EKSAMEN MEK 4540,
2 Juni 2006
3 Timer

Oppgave 1

a) Stivhet og styrke

	E [GPa]	σ_{TS} [MPa]
Epoxy	2 - 4	50 - 130
Polyester	2 - 4	30 - 100
Glassfiber	72	2 - 4000
Karbonfiber	250 - 400	2 - 3000

b) Antakelser for blandingsregelen

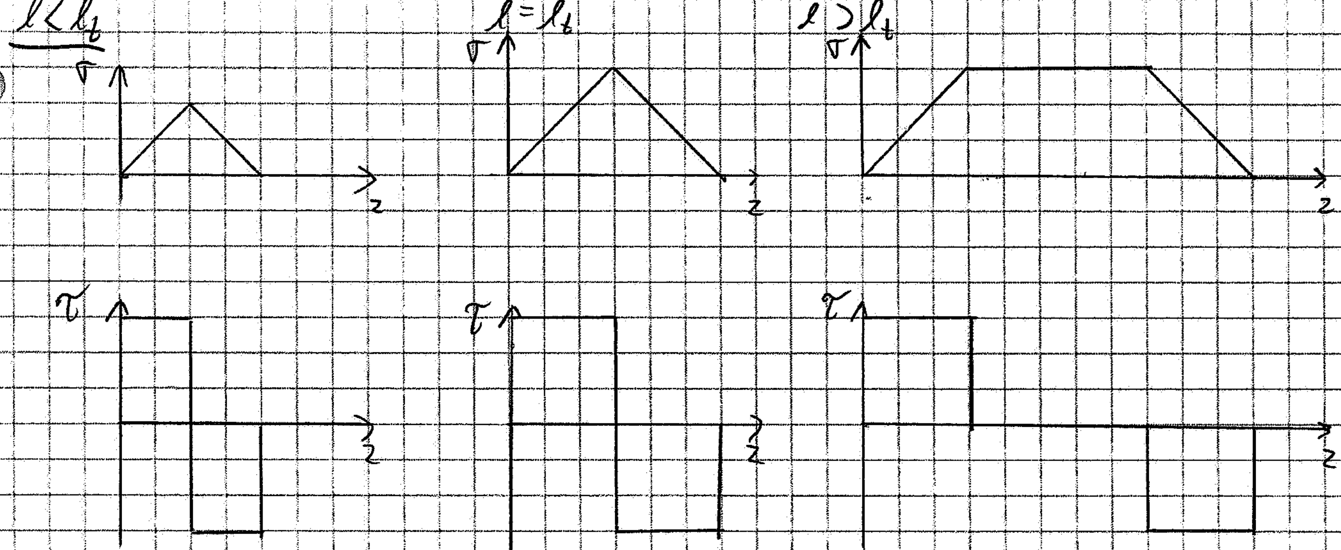
* Perfekt heft mellom fiber og matrise
* Uniforme, kontinuerlige og parallelle fibere
* Lineært elastisk fiber og matrise

c) Lastoverføringslengde

Den nødvendige fiberlengden for at spenningen i fiberen skal kunne nå samme spenning som en kontinuerlig fiber ved samme last.

Kritisk lengde

Den minste fiberlengden som er nødvendig for at fiberen skal kunne ta opp bruddlasten



d) A angir sammenhengen mellom i-planet krefter (normal og skjær) og midtplanstøyninger

B angir kobling mellom normalkrefter og krumning samt mellom momenter og midtplanstøyninger

D angir sammenhengen mellom momenter og krumning

A_{16} og A_{26} angir kobling mellom normalkrefter og i-planet skjær samt skjærkrefter og midtplans-tøyning

D_{16} og D_{26} angir på samme måte kobling mellom bøyemomenter og vridning samt vridningsmoment og bøyning.

Oppgave 2

a) Hooke's lov for spesialortotropt materiale

Anta:

$\sigma_T = \tau_{LT} = 0,\ \sigma_L \neq 0$

$\Downarrow$

$\varepsilon_L = \dfrac{\sigma_L}{E_L}$

$\varepsilon_T = -\nu_{LT}\,\varepsilon_L = -\nu_{LT}\dfrac{\sigma_L}{E_L}$

$\gamma_{LT} = 0$

$\sigma_L = \tau_{LT} = 0,\ \sigma_T \neq 0$

$\Downarrow$

$\varepsilon_L = -\nu_{TL}\,\varepsilon_T = -\nu_{TL}\dfrac{\sigma_T}{E_T}$

$\varepsilon_T = \dfrac{\sigma_T}{E_T}$

$\gamma_{LT} = 0$

$\sigma_L = \sigma_T = 0,\ \tau_{LT} \neq 0$

$\varepsilon_L = 0$

$\varepsilon_T = 0$

$\gamma_{LT} = \dfrac{\tau_{LT}}{G_{LT}}$

Superponerer spenningstilstandene og får

$$\left.\begin{aligned}
\varepsilon_L &= \frac{\sigma_L}{E_L} - \nu_{TL}\frac{\sigma_T}{E_T}\\
\varepsilon_T &= -\nu_{LT}\frac{\sigma_L}{E_L} + \frac{\sigma_T}{E_T}\\
\gamma_{LT} &= \frac{\tau_{LT}}{G_{LT}}
\end{aligned}\right\} \Rightarrow \begin{bmatrix}\varepsilon_L\\ \varepsilon_T\\ \gamma_{LT}\end{bmatrix} = \underbrace{\begin{bmatrix}\frac{1}{E_L} & -\frac{\nu_{TL}}{E_T} & 0\\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & 0\\ 0 & 0 & \frac{1}{G_{LT}}\end{bmatrix}}_{=[S]}\begin{bmatrix}\sigma_L\\ \sigma_T\\ \tau_{LT}\end{bmatrix}$$

Antall uavhengige elastiske konstanter er 4. Symmetri av tøyningstensoren krever at $S_{12} = S_{21} \Rightarrow \dfrac{\nu_{LT}}{E_L} = \dfrac{\nu_{TL}}{E_T}$

b)

$$[S] = \begin{bmatrix} \frac{1}{35000} & -\frac{0.3}{35000} & 0 \\ -\frac{0.3}{35000} & \frac{1}{7000} & 0 \\ 0 & 0 & \frac{1}{5000} \end{bmatrix} MPa^{-1} = \begin{bmatrix} 2.86 & -0.86 & 0 \\ -0.86 & 14.29 & 0 \\ 0 & 0 & 20 \end{bmatrix} \times 10^{-5} MPa$$

$$[Q] = [S]^{-1} = \begin{bmatrix} 35.6 & 2.14 & 0 \\ 2.14 & 7.13 & 0 \\ 0 & 0 & 5 \end{bmatrix} \times 10^{3} MPa$$

c)

$$[Q_{45}'] = [T_{45}]^{-1} [Q'] [T_{45}]$$

der $Q_{66}' = 2Q_{66}$

$[Q']$ gjelder for tensorielle tøyninger

$$[T_{45}] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & -1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

$$\Rightarrow [Q_{45}'] = \begin{bmatrix} 16.8 & 6.8 & 14.3 \\ 6.8 & 16.8 & 14.3 \\ 7.1 & 7.1 & 19.2 \end{bmatrix} \times 10^{3} MPa$$

$$\Rightarrow [Q_{45}] = \begin{bmatrix} 16.8 & 6.8 & 7.1 \\ 6.8 & 16.8 & 7.1 \\ 7.1 & 7.1 & 9.62 \end{bmatrix} \times 10^{3} MPa$$

$[Q_{45}]$ gjelder for ingeniørtøyninger

$[Q_{90}]$ finnes enklest ved å bytte x og y-retning i $[Q]$

$$\Rightarrow \quad [Q_{90}] = \begin{bmatrix} 7.13 & 2.14 & 0 \\ 2.14 & 35.6 & 0 \\ 0 & 0 & 5 \end{bmatrix} \times 10^3 \text{ MPa}$$

d) Last-deformasjonskurve

Trenger $[A]$ for laminatet

$$[A] = 10 * [Q] \times 0.1\,mm + 10 * [Q_{90}] \times 0.1\,mm$$

$$= [Q] + [Q_{90}] = \begin{bmatrix} 42.7 & 4.28 & 0 \\ 4.28 & 42.7 & 0 \\ 0 & 0 & 10 \end{bmatrix} \times 10^3 \; N/mm$$

Bruddtøyningen finnes fra de målte spenningene

$$\varepsilon_{Lu} = \frac{\sigma_{Lu}}{E_L} = \underline{2.3 \times 10^{-2}}$$

$$\varepsilon_{Tu} = \frac{\sigma_{Tu}}{E_T} = \underline{2.9 \times 10^{-3}}$$

Ser at tverrlagene vil ryke først (ved $\varepsilon_x = 2.9 \times 10^{-3}$)
Lasten er da:

$$\begin{bmatrix} N_x \\ 0 \\ 0 \end{bmatrix} = [A] \begin{bmatrix} 0.0029 \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Løser ligningssettet og finner

$$N_x = 123 \; N/mm$$

N_y $[A]$ blir (antar $[Q_{90}]=0$ etter brudd)

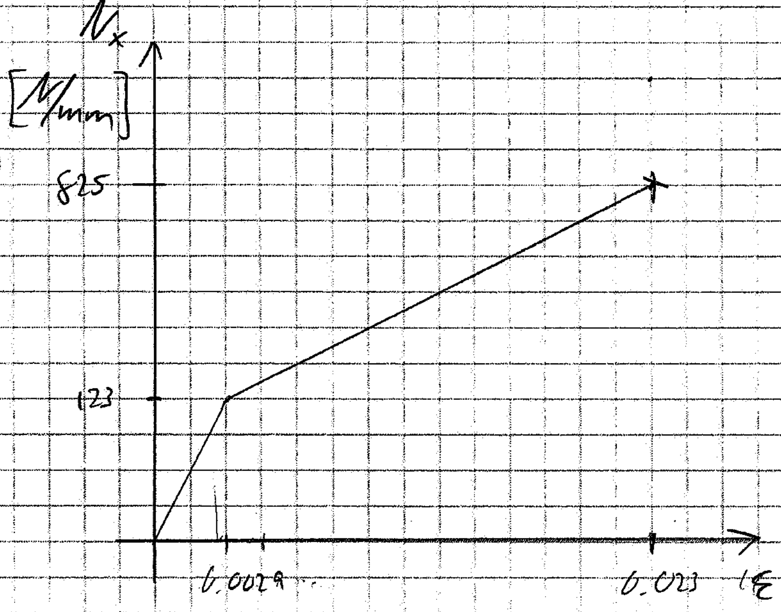
$$[\bar{A}] = 10\,[Q] \times 0.1\,mm = \begin{bmatrix} 35.6 & 2.14 & 0 \\ 2.14 & 7.13 & 0 \\ 0 & 0 & 5 \end{bmatrix} \times 10^3\ MPa$$

$$\begin{bmatrix} \Delta N_x \\ 0 \\ 0 \end{bmatrix} = [\bar{A}] \begin{bmatrix} 0.023 - 0.0029 \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Løser likningssettet og finner

$$\underline{\underline{\Delta N_x = 702\ N/mm}}$$

Last-deformasjonskurven blir da



MEK 4540 : Oppgave nr. 3 ①

DEL A

(a) • High bending stiffness (for given weight)
• High bending strength (for given weight)
• Smoothe surfaces both sides since no stiffening is needed.

(b) "Partial deflections" is an expression used when deformations due to bending and shear are considered separately and then superimposed to give the total deformation. (The approach is exact for some cases but only approximate for others.)

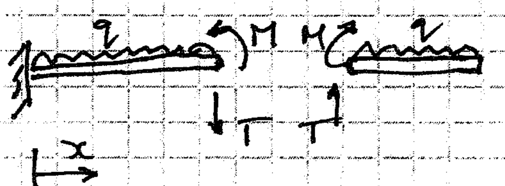
(c) • Tensile or compressive (or other) failure of faces giving yielding or fracture
• Global buckling, which generally involves a mixture of bending and shear. (Bending buckling and shear crimping are limiting cases when one or the other dominates, i.e. for very slender or stocky columns.)
• Face sheet wrinkling or dimpling
• Local failure due to concentrated loads or at joints

DEL B

The simplest solution is to use partial deflections so

$$w = w_b + w_s$$

where w_b is obtained from the classical bending solution and w_s is the purely shear deformation.



By equilibrium of right-hand part,

$$M = q\frac{(L-x)^2}{2}, \quad T = q(L-x)$$

Bending: $-D\frac{d^2w_b}{dx^2} = M = -q\frac{(L-x)^2}{2}$ i.e. $D\frac{d^2w_b}{dx^2} = q\frac{1}{2}(L-x)^2$

Integrate: $D\frac{dw_b}{dx} = -\frac{1}{6}q(L-x)^3 + A$

Again: $Dw_b = \frac{1}{24}q(L-x)^4 + Ax + B$

Boundary conditions:

$$\frac{dw_b}{dx} = 0 \text{ at } x = 0 \rightarrow A = \frac{1}{6} q L^3$$

$$w_b = 0 \text{ at } x = 0 \rightarrow 0 = \frac{1}{24} q L^4 + \cancel{\frac{1}{6} q L^3 \times 0} + B$$

$$B = -\frac{1}{24} q L^4$$

Deflection at $x = L$:

$$\delta_b = w_b(L) = \frac{1}{D}\left\{AL + B\right\} = \frac{1}{D}\left(\frac{1}{6} q L^4 - \frac{1}{24} q L^4\right)$$

$$= \frac{1}{D} \cdot \frac{q L^4}{8}$$

Shear:

$$\gamma = \frac{T}{S} = \frac{q}{S}(L - x)$$

$$w_s = \int_0^x \gamma \, dx$$

$$\delta_s = w_s(L) = \int_0^L \frac{q}{S}(L - x)\, dx = \frac{q}{S}\left[Lx - \frac{1}{2}x^2\right]_0^L$$

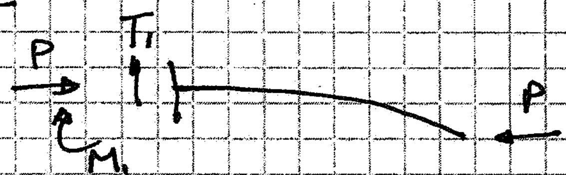
$$= \frac{q}{S} \cdot \frac{1}{2} L^2$$

Total deflection

$$\delta = \delta_b + \delta_s = \underline{\underline{\frac{q L^4}{8D}\left(1 + \frac{4D}{S L^2}\right)}}$$

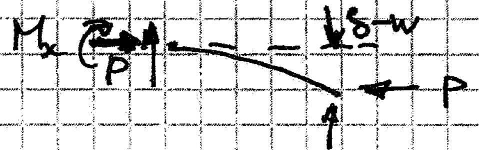
DEL C

(a)



Vertical equilibrium of forces on entire beam: T_1 is the only force so $T_1 = 0$

(b)



Moment equilibrium of part to right of cut: Take moments about cut.

$$M_x + P(\delta - w) = 0$$

$$M_x = Pw - P\delta$$

We have $\quad w = w_b + w_s$

so $$\frac{d^2w}{dx^2} = \frac{d^2w_b}{dx^2} + \frac{d^2w_s}{dx^2}$$

But $\quad -D\dfrac{d^2w_b}{dx^2} = M_x \quad$ (bending equation)

and $\quad \dfrac{dw_s}{dx} = \dfrac{1}{S}T_x \quad$ (shear equation)

so $\quad \dfrac{d^2w_s}{dx^2} = \dfrac{1}{S}\dfrac{dT_x}{dx} = \dfrac{1}{S}\dfrac{d^2M_x}{dx^2} \quad$ (since $T_x = \dfrac{dM_x}{dx}$)

Hence $$\frac{d^2w}{dx^2} = -\frac{M_x}{D} + \frac{1}{S}\frac{d^2M_x}{dx^2}$$

$$= \frac{P\delta - Pw}{D} + \frac{1}{S}P\frac{d^2w}{dx^2}$$

Rearrange: $$\left(1 - \frac{P}{S}\right)\frac{d^2w}{dx^2} + \frac{P}{D}w = \frac{P}{D}\delta$$

i.e. $$\frac{d^2w}{dx^2} + \frac{P}{D}\left(\frac{S}{S-P}\right)w = \frac{P}{D}\left(\frac{S}{S-P}\right)\delta$$

With $a^2 = \dfrac{P}{D}\left(\dfrac{S}{S-P}\right)$ this becomes

$$\frac{d^2w}{dx^2} + a^2w = a^2\delta$$

Solution is

$$w = C_1 \sin ax + C_2 \cos ax + \delta$$

Boundary conditions are $\frac{dw_b}{dx} = 0$ at $x = 0$

$w = 0$ at $x = 0$

But $\frac{dw_s}{dx} = 0$ at $x = 0$ (see footnote to question) so first condition is $\frac{dw}{dx} = 0$ at $x = 0$.

This gives $C_1 a \cos(0) + -C_2 \sin(0) = 0$

i.e. $C_1 = 0$

$w = 0$ at $x = 0$ gives $0 = C_2 + \delta$, i.e. $C_2 = -\delta$

We note also that $w(L) = \delta$, so

$$\delta = \cancel{C_1 \sin aL} + C_2 \cos aL + \delta$$

This gives $C_2 \cos aL = 0$

so $C_2 = 0$ (trivial solution) or $\cos aL = 0$

Non-trivial solution possible only if $aL = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

i.e. $aL = \frac{2n-1}{2}\pi \quad n = 1, 2, 3, \ldots$

But $a^2 = \frac{P}{D}\left(\frac{S}{S-P}\right)$ so P_{kr} given by

$$(S-P) D a^2 = PS \quad \text{i.e.} \quad P(S + Da^2) = SDa^2$$

$$P_{kr} = \frac{SDa^2}{S + Da^2} = \frac{\left(\frac{2n-1}{2}\right)^2 \pi^2 SD/L^2}{S + \left(\frac{2n-1}{2}\right)^2 \pi^2 D/L^2}$$

Lowest given by $n = 1$

$$\text{so} \quad P_{kr} = \frac{\frac{1}{4}\pi^2 SD/L^2}{S + \frac{1}{4}\pi^2 D/L^2} = \frac{\pi^2 D}{4L^2 + \pi^2 D/S}$$