MAT2410 - Introduction to Complex Analysis

Assignment 2

To be returned no later than the 27th October 2016 at 14:30.

Problem 1. Find complex numbers a, b, c, d with $ad \neq bc$ such that the Möbius transformation

$$f(z) = \frac{az + b}{cz + d}$$

satisfies $f(\infty) = i$, $f(i) = \infty$, $f(1) = 2$.

Problem 2. Let

$$\gamma(t) := \pi t + (t^2 - t)i, \quad 0 \leq t \leq 1.$$

Compute the line integral

$$\int_\gamma z \sin z \, dz.$$

Problem 3. Let

$$f(z) := \frac{z^2 + 3z}{e^z - 1}.$$

(i) Show that $f(z)$ can be expressed as a power series in some punctured disk around the origin, i.e. for some $r > 0$ one has

$$f(z) = \sum_{k=0}^{\infty} a_k z^k \quad \text{for } 0 < |z| < r.$$

(ii) Compute the coefficients a_0, a_1, a_2.

(iii) Determine the radius of convergence of the series $\sum_{k=0}^{\infty} a_k z^k$.

Continued on the next page!

Problem 4. Let $f(z)$ be an entire function.

(i) Show that the function $g(z) := \overline{f(\bar{z})}$ is analytic.

(ii) Suppose f maps $\mathbb{R}$ into $\mathbb{R}$. Show that

$$f(\bar{z}) = \overline{f(z)} \quad \text{for all } z \in \mathbb{C}.$$

Problem 5. Use the Cauchy integral formula to compute the line integral

$$\int_{|z|=4} \frac{\cos z}{z^3 + \pi z^2}\, dz.$$