UNIVERSITY OF OSLO

Faculty of mathematics and natural sciences

Examination in	MAT2700 — Introduction to Mathematical Finance and Investment Theory.
Day of examination:	Thursday, December 13th, 2012.
Examination hours:	14.30 – 18.30

This problem set consists of 2 pages.

Appendices:	None.
Permitted aids:	None.

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

All items (1a, 1b etc.) count 10 points.

Problem 1

We shall study a two-period marked with $r = \frac{1}{4}$, $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, and a single risky asset S given by

	$\omega = \omega_1$	$\omega = \omega_2$	$\omega = \omega_3$	$\omega = \omega_4$
$S(\omega, 0)$	32	32	32	32
$S(\omega, 1)$	70	70	30	30
$S(\omega, 2)$	100	75	50	0

a) Find the discounted process S^*.

b) Show that the probability measure $Q(\omega_1) = \frac{1}{8}, Q(\omega_2) = \frac{1}{8}, Q(\omega_3) = \frac{9}{16}, Q(\omega_4) = \frac{3}{16}$ is the unique risk neutral measure. Is the market arbitrage free and/or complete?

c) Find the value $V_0(X)$ at time 0 of the European call option

$$X = (S(\omega, 2) - 25)^+$$

d) Find a trading strategy replicating (attaining) the claim X above.

e) Find the value process $Z(\omega, t)$ of the American put option

$$Y(\omega, t) = (35 - S(\omega, t))^+$$

Should the option be exercised early? (In this problem you may want to use the dynamic programming equation

$$Z_{t-1} = \max\{Y_{t-1}, E_Q[Z_t B_{t-1}/B_t \,|\, \mathcal{F}_{t-1}]\}$$

for American options.)

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Problem 2

Consider a single period market $\mathcal{M}_1$ with $r = \frac{1}{4}$, $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $P(\omega_i) = \frac{1}{3}$ for $i = 1, 2, 3$, and a single risky asset S_1 given by

	$\omega = \omega_1$	$\omega = \omega_2$	$\omega = \omega_3$
$S_1(\omega, 0)$	9	9	9
$S_1(\omega, 1)$	15	10	5

a) Find all risk neutral measures for $\mathcal{M}_1$.

b) Explain that the market is not complete, and find a description of the attainable claims. Is the claim X given by $X(\omega_1) = 10, X(\omega_2) = 15, X(\omega_3) = 5$ attainable?

We add a new, risky asset S_2 given by

	$\omega = \omega_1$	$\omega = \omega_2$	$\omega = \omega_3$
$S_2(\omega, 0)$	8	8	8
$S_2(\omega, 1)$	10	15	5

The new, extended market is called $\mathcal{M}_2$.

c) Show that $\mathcal{M}_2$ is complete.

d) The utility of a consumption process (C_0, C_1) is given by

$$\ln(C_0) + E[\ln(C_1)]$$

Find the optimal, admissible investment plan (C_0, C_1) for market $\mathcal{M}_2$ when the initial wealth is $\nu = 300$.

Problem 3

Consider an arbitrage free, but incomplete, single period market $\mathcal{M}$ and an unattainable contingent claim X.

a) What do we mean by $V_-(X)$ and $V_+(X)$? Explain why $V_-(X) < V_+(X)$ in this case. (It suffices to refer to theory from the textbook; you don't have to prove anything).

We assume that $X > 0$ and that a is a positive, real number. We extend $\mathcal{M}$ to a new market $\mathcal{M}^+$ by adding a new asset S given by $S(0) = a$ and $S(\omega, 1) = X(\omega)$ for all $\omega \in \Omega$.

b) Show that if $a < V_-(X)$ or $a > V_+(X)$, then the extended market $\mathcal{M}^+$ has arbitrage opportunities.

c) Show that if $V_-(X) < a < V_+(X)$, then the extended market $\mathcal{M}^+$ is arbitrage free.

THE END