MAT2700 - MANDATORY ASSIGNMENT, FALL 2013

REMINDER: The assignment must be handed in before 15:00 on Thursday 31 October, 2013, at the reception of the Department of Mathematics, in the 7th floor of Niels Henrik Abels hus, Blindern. Be careful to give reasons for your answers. To have a passing grade you must have correct answers to at least 50% of the questions.

Exercise 1. Consider a single period market consisting of a probability space $\Omega = \{\omega_1, \omega_2, \omega_3\}$, probability measure

$$P(\omega) = \begin{cases} 1/3, & \omega = \omega_1, \\ 1/3, & \omega = \omega_2, \\ 1/3, & \omega = \omega_3, \end{cases}$$

bank account with $B_0 = 1$, $r = 0.1$, and one risky asset S_1 with

$$S_1(0) = 4, \qquad S_1(1)(\omega) = \begin{cases} 1.1, & \omega = \omega_1, \\ 2.2, & \omega = \omega_2, \\ 3.3, & \omega = \omega_3. \end{cases}$$

1a. Determine the discounted gain G^*.

1b. What is the definition of a dominant trading strategy? Show that a dominant trading strategy exists if and only if there exists a trading strategy satisfying $V(0) = 0$ and $V(1)(\omega) > 0$ for all $\omega \in \Omega$.

Show that $H = (H_0, H_1)^T$ with $H_0 = 4$ and $H_1 = -1$ is a dominant trading strategy.

1c. Determine all risk-neutral probability measures Q (if any), argue directly from the definition of a risk neutral probability.

Exercise 2. Consider a (single period) market in which a stock S, over the next year, can go up in value to NOK 75 (with probability of 60%) or down to NOK 30 (with probability of 40%). The stock is currently trading at NOK 50. The risk-free return (interest rate) is 5%.

2a. Let X be a call option that expires in one year with an exercise price NOK 55. Determine the arbitrage-free price of X.

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2b. Determine the replicating investment strategy for the claim X from part (a). What is the initial portfolio value?

2c. What is the mean (expected) return $\bar{R}_S$ from the stock? What is the mean return $\bar{R}$ from the replicating portfolio found in part (b). Indicate why Q is called risk-neutral.

2d. Compute the state price density L. Determine the trading strategy H' that generates L. Suppose $H = (-12.6984, 0.4444)^T$. Compute the beta of the trading strategy H with respect to the trading strategy H'.

2e. Use the call-put parity to compute the arbitrage-free price of a put option with exercise price NOK 55. Determine the replicating portfolio for the put and compute the corresponding initial portfolio value.

Exercise 3. Consider a single period market consisting of a probability space $\Omega = \{\omega_1, \omega_2, \omega_3\}$, probability measure

$$P(\omega) = \begin{cases} 2/3, & \omega = \omega_1, \\ 1/6, & \omega = \omega_2, \\ 1/6, & \omega = \omega_3, \end{cases}$$

bank account with $B_0 = 1$, $r = 1/10$, and one risky asset S_1 with

$$S_1(0) = 100, \qquad S_1(1)(\omega) = \begin{cases} 165, & \omega = \omega_1, \\ 110, & \omega = \omega_2, \\ 55, & \omega = \omega_3. \end{cases}$$

3a. Determine all risk-neutral probabilities Q. Is the market complete?

3b. Determine all attainable claims X.

3c. Compute the arbitrage-free prices of the claim $X = (10, 5, 10)^T$.

Exercise 4. Consider the market model in Exercise 2. We denote by $u(w)$ the utility function

$$u(w) = \ln w, \qquad w > 0.$$

Use the (two-steps) risk-neutral computational approach to solve the problem of maximizing expected utility of terminal wealth, with initial wealth $V(0) = \nu$ for a given positive number ν:

$$\max_{H \in \mathbf{R}^2} E\left[u\left(V(1)\right)\right], \qquad V(0) = \nu,$$

where $V(1) = H_0 B(1) + H_1 S(1)$, and "solve" means finding the optimal trading strategy $H = (H_0, H_1)^T$.