γ; note $G(z)\equiv 0$.

Exercises 4.6, Page 219

3. Hint: Apply the Cauchy estimates to the disk $\{\zeta : |\zeta - z| \le r - |z|\}$.

5. $|e^{f(z)}| \le e^M$, so $e^{f(z)} \equiv$ constant, which implies $f'(z)e^{f(z)} \equiv 0$. Thus $f'(z) \equiv 0$.

7. If f is entire and $|f(z)| \le M|z|^n$ for $|z| > r_0$, where n is a nonnegative integer, then f must be a polynomial of degree at most n.

15. Hint: Suppose f does not vanish and apply the maximum modulus principle as well as the minimum modulus principle (Prob. 14).

17. $\dfrac{9\sqrt{2}}{8}$

Exercises 4.7, Page 225

1. $\phi(z) \equiv -5$

5. Consider $\phi_1(x, y) = y$ and $\phi_2(x, y) \equiv 0$ in the upper half-plane.

11. 3

15. $\dfrac{1}{\pi}\left[\tan^{-1}\left(\dfrac{1-x}{y}\right) - \tan^{-1}\left(\dfrac{-1-x}{y}\right)\right]$ for $y > 0$.

CHAPTER 5

Exercises 5.1, Page 239

1. **(a)** $\dfrac{9}{10} + i\,\dfrac{3}{10}$ **(b)** $3(1-i)$ **(c)** $\dfrac{3}{5}$ **(d)** $\dfrac{-2+i}{5 \cdot 2^{13}}$ **(e)** $\dfrac{9}{8}$ **(f)** -1

3. Hint: If $z_n, z_{n+1}, z_{n+2}, \ldots$ are within ε of their limit L, how far apart can any two z_j be?

5. Apply Prob. 3.

7. **(a)** diverges **(b)** converges **(c)** diverges **(d)** diverges **(e)** converges **(f)** diverges

9. Hint: How does $|z|$ compare with $|x| + |y|$?

11. **(a)** $|z| < 1$ **(b)** $|z - i| < 2$ **(c)** all z **(d)** $|z + 5i| < 1$

17. Hint: Apply Log to the inequality $x^n < \dfrac{1}{2}$.

Exercises 5.2, Page 249

3. Hint: Use the chain rule to find the derivatives of the composite functions.

5. **(a)** $\sum_{j=0}^{\infty}(-z)^j$, $|z|<1$ **(b)** $\sum_{j=0}^{\infty}\frac{(-1)^j z^{2j}}{j!}$, all z

(c) $\sum_{j=0}^{\infty}\frac{(-1)^j 3^{2j+1} z^{2j+4}}{(2j+1)!}$, all z

(d) $\sum_{j=0}^{\infty}\frac{i^j\left[1+(-1)^j\right]-i}{j!}z^j$, all z

(Note that $\left[1+(-1)^j\right]$ vanishes for odd j.)

(e) $i+\sum_{j=1}^{\infty}\frac{2}{(1-i)^{j+1}}(z-i)^j$, $|z-i|<\sqrt{2}$

(f) $\frac{1}{\sqrt{2}}\left\{1-\left(z-\frac{\pi}{4}\right)-\frac{1}{2!}\left(z-\frac{\pi}{4}\right)^2+\frac{1}{3!}\left(z-\frac{\pi}{4}\right)^3+\frac{1}{4!}\left(z-\frac{\pi}{4}\right)^4-\cdots\right\}$, all z

(g) $\sum_{j=1}^{\infty} jz^j\left(=z\frac{d}{dz}\frac{1}{1-z}\right)$, $|z|<1$

7. $2\sum_{j=0}^{\infty}\frac{z^{2j+1}}{2j+1}$, $|z|<1$

11. **(a)** $1+z-\frac{z^3}{3}+\cdots$ **(b)** $-1-2z-\frac{5z^2}{2}-\ldots$

(c) $1+\frac{z^2}{2}+\frac{5z^4}{24}+\cdots$ **(d)** $z-\frac{z^3}{3}+\frac{2z^5}{15}+\cdots$

13. $\frac{z(1+z)}{(1-z)^3}$

17. $f(z)=(1-z)^{-1}$ is not analytic at $z=1$.

19. nine terms ($n=0$ to 8)

Exercises 5.3, Page 258

3. **(a)** $|z|=1$ **(b)** $|z-1|=\frac{1}{2}$ **(c)** $|z|=0$ **(d)** $|z-i|=3$

(e) $|z+2|=\frac{1}{\sqrt{10}}$ **(f)** $|z|=2$

5. **(a)** $\frac{6!6^3}{3^6}$ **(b)** $2\pi i$ **(c)** 0 **(d)** 0

7. $z + \dfrac{z^3}{3} + \dfrac{z^5}{10}$

9. Hint: The polynomials are analytic inside and on C.

11. **(b)** Hint: Argue that two analytic functions that agree on a real interval must have identical derivatives.

13. **(a)** $\sum_{k=0}^{\infty} \dfrac{z^{2k}}{2^k k!} = e^{z^2/2}$

(b) $1 + z - \dfrac{4}{2!}z^2 - \dfrac{4}{3!}z^3 + \dfrac{4^2}{4!}z^4 + \dfrac{4^2}{5!}z^5 + \cdots$

$= \left[1 - \dfrac{(2z)^2}{2!} + \dfrac{(2z)^4}{4!} - \cdots\right] + \dfrac{1}{2}\left[(2z) - \dfrac{(2z)^3}{3!} + \dfrac{(2z)^5}{5!} - \cdots\right]$

$= \cos 2z + \dfrac{1}{2}\sin 2z$

(c) $\sum_{k=0}^{\infty} (k+1)z^{2k} = \dfrac{1}{(1-z^2)^2}$

15. **(a)** $\sum_{k=0}^{\infty} \dfrac{(-1)^k}{(2k+1)!}\left[\int_{-1}^{2} t^{2k+1} g(t)\, dt\right] z^{2k+1}$

(b) Hint: Differentiate (a) termwise.

Exercises 5.4, Page 266

1. **(a)** 2 **(b)** ∞ **(c)** 0 **(d)** ∞

3. **(a)** 2 **(b)** 1 **(c)** $\dfrac{1}{3}$

(d) e (Hint: Use the ratio test.) **(e)** 1 **(f)** 1

5. **(a)** R **(b)** R^4 **(c)** $\sqrt{R}$ **(d)** R

(e) ∞ (if $R > 0$)

9. $f(z) = a_0 + \sum_{j=0}^{\infty} z^{2^j}$, $|z| < 1$

11. $1, \zeta, \dfrac{(3\zeta^2 - 1)}{2}, \dfrac{(5\zeta^3 - 3\zeta)}{2}$

Exercises 5.5, Page 276

1. **(a)** $\sum_{j=-1}^{\infty} (-1)^{j+1} z^j$ **(b)** $\sum_{j=2}^{\infty} (-1)^j z^{-j}$

(c) $-\sum_{j=-1}^{\infty} (z+1)^j$ **(d)** $\sum_{j=2}^{\infty} (z+1)^{-j}$

3. **(a)** $\frac{1}{3}\sum_{j=0}^{\infty}\left[(-1)^j - \left(\frac{1}{2}\right)^j\right]z^j$

(b) $\frac{1}{3}\sum_{j=1}^{\infty}(-1)^{j-1}z^{-j} - \frac{1}{3}\sum_{j=0}^{\infty}\left(\frac{1}{2}\right)^j z^j$

(c) $\frac{1}{3}\sum_{j=1}^{\infty}\left[(-1)^{j-1} + 2^j\right]z^{-j}$

5. $\frac{\frac{5}{4}}{(z-4)^3} + \sum_{j=-2}^{\infty}(-1)^{j+1}\left(\frac{1}{4}\right)^{j+4}(z-4)^j$

7. **(a)** $\frac{1}{z^2} + \frac{1}{z^3} + \frac{3}{2z^4} + \cdots$ **(b)** $\frac{1}{z} - \frac{1}{2} + \frac{z}{12} + \cdots$

(c) $\frac{1}{z} + \frac{z}{6} + \frac{7z^3}{360} + \cdots$ **(d)** $\frac{1}{e}\left[1 + z + \frac{z^2}{2!} + \cdots\right]$

9. $\frac{1}{2} < |z| < 2$

11. $\sum_{j=n}^{\infty}\alpha^{j-n}\frac{(j-1)!}{(j-n)!(n-1)!}z^{-j}$

13. Hint: Use Eq. (1).

Exercises 5.6, Page 285

1. **(a)** pole of order 2 at 0, removable singularity at -1

(b) essential singularity at 0 **(c)** simple poles at $\pm i$

(d) simple poles at $2n\pi i$ ($n = 0, \pm 1, \pm 2, \ldots$)

(e) simple poles at $\frac{2n+1}{2}\pi$ ($n = 0, \pm 1, \pm 2, \ldots$)

(f) essential singularity at 0

(g) removable singularity at 0

(h) essential singularity at 0, simple poles at $\frac{1}{n\pi}$ ($n = \pm 1, \pm 2, \ldots$)

3. Possible answers are:

(a) $\frac{(z-i)^2}{(z-2+3i)^5}$ **(b)** $ze^{1/(z-1)}$ **(c)** $\frac{(\sin z)e^{1/(z-i)}}{z(z-1)^6}$ **(d)** $\frac{e^{1/[z(z-1)]}}{(z-1-i)^2}$

5. **(a)** false **(b)** true **(c)** true **(d)** false **(e)** true

7. essential

9. Yes; $e^{1/z}$ is bounded on the negative real axis.

11. Hint: If, say, Re $f \leq M$ then $e^{f(z)}$ would have a removable singularity. Now take the log.

13. Hint: Choose a tiny contour in the formula for a_{-j}.

19. **(b)** $0 + (\frac{1}{6} - \frac{2}{\pi^2})z + (0)z^2 + (\frac{7}{360} - \frac{2}{\pi^4})z^3 + (0)z^4 + \cdots$, radius $= 2\pi$

(c) $\cdots + \frac{-2\pi^2}{z^3} - \frac{1}{z} + (\frac{1}{6} - \frac{2}{\pi^2})z + (\frac{7}{360} - \frac{2}{\pi^4})z^3 + \cdots$

Exercises 5.7, Page 290

1. **(a)** essential singularity **(b)** essential singularity **(c)** analytic **(d)** zero of order 2 **(e)** pole of order 2 **(f)** essential singularity **(g)** essential singularity **(h)** not isolated **(i)** analytic

3. **(a)** $1 + 2\sum_{j=1}^{\infty}(-1)^j/z^j$, $|z| > 1$ **(b)** $\sum_{j=0}^{\infty}(-1)^j/z^{2j}$, $|z| > 1$

(c) $\sum_{j=0}^{\infty} i^j/z^{3j+3}$, $|z| > 1$.

5. (deg Q) $-$ (deg P)

7. Observe $\oint_{|z|=1} \frac{dz}{z} = 2\pi i$

11. deg(P) - deg(Q) = 1, 0, or -1

Exercises 5.8, Page 301

1. z^2

3. $\sin\frac{1}{1-z}$ vanishes at $z = 1 - \frac{1}{n\pi}$, $n = 1, 2, \ldots$

5. all values except $0 \leq \alpha < 1$

7. They both sum to $\frac{1}{1-z}$.

9. **(a)** no **(b)** yes **(c)** yes **(d)** no **(e)** yes **(f)** yes

11. $f(z) = zg'(z)$

15. If $\phi(x, y) \to 0$, it can be harmonically extended as an odd function of y. If $\partial\phi/\partial y \to 0$, ϕ can be harmonically extended as an even function of y.

CHAPTER 6

Exercises 6.1, Page 313

1. **(a)** $\text{Res}(2) = e^6$ **(b)** $\text{Res}(1) = -2$, $\text{Res}(2) = 3$ **(c)** $\text{Res}(0) = 0$
(d) $\text{Res}(-1) = -6$ **(e)** $\text{Res}(0) = 1$, $\text{Res}(-1) = -5/2e$
(f) $\text{Res}(0) = \dfrac{1}{3}$
(g) $\text{Res}\left[\pm\dfrac{(2n+1)\pi}{2}\right] = -1$, $n = 0, 1, 2, \ldots$
(h) $\text{Res}(n\pi) = (-1)^n(n\pi - 1)$, $n = 0, \pm 1, \pm 2, \ldots$
(i) $\text{Res}(1) = -2$

3. **(a)** $\pi i \sin 2$ **(b)** $\dfrac{\pi i\,(e^2 - 1)}{4}$ **(c)** $-8\pi i$
(d) $\pi i\left[\dfrac{(2-5i)e^{2i}}{58} - \dfrac{12-5i}{50}\right]$ **(e)** $\dfrac{\pi i}{3}$ **(f)** 0 **(g)** 0

5. no; yes ($1/z^2$, for example)

7. $2\pi i$

Exercises 6.4, Page 336

5. $\dfrac{\pi \sin 3}{2e^3}$

7. $\dfrac{\pi}{3e}\left(1 - \dfrac{1}{2e}\right)$

9. $\dfrac{i\pi}{e^6}$

11. $m > 0$, $\deg P < \deg Q$

Exercises 6.5, Page 344

1. **(a)** $\dfrac{i\pi}{2}$ **(b)** $\dfrac{3\pi i e^{3i}}{8}$ **(c)** 0 **(d)** $-\pi i$

9. $\dfrac{3\pi}{4}$

11. $-\pi\cot(a\pi)$

Exercises 6.6, Page 354

9. **(a)** $\pi/4 - \text{Log}\sqrt{2}$ **(b)** $2\pi\sqrt{3}/9$

13. $2\pi\sqrt{3}/3$

Exercises 6.7, Page 364

1. (a), (c), (e), and (f)

3. 1

7. Hint: Compare with $f(z) = 27$.

9. 4

13. Easy: let $h(z) = -f(z)$

15. Hint: f and $f + h$ have the same poles; now apply Prob. 14.

21. A "zero" for $F(z)$ is a "minus 1" for $P(z)$; apply the argument principle.

CHAPTER 7

Exercises 7.1, Page 374

1. $\text{Log}\,|w| + \text{Arg}\,w$; $e^x \cos y + e^x \sin y$

3. $a_4 + \dfrac{1}{\pi}\sum_{k=1}^{3}(a_k - a_{k+1})\,\text{Arg}\,(z - x_k)$

5. $\dfrac{1}{2} - \dfrac{1}{2}\dfrac{x^2 + y^2 - 1}{(1 + x)^2 + y^2}$

7. These are the Cauchy-Riemann equations for $f^{-1}(w)$.

Exercises 7.2, Page 382

1. **(a)** 1; $f(-1 + \zeta) = f(-1 - \zeta)$ **(b)** 1; $f(n\pi + \zeta) = f(n\pi - \zeta)$
(c) 2; $f(r) = f\left(re^{i2\pi/3}\right) = f\left(re^{i4\pi/3}\right)$

3. Angles increase (decrease) for $\alpha > 1$ ($\alpha < 1$).

5. pure imaginary constants

11. **(a)** the whole upper half-plane: $\text{Im}\,w > 0$
(b) the whole plane minus the logarithmic spiral $\rho = e^{\phi}, -\infty \leq \phi < \infty$
(c) $\{w : |w| < 1, \text{Im}\,w > 0\}$
(d) $\{w : |w| > 1, \text{Im}\,w > 0\}$

(e) the upper half-annulus $\{w : e < |w| < e^2, \operatorname{Im} w > 0\}$

(f) $\{w : |w| > 1\}$ and $\{w : |w| < 1\}$

13. **(a)** the upper half-plane: $\operatorname{Im} w > 0$ **(b)** fourth quadrant

(c) the whole plane minus the real intervals $(-\infty, -1]$, $[1, \infty)$

(d) the interior of the ellipse $(u^2/\cosh^2 1) + (v^2/\sinh^2 1) = 1$ excluding the real segments $[-\cosh 1, -1]$, $[1, \cosh 1]$

Exercises 7.3, Page 392

1. $w = 3iz + 5$

3. **(a)** $\{w : |w - 2 + 2i| \le 1\}$ **(b)** $\{w : |w - 6i| \le 3\}$

(c) $\{w : \operatorname{Re}(w) \le \frac{1}{2}\}$

(d) $\{w : \operatorname{Re}(w) \ge \frac{3}{2}\}$ **(e)** $\{w : |w - \frac{2}{3}| \le \frac{1}{3}\}$

5. $w = e^{3\pi i/4}\left(\dfrac{z+i}{z-1}\right)$

7. **(a)** $w = iz$ **(b)** $w = \dfrac{2z}{z+1}$ **(c)** $w = \dfrac{z+i}{z-1}$ **(d)** $w = \dfrac{z+1}{z-1}$

9. the region exterior to both of the circles $C_1 : |w - (1-i)/2| = 1/\sqrt{2}$ and $C_2 : |w - (1+i)/2| = 1/\sqrt{2}$

11. $w = \exp\left(4\pi\left[-i\left(\dfrac{1}{z-2} + \dfrac{1}{4}\right)\right]\right)$

Exercises 7.4, Page 403

1. $z - 2$

3. **(a)** $\dfrac{4-3i}{25}$ **(b)** $\dfrac{7-i}{6}$ **(c)** $\dfrac{5-2i}{3}$

5. $\lambda > 0$

7. No. (This would violate the symmetry principle.)

9. 0

17. $w = \dfrac{i-2z}{2+iz}e^{i\theta}$ (any real θ)

19. $w = \dfrac{az+b}{cz+d}$ with a, b, c, d real and $ad - bc > 0$

21. **(b)** No. (A shift, for example, does not commute with an inversion.)

Exercises 7.5, Page 416

1. $w = A(z - x_1)^2 - 1$, where $A < 0$

3. $w = \dfrac{i}{2} - \dfrac{i}{\pi}\sin^{-1} z$

5. $w = -\dfrac{2}{\pi}\left(\sin^{-1} z + z\sqrt{1 - z^2}\right)$

7. $w = \dfrac{i}{\pi}\sqrt{z^2 - 1} + \dfrac{\sin^{-1} z}{\pi} + \dfrac{1}{2}$

9. $w = \dfrac{1}{\pi}\operatorname{Log}\left(\dfrac{z - x_2}{x_2 - x_1}\right)$, where $x_1 < x_2$

11. The analytic continuation of the S-C transformation across the interval (x_{j-1}, x_j) maps the lower half-plane onto the figure obtained by reflecting the polygon P through the mirror containing $f(x_{j-1})$ and $f(x_j)$.

Exercises 7.6, Page 430

1. $\phi(x, y) = \dfrac{2}{\pi}\operatorname{Arg}\left(\dfrac{1 + z}{1 - z}\right)$

3. $T(x, y) = \dfrac{2}{\pi}\operatorname{Arg}\left(\dfrac{1 + z^2}{1 - z^2}\right) - 1$

5. $\phi(x, y) = \dfrac{1}{\pi}\operatorname{Arg}\left(\dfrac{\sqrt{z^2 + 1} - 1}{\sqrt{z^2 + 1} + 1}\right)$

7. **(a)** $T(x, y) = 1 - \dfrac{1}{\pi}[\operatorname{Arg}(\cos(\pi i z) + 1) + \operatorname{Arg}(\cos(\pi i z) - 1)]$

(b) $T(x, y) = \dfrac{2}{\pi}\operatorname{Arg}(e^{\pi z} + 1) - \dfrac{1}{\pi}\operatorname{Arg}(e^{\pi z})$

9. $\phi(x, y) = (\operatorname{Log} s)^{-1}\operatorname{Log}\left|\dfrac{4 - \lambda z}{4\lambda - z}\right|$, where $\lambda = \dfrac{19 + \sqrt{105}}{16}$ and $s = \dfrac{13 - \sqrt{105}}{8}$.

11. A parametric representation of the streamlines is obtained by holding y constant in the S-C mapping equation $w = f(x + iy)$.

Exercises 7.7, Page 439

1. isotherms: $z(t) = g(a + it)$, $t \geq 0$, $-\frac{\pi}{2} < a < \frac{\pi}{2}$, where $g(w) = \frac{1}{2} + \frac{w - \cos w}{\pi}$ maps the half-strip $-\frac{\pi}{2} < u < \frac{\pi}{2}$, $v > 0$ onto the given region.

3. $T(z) = \dfrac{2}{\pi}\operatorname{Re}\left[\sin^{-1}\left(z^2\right)\right]$

5. $T(z) = -\frac{20}{\pi}\operatorname{Re}\left[\sin^{-1}\left(-e^{-\pi z}\right)\right]$

7. streamlines: $\operatorname{Im}\left(z^{\pi/\alpha} + z^{-\pi/\alpha}\right) = \text{constant}$

CHAPTER 8

Exercises 8.1, Page 459

1. **(a)** $\frac{i}{8}\left[-3e^{it} + 3e^{-it} + e^{3it} - e^{-3it}\right]$

(b) $\sum_{n=-\infty}^{\infty} \frac{12(-1)^n}{\pi\,(9-4n^2)\,(1-4n^2)}\, e^{i2nt/3}$

(c) $c_n = \frac{2(-1)^n}{n^2}$ for $n \neq 0$; $c_0 = \frac{\pi^2}{3}$

(d) $c_n = (-1)^n\left[\frac{i\pi}{n} - \frac{2i}{\pi n^3}\right] + \frac{2i}{\pi n^3}$ for $n \neq 0$; $c_0 = 0$

3. (a), (b)

5. $\overline{c}_n = c_{-n}$

7. **(a)** $\frac{1}{8} + \frac{\cos 2t}{2} - \frac{\cos 4t}{104}$

(b) $\frac{\pi^2}{3} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{c_n e^{int}}{1 + in - n^2}$, c_n as in Prob. 1(c).

(c) $\sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{c_n e^{int}}{2 + 4in - n^2}$, c_n as in Example 4.

11. $u(x,t) = \sum_{n=1}^{\infty} b_n \sin nx \cos nt + \sum_{n=1}^{\infty} c_n \sin nx \sin nt$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f_1(\xi)\sin n\xi\, d\xi$$

$$c_n = \frac{2}{\pi n}\int_0^{\pi} f_2(\xi)\sin n\xi\, d\xi$$

Exercises 8.2, Page 473

1. **(a)** $G(\omega) = \frac{1}{\pi}\left(\frac{1}{1+\omega^2}\right)$ **(b)** $G(\omega) = \frac{1}{2\sqrt{\pi}}\, e^{-\omega^2/4}$ **(c)** $G(\omega) = \frac{-i\omega e^{-\omega^2/4}}{4\sqrt{\pi}}$

(d) $G(\omega) = \begin{cases} \frac{1}{2} & \text{if } |\omega| < 1 \\ 0 & \text{if } |\omega| > 1 \\ \frac{1}{4} & \text{if } \omega = \pm 1 \end{cases}$ **(e)** $G(\omega) = \begin{cases} -\frac{i}{2}\sin\omega & \text{if } |\omega| \leq \pi \\ 0 & \text{if } |\omega| \geq \pi \end{cases}$

3. **(a)** $G(\omega) = \dfrac{e^{-|\omega|/\sqrt{2}}}{2\sqrt{2}}\left(\cos\dfrac{|\omega|}{\sqrt{2}} + \sin\dfrac{|\omega|}{\sqrt{2}}\right)$

(b) $G(\omega) = -\dfrac{ie^{-|\omega|/\sqrt{2}}}{2}\sin\dfrac{\omega}{\sqrt{2}}$

(c) $G(\omega) = \dfrac{1}{2\sqrt{\pi}}e^{-\omega^2/4}$

Exercises 8.3, Page 484

1. **(a)** $\dfrac{3s}{s^2+4} - \dfrac{8}{s+2}$ **(b)** $\dfrac{2}{s} - \dfrac{\pi}{(4-s)^2+\pi^2}$ **(c)** $\dfrac{1}{s}(1-e^{-s})$

(d) $\dfrac{1}{s}(e^{-s} - e^{-2s})$ **(e)** $\dfrac{2}{s(s^2+4)}$ **(f)** $\sqrt{\dfrac{\pi}{s}}$

3. **(a)** $\dfrac{1}{2}\sin(2t)$ **(b)** $4te^t$ **(c)** $e^{-2t} - te^{-2t}$

(d) $\dfrac{1}{2}\left[1 - 2e^{-t} + e^{-2t}\right]$ **(e)** $e^{-2t}\left(\cos\sqrt{3}t + \dfrac{1}{\sqrt{3}}\sin\sqrt{3}t\right)$

5. **(a)** $f(t) = \dfrac{1}{2}e^{3t} + \dfrac{5}{2}e^t$

(b) $f(t) = 4e^{2t} - 3e^{3t}$

(c) $f(t) = \dfrac{28}{39}e^{2t} - \dfrac{5}{6}e^{-t} - \dfrac{1}{13}e^{-t}\sin 2t + \dfrac{3}{26}e^{-t}\cos 2t$

(d) $f(t) = \begin{cases} 0 & \text{if } 0 \le t \le 3 \\ \frac{1}{2} + \frac{1}{2}e^{2t-6} - e^{t-3} & \text{if } 3 \le t \le 6 \\ \frac{1}{2}e^{2t-6} - \frac{1}{2}e^{2t-12} + e^{t-6} - e^{t-3} & \text{if } t \ge 6 \end{cases}$

7. **(b)** Hint: Consider the partial fraction expansion of $F(s)$, and compare with the table entries.

9. **(b)** i. $x(t) = \cos\sqrt{3}t$, $y(t) = -\cos\sqrt{3}t$;

ii. $x(t) = y(t) = \cos t$;

iii. $x(t) = \dfrac{1}{2}\left(\cos t + \cos\sqrt{3}t\right)$, $y(t) = \dfrac{1}{2}\left(\cos t - \cos\sqrt{3}t\right)$.

(c) (i), (ii)

Exercises 8.4, Page 494

3. Multiply the radii by $|\alpha|$.

5. **(a)** $a(n) = -(-3)^{-n}(n \le -1)$, 0 otherwise

(b) $a(n) = \left(-\dfrac{1}{3}\right)^n (n \ge 0)$, 0 otherwise

(c) $a(n) = (-1)^n 2^{n+3} (n \leq -4)$, 0 otherwise

(d) $a(n) = (-2)^{n+3} (n \geq -3)$, 0 otherwise

(e) $a(n) = -\left(\frac{1}{2}\right)^n (n \geq 1), -3^{n-1} (n \leq -1), -\frac{1}{3} (n = 0)$

(f) $a(n) = 4\left(-\frac{1}{2}\right)^n - 3\left(-\frac{1}{4}\right)^n (n \geq 0)$, 0 otherwise

(g) $a(n) = 4\left(\frac{1}{2}\right)^n + 2n - 4 (n \geq 0)$, 0 otherwise

(h) $a(n) = \frac{1}{\alpha^n}\left[\alpha^{-1} - \alpha\right] (n \geq 1), \frac{1}{\alpha} (n = 0)$, 0 otherwise

7. **(a)** 2^{-n+1} **(b)** $\frac{1}{3} + \frac{2}{3}(-2)^n$ **(c)** $\frac{1}{2} + 2^{n+1} - \frac{3^n}{2}$

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3. Hint: Use Lemma 4, Section 6.5.

7. **(b)** p.v. $\frac{1}{x - x_0} - i\pi\delta(x - x_0)$