

MAT4730/9730

Mathematical Finance

EXAM

The exam is held at the blackboard for a duration of about 30-40 minutes in all.

A selection of topics from the list here below will be asked. The candidate can organize his/her reply bearing in mind:

- to center the topic addressing the major results
- to be prepared on the proofs of the statements discussed or left as exercise
- to be prepared on the notions that are embedded in the topic addressed.

During the exam the candidate could be asked to use the notions and techniques acquired in the course, e.g. short exercise.

Hereafter is a list of topics which are going to be tested during the exam.

TOPICS

Stochastic calculus

- Local martingales and properties used during the course
- Extended Itô integration compared with classical Itô integration
- Doob-Meyer decomposition theorem (statement)
- Martingale and local-martingale representation theorems

Single agent consumption/investment optimization

- Market model framework with Brownian noise, major stochastic processes of interest
- Self-financing and Gamma-financing portfolios
- Single agent consumption/investment scheme: utility functions and the preference structure, the optimization problem, admissibility, budget constrain
- The concept of replicable claim in a classical market with no consumption and in the presence of consumption (use of self-financing versus gamma-financing strategies)
- The use of budget constrain in the replication result in a market with consumption
- From the original single agent optimization problem to the auxiliary problem and return
- Solution of the auxiliary problem and Lagrange multipliers method

Pricing in an incomplete market due to constrains on portfolios

- Non arbitrage pricing in classical market models with no consumption and no constraint on the portfolios. Concept of interval of admissible non-arbitrage prices
- Definition of constrains over the portfolios. The concept of admissibility and budget constrain in presence of consumption.
- In a market with consumption and constrains over the portfolios, the concept of upper-hedging price
- Representation of the convex constrain on portfolios with support functions
- From an incomplete market with convex constrains on portfolios to a family of auxiliary markets and return
- Characterization of the upper-hedging price in general and in the case of European options with payoff function of polynomial growth

Single agent consumption/investment optimization with constraints on the portfolios

- The optimization problem with constraints on portfolios: admissibility
- From the original problem to a family of constraint-free problems and return

Introduction to: Equilibrium in a complete market

- The problem of equilibrium, the agents, the optimization of each agent (continuous endowment, consumption, portfolios, admissibility, budget constraints, feasibility constraints, preference structure, the single agent optimization problem)
- Solution of the single agent optimization problem
- Definition of equilibrium market
- Characterization of the equilibrium, existence result (representative agent)

Cut-through questions

- The use of martingale and local-martingale representation theorem in perfect replication type results
- Supermartingales and budget constraints and overview
- The use of the support function in the representation of portfolios constraints in pricing and optimization problems