UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Examination in:	MAT2700 — Introduction to mathematical finance and investment theory.
Day of examination:	Thursday, December 13, 2007.
Examination hours:	14.30 – 17.30.

This examination set consists of 3 pages.

Appendices:	None.
Permitted aids:	None.

Make sure that your copy of the examination set is complete before you start solving the problems.

Problem 1.

Let us consider the following *complete* multi-period market model:

- Probability space $(\Omega, \mathcal{F}, P)$ with sample space $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and probability measure

$$P(\omega) = \begin{cases} 1/4, & \omega = \omega_1, \\ 1/4, & \omega = \omega_2, \\ 1/4, & \omega = \omega_3, \\ 1/4, & \omega = \omega_4, \end{cases}$$

- A bank account with interest rate $r = 0$

- One risky security with price process given by

$S(t,\omega)$	$t=0$	$t=1$	$t=2$
$\omega = \omega_1$	5	6	10
$\omega = \omega_2$	5	6	4
$\omega = \omega_3$	5	4	8
$\omega = \omega_4$	5	4	2

(Continued on page 2.)

With the above data, find the optimal portfolio optimizing the problem

$$\max_{H \in \mathbb{H}} E\big[u(V_2)\big]$$
$$V_0 = v$$

for the utility function

$$u(w) = 12w - \frac{w^2}{2}, \qquad 0 < w < 12.$$

Here $\mathbb{H}$ is the set of all self-financing trading strategies and V_t, $t = 0, 1, 2$, is the value process corresponding to the strategy $H \in \mathbb{H}$. Let $v = 2$.
Apply either dynamic programming or martingale methods (i.e. Lagrange multipliers method).

Problem 2.

Let us consider the following single period market model:

- A probability space $(\Omega, \mathcal{F}, P)$ with sample space $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ and probability measure $P(\omega) > 0$, $\omega \in \Omega$
- A bank account with interest rate $r = 1/6$.
- Two risky securities with prices given by

	$S_1(0,\omega)$	$S_1(1,\omega)$	$S_2(0,\omega)$	$S_2(1,\omega)$
$\omega = \omega_1$	7	35/3	5	7
$\omega = \omega_2$	7	7	5	7
$\omega = \omega_3$	7	35/6	5	14/3
$\omega = \omega_4$	7	35/3	5	14/3

1. Describe the set of all the replicable contingent claims.
2. Consider the following look-back option

$$X_{look-back} := \max\big\{0, S_1(0) - 8, S_1(1) - 8\big\}.$$

   2a. Is it replicable?

   2b. Find all non-arbitrage prices of $X_{look-back}$.
3. Consider the following put option

$$X_{put} = \max\big\{0, 8 - S_2(1)\big\}.$$

   3a. Is it replicable?

   3b. Find all non-arbitrage prices of X_{put}.

   3c. Find a replicating strategy for X_{put}.

(Continued on page 3.)

4. Consider the following call option

$$X_{call} = \max\big\{0, S_2(1) - 8\big\}.$$

   4a. Is it replicable?

   4b. Find all non-arbitrage prices of X_{call}.

5. For which values of e is the following look-back option

$$X_{look-back} := \max\big\{0, S_2(0) - e, S_2(1) - e\big\}.$$

   replicable?

Problem 3.

1. Give the definition of a martingale process M_t, $t = 0, 1, ..., T$, with respect to the filtration $\mathbb{F}$.

2. Take the market model of Exercise 1 into account. Compute the filtration $\mathbb{F}$ generated by the price process $S(t)$, $t = 0, 1, 2$.

3. Consider the market model of Exercise 1. Compute the variables of the stochastic process A_t, $t = 0, 1, 2$, such that

   - $A_0 = 0$,
   - A_t, $t = 0, 1, 2$, is predictable with respect to the filtration $\mathbb{F}$ generated by the price process,
   - $M_t := S^2(t) - A_t$, $t = 0, 1, 2$, is a martingale with respect to the filtration $\mathbb{F}$ generated by the price process.

END