MEK 4540 - 2009

1(a) Thermoplastics : Soften or melt on heating - weak intermolecular bonds. Can be melted and solidified again. Semi-crystalline or amorphous.

Eg: polyethylene, PEEK, ...

Thermosets : Decompose on heating - cross-linked polymers. Once cured they cannot be reshaped. Cured by addition of initiator / hardener

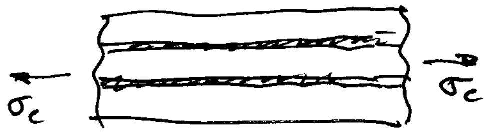
Eg: polyester, epoxy, vinylester

(b) Production methods: Can choose between spray lay-up, hand lay-up, various injection/infusion methods, filament winding, ...

Advantages and disadvantages can be associated with criteria for choice of method such as:

- Production volume
- Size and shape of component
- Requirements to properties (or variability of these)
- Surface finish
- Conditions wrt health, safety, environment

(c) For E_L:



Apply stress σ_c in fibre direction:

$$\text{Force} = \sigma_c A = \sigma_f V_f A + \sigma_m (1-V_f) A$$

A = cross-sectional area.

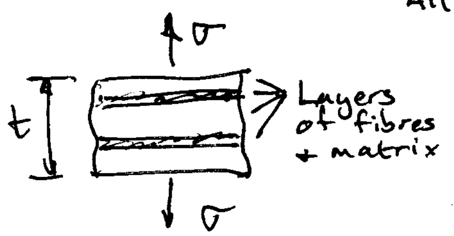
Fibres and matrix experience same strain ϵ

Divide equation by $A\epsilon$:

$$\frac{\sigma_c}{\epsilon} = \frac{\sigma_f V_f}{\epsilon} + \frac{\sigma_m}{\epsilon}(1-V_f) A$$

$$E_L = E_f V_f + E_m (1-V_f)$$

For E_T:



All layers experience same stress σ

$$\text{Total extension in T direction} = \epsilon_c t = \epsilon_f (V_f t) + \epsilon_m (1-V_f) t$$

Divide both sides by σt:

$$\frac{\epsilon_c}{\sigma} = \frac{\epsilon_f}{\sigma} V_f + \frac{\epsilon_m}{\sigma}(1-V_f)$$

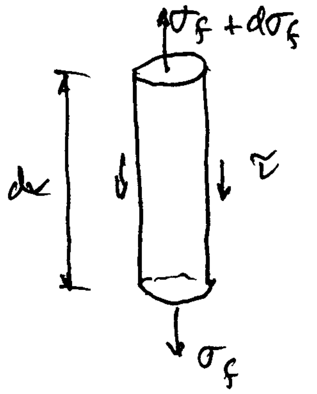
$$\frac{1}{E_c} = \frac{V_f}{E_f} + \frac{1-V_f}{E_m}$$

1(c) - contd. Assumptions for E_L: continuous, parallel fibres, perfect bond between fibres and matrix. These are OK and formula quite accurate.

Assumptions for E_T: Composite can be considered as separate, uniform layers of fibre and matrix materials. Not a very good approximation - gives only a very rough estimate. Halpin-Tsai equations are better.
(Assume also perfect bond)

Could also mention that voids are neglected for both E_L and E_T.

(d)



Equilibrium of forces:

$$(\cancel{\sigma_f} + d\sigma_f - \cancel{\sigma_f})\pi\left(\frac{d}{2}\right)^2 - \tau\pi d\, dx = 0$$

$$\frac{d\sigma_f}{dx} = \frac{4\tau}{d}$$

Assume matrix is perfectly plastic: $\tau = \tau_y$

and $\sigma_f = 0$ at fibre end, $x = 0$

Then $$\int_0^x d\sigma_f = \int_0^x \frac{4\tau_y}{d}\, dx$$

$$\sigma_f = \frac{4\tau_y}{d} x$$

Strain $$\epsilon_f = \frac{\sigma_f}{E_f} = \frac{4\tau_y}{d E_f} x$$

In a long-fibre composite $\epsilon_f = \epsilon_c = \dfrac{\sigma_c}{E_c}$

These are equal when $$\frac{4\tau_y}{d E_f} x = \frac{\sigma_c}{E_c}$$

so $$x = \frac{E_f}{E_c} \cdot \frac{d\sigma_c}{4\tau_y}$$

This is the length of fibre used to build up the same stress as in a long-fibre composite, from one end along the fibre. The same length is needed from the other end, so l_t = twice this value of x:

$$l_t = \frac{E_f}{E_c} \cdot \frac{d\sigma_c}{2\tau_y}.$$

2 A (a) To build up the compliance matrix $[S]$:

σ_L alone gives $\epsilon_L = \dfrac{\sigma_L}{E_L}$; $\epsilon_T = -\nu_{LT}\epsilon_L = -\nu_{LT}\dfrac{\sigma_L}{E_L}$; $\gamma_{LT} = 0$

σ_T alone gives $\epsilon_T = \dfrac{\sigma_T}{E_T}$; $\epsilon_L = -\nu_{TL}\epsilon_T = -\nu_{TL}\dfrac{\sigma_T}{E_T}$; $\gamma_{LT} = 0$

τ_{LT} alone gives $\epsilon_L = \epsilon_T = 0$, $\gamma_{LT} = \dfrac{\tau_{LT}}{G_{LT}}$

These are all superposed to give:

$$\left.\begin{aligned} \epsilon_L &= \frac{\sigma_L}{E_L} - \nu_{TL}\frac{\sigma_T}{E_T} \\ \epsilon_T &= \frac{\sigma_T}{E_T} - \nu_{LT}\frac{\sigma_L}{E_T} \\ \gamma_{LT} &= \frac{1}{G}\tau_{LT} \end{aligned}\right\} \Rightarrow \begin{bmatrix} \epsilon_L \\ \epsilon_T \\ \gamma_{LT} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & 0 \\ -\frac{\nu_{LT}}{E_T} & \frac{1}{E_T} & 0 \\ 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix}}_{=[S]} \begin{bmatrix} \sigma_L \\ \sigma_T \\ \tau_{LT} \end{bmatrix}$$

(b) A-matrix gives relation between in-plane forces (both normal and shear forces) and in-plane strains at mid-surface of laminate

B-matrix gives coupling relations between in-plane forces and curvatures/twisting deformations, and between moments (both bending moments and twisting moments) and mid-surface strains.

D-matrix gives relation between bending and twisting moments and curvature/twist deformation.

$B = 0$ if the laminate has a symmetrical layup.

A_{16}, A_{26} represent coupling between normal (direct) in-plane forces and in-plane shear deformation and similarly between in-plane shear forces and extensional deformation (tension or compression). They are zero if, for every ply with direction θ from a given axis direction, there is an identical ply with direction $-\theta$.

D_{16}, D_{26} represent, similarly, coupling between bending moments and torsional deformation, and between twisting moments and bending curvatures. They are zero if, for every ply with direction θ at distance h above the middle surface, there is an identical ply with direction $-\theta$ at distance h below the middle surface.

2 B (a) Substitute values into [S]:

$$[S] = \begin{bmatrix} 0.03745 & -0.01086 & 0 \\ -0.01086 & 0.11905 & 0 \\ 0 & 0 & 0.28571 \end{bmatrix} GPa^{-1}$$

$$[Q] = [S]^{-1} = \begin{bmatrix} 27.426 & 2.502 & 0 \\ 2.502 & 8.628 & 0 \\ 0 & 0 & 3.5 \end{bmatrix} GPa$$

To find $[\bar{Q}]$ for 90° ply, we can apply the T-transformations but in fact all that has happened is that L and T directions are reversed, so for 90° plies,

$$[\bar{Q}] = \begin{bmatrix} 8628 & 2.502 & 0 \\ 2.502 & 27.426 & 0 \\ 0 & 0 & 3.5 \end{bmatrix} GPa$$

For 0° plies, $[\bar{Q}] = [Q]$.

To find [A]:

$$[A] = 2 \times 1.2 \left\{ [\bar{Q}_0] + [\bar{Q}_{90}] \right\}$$

$$= \begin{bmatrix} 86.53 & 12.01 & 0 \\ 12.01 & 86.53 & 0 \\ 0 & 0 & 16.8 \end{bmatrix} GPa\ mm$$

i.e. kN mm or Nm.

(c)
$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = [A] \begin{bmatrix} 1.5 \\ 0.5 \\ 0.8 \end{bmatrix} \times \cancel{10^3} \times \cancel{10^{-3}}\ Nmm = \begin{bmatrix} 135.8 \\ 61.3 \\ 13.4 \end{bmatrix} Nmm$$

(d) To find E_x, apply $N_x \neq 0$, $N_y = 0$, $N_{xy} = 0$, solve for $\epsilon_x, \epsilon_y, \gamma_{xy}$.
Then $E_x = \frac{N_x}{t\epsilon_x}$ where $t = 4 \times 1.2 = 4.8$ mm

Similarly for E_y apply $N_y \neq 0$, $N_x = N_{xy} = 0$ and find $E_y = \frac{N_y}{t\epsilon_y}$.
+ for G_{xy} apply $N_{xy} \neq 0$, $N_x = N_y = 0$ and find $G_{xy} = \frac{N_{xy}}{t\epsilon_y}$.

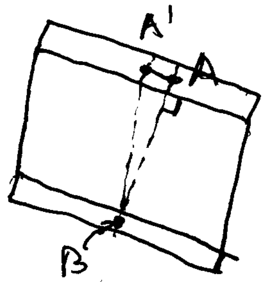
3 A Failure mechanisms:

- Yield or fracture in faces under tension or compression
- Yield or fracture in core under transverse shear.
- Local deformation, with yield or fracture, caused by concentrated loads or at connections
- Global buckling under axial compression – normally a combination of flexural and shear deformation, but can in extreme cases involve predominantly either flexural buckling or shear crimping.
- Local buckling of face sheet in compression (wrinkling with foams, balsa, etc as core, dimpling with honeycomb).

Sketches as provided in text book and lecture notes.

B (a) w_b = contribution to transverse displacement w from bending alone.
w_s = " – – – – – shear def. alone.

D is the flexural stiffness as in part A(b)
S is the shear stiffness, defined as T_x/γ' where



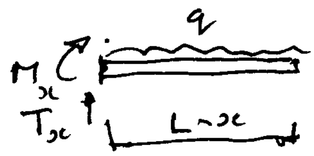
γ' = angle $A\hat{B}A'$ where A, B, A' are points in mid-surfaces of faces.

The first equation just says that $w(x)$ can be made up from contributions from bending and shear deformation. The second equation is the bending moment – curvature relation as for an ordinary beam.

The 3rd equation comes from moment equilibrium of an element of beam and says that the shear force is equal to the bending moment gradient. The 4th equation relates the shear deformation to the shear force. The shear deformation can be split into 2 component parts – the part that gives a transverse displacement w_s (or rather displacement gradient $\frac{dw_s}{dx}$) and a part that just involves relative movement of the two faces in the plane parallel to the undeformed beam. The shear strain in the core associated with this second part is γ_0.

$\gamma_0 = 0$ if either or both ends of the beam are built-in. It can also be zero in other cases, eg. if the beam is symmetrically supported with symmetrical loading.

(b) Free body diagram of beam and of portion to right of arbitrary section:



$T_x = q(L-x)$

$M_x = -\dfrac{q(L-x)^2}{2}$

$\left.\begin{array}{l} R_A = qL \\ M_A = \dfrac{qL^2}{2} \end{array}\right\}$ by equilibrium

Solve for w_b : $-D\dfrac{d^2w_b}{dx^2} = M_x = -\dfrac{q}{2}(L-x)^2$

$$-D\frac{dw_b}{dx} = +\frac{q}{6}(L-x)^3 + C_1$$

$$-D\,w_b = -\frac{q}{24}(L-x)^4 + C_1x + C_2$$

Solve for w_s: End A is built-in so $\gamma_0 = 0$

Then $\quad S\dfrac{dw_s}{dx} = T_x = q(L-x)$

$$S\,w_s = -\tfrac{1}{2}q(L-x)^2 + C_3$$

Boundary conditions: $\dfrac{dw_b}{dx} = 0$ at $x = 0$ $\rightarrow C_1 = -\dfrac{q}{6}L^3$

and $w(0) = 0$

$$w = w_b + w_s = \frac{q}{24D}(L-x)^4 - \frac{C_2}{D} + \frac{C_3}{S} - \frac{1}{2}\frac{q}{S}(L-x)^2 + \frac{q}{6D}L^3x$$

$w(0) = 0 \rightarrow \quad 0 = \dfrac{qL^4}{24D} - \dfrac{qL^2}{2S} + \left(\dfrac{C_3}{S} - \dfrac{C_2}{D}\right)$

so $\left(\dfrac{C_3}{S} - \dfrac{C_2}{D}\right) = \dfrac{qL^2}{2S} - \dfrac{qL^4}{24D}$

This gives

$$w(x) = \frac{qL^4}{24D}\left\{\left(1-\frac{x}{L}\right)^4 - 1\right\} + \frac{qL^2}{2S}\left\{1 - \left(1-\frac{x}{L}\right)^2\right\} + \frac{qL^3}{6D}x$$

$$= \frac{qL^4}{24D}\left\{\left(1-\frac{x}{L}\right)^4 + 4\frac{x}{L} - 1\right\} + \frac{qL^2}{2S}\left(\frac{2x}{L} - \frac{x^2}{L^2}\right)$$

This can be simplified further, but we proceed to find $w(L)$:

$$\delta = w(L) = \frac{qL^4}{24D}(+4-1) + \frac{qL^2}{2S}(2-1)$$

$$= \frac{qL^4}{8D} + \frac{qL^2}{2S}$$

$$= \frac{qL^4}{8D}\left(1 + \frac{4D}{L^2S}\right)$$

(c) If B were simply supported we'd have to include a vertical reaction force R_B, which cannot be found from equilibrium as the beam is now statically indeterminate.

The expressions for M_x and T_x would now include R_B. However, we can just proceed as part (b), finding expressions for w_b and w_s. We still have $\frac{dw_b}{dx} = 0$ at $x = 0$, but now we have to apply both $w(0) = 0$ and $w(L) = 0$, to get the unknown constants of integration and in addition R_B.