MAT2700: Mandatory Assignment, Fall 2012

Deadline: *You must turn in your paper before Thursday, November 1, 2012, 2.30 p.m. at the 7th floor of the Niels Henrik Abel Building. Remember to use the official front page available at*

http://www.mn.uio.no/math/studier/admin/obligatorisk-innlevering/obligforside.pdf

If you due to illness or other circumstances need extra time, you must apply for an extension to studieinfo@math.uio.no *before the deadline. Remember that illness has to be documented by a medical doctor! See*

http://www.mn.uio.no/math/studier/admin/obligatorisk-innlevering/obligregelverk-eng.html

for more information about the rules for mandatory assignments.

Instructions: *The assignment is compulsory, and students who do not get their paper accepted, will not get access to the final exam. To get the assignment accepted, you need a score of at least 60%. In the evaluation, credit will be given for a clear and well-organized presentation. All questions (points a), b) etc.) have equal weight. Students who do not get their original paper accepted, but who have made serious and documented attempts to solve the problems, will get* one *chance of turning in an improved version.*

In solving the problems you may collaborate with others and use tools of all kinds (you may, e.g., use MATLAB or a similar program to solve equations, but you have to explain how the solutions are obtained). However, the paper you turn in should be written by you *(by hand or computer) and should reflect* your *understanding of the material. If we are not convinced that you understand your own paper, we may ask you to give an oral presentation.*

The assignment has only one problem consisting of eight parts a)-h).

Problem

We shall study a market $\mathcal{M}_1$ with a bank account process B and two securities S_1, S_2 specified by:

(i) $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, $P(\omega_1) = \frac{1}{3}$, $P(\omega_2) = \frac{1}{6}$, $P(\omega_3) = \frac{1}{6}$, $P(\omega_4) = \frac{1}{3}$.

(ii) $r = \frac{1}{4}$, $S_1(0) = 2$, $S_2(0) = 4$ and

	$\omega = \omega_1$	$\omega = \omega_2$	$\omega = \omega_3$	$\omega = \omega_4$
$S_1(\omega, 1)$	3	3	2	2
$S_2(\omega, 1)$	3	6	5	6

a) Compute the returns R_1 and R_2 of the two stocks. Find all risk free measures and explain why the market is arbitrage free, but incomplete.

b) Describe all attainable contingent claims $X : \Omega \to \mathbb{R}$, and use the description to show that the claim given by $X(\omega_1) = 8$, $X(\omega_2) = 5$, $X(\omega_3) = 4$, $X(\omega_4) = 3$ is attainable.

c) Find a strategy (H_0, H_1, H_2) that replicates the contingent claim X above.

d) Show that the contingent claim given by $Y(\omega_1) = 3$, $Y(\omega_2) = 3$, $Y(\omega_3) = 2$, $Y(\omega_4) = 3$ is not attainable, and find the upper and lower value $V_+(Y)$ and $V_-(Y)$ of Y.

e) We create a new market $\mathcal{M}_2$ by adding a new security S_3 to $\mathcal{M}_1$. The new security is given by $S_3(0) = 6$ and $S_3(\omega_1, 1) = 9$, $S_3(\omega_2, 1) = 9$, $S_3(\omega_3, 1) = 9$, $S_3(\omega_4, 1) = 4$. Show that the new market is complete and find the time zero value $V(Y)$ of the above claim Y in the new market.

f) Find the optimal consumption process (C_0, C_1) in the new market $\mathcal{M}_2$ when the initial wealth is $\nu = 200$ and the utility function is $u(c) = \ln c$.

g) What is the trading strategy corresponding to the optimal consumption process in f)?

h) Assume that V_1 is the value at time 1 of a portfolio in *the original market* $\mathcal{M}_1$. Find the maximal value of $\mathrm{E}(\ln(V_1))$ when we assume that the value V_0 of the portfolio at time 0 is $\nu > 0$.

THE END