4.1 Fibres: Diameter $d = 0.03$ mm, $E_f = 70$ GPa.

Matrix: $E_m = 3.5$ GPa, Yield strength $\sigma_y = 28$ MPa.

Fibre volume content $V_f = 0.4$.

We need the shear yield strength of the matrix, $\tau_y$. For a ductile material obeying the von Mises yield criterion, $\tau_y = \sigma_y/\sqrt{3}$,
so $\tau_y = 28/\sqrt{3} = 16.17$ MPa

(a) Calculation of load transfer length $l_t$:

With rigid-plastic matrix, fibre stress at distance $z$ from fibre end is given by

$$\sigma_f = \frac{2\tau_y z}{r} \qquad r = \text{fibre radius} = d/2$$

Applies to both ends so for very short fibre of length $l$, $\sigma_f$ is maximum at $z = l/2$, giving

$$(\sigma_f)_{max} = \frac{\tau_y l}{r}$$

As $l$ is increased, $(\sigma_f)_{max}$ increases but it can never exceed $\sigma_f$ that would occur in a long-fibre UD composite with same $V_f$ subjected to same composite stress $\sigma_c$.

Thus $\dfrac{(\sigma_f)_{max}}{E_f} = \epsilon_f = \epsilon_c = \dfrac{\sigma_c}{E_c}$ where $E_c$ is modulus of a long-fibre UD composite, given by

$$E_c = E_f V_f + E_m(1 - V_f) = 30.1 \text{ GPa}$$

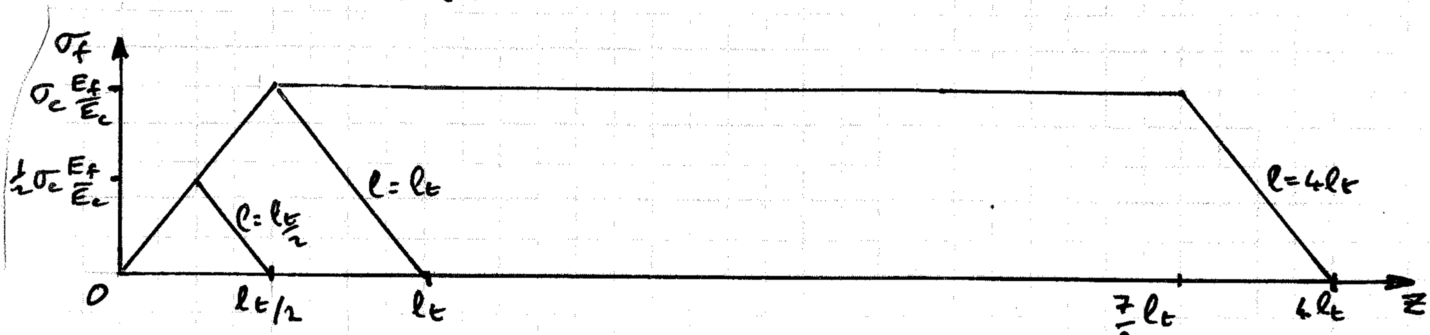
The length $l$ that is just sufficient to give this condition is the load transfer length $l_t$. Thus $l_t$ is given by

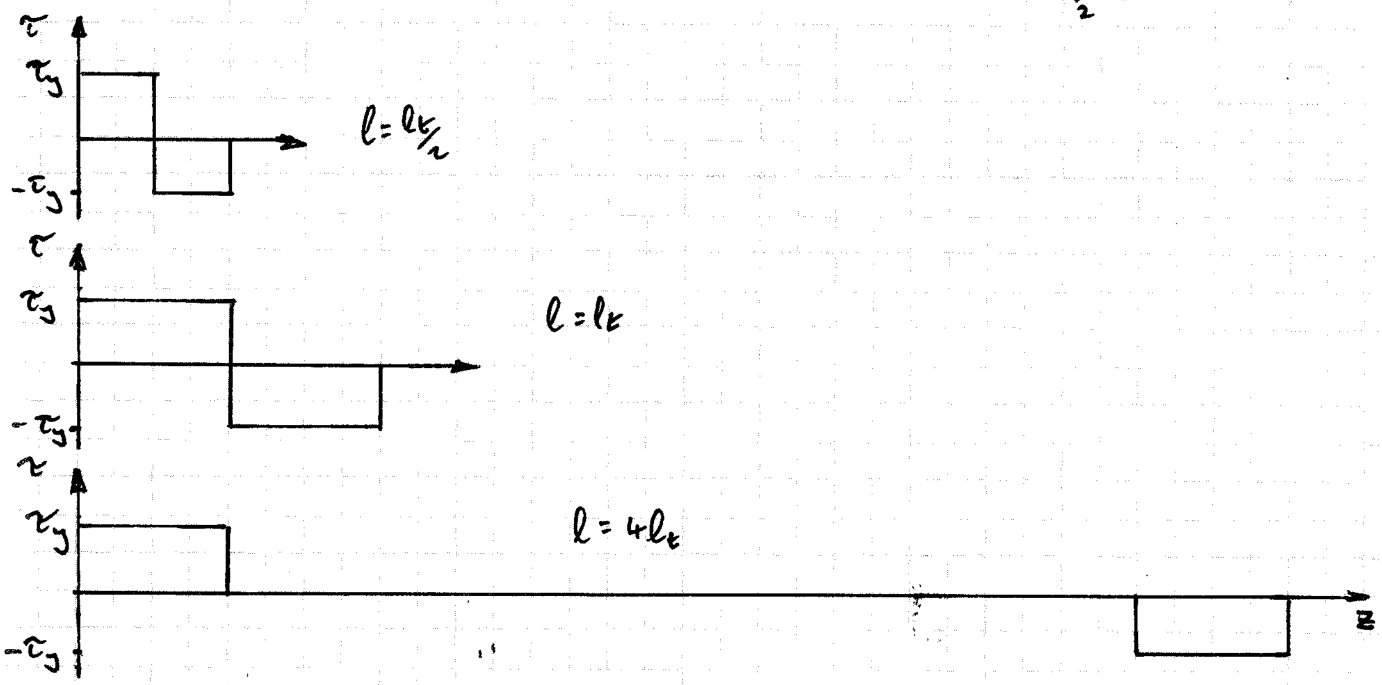
$$\frac{\tau_y l_t}{r} = \frac{E_f}{E_c}\sigma_c, \quad \text{i.e. } l_t = \frac{E_f}{E_c}\cdot\frac{r\sigma_c}{\tau_y} = \frac{E_f}{E_c}\cdot\frac{d}{2}\cdot\frac{\sigma_c}{\tau_y}$$

For $\sigma_c = 70$ MPa, $l_t = \underline{\underline{0.151 \text{ mm}}}$

For $\sigma_c = 210$ MPa, $l_t = \underline{\underline{0.453 \text{ mm}}}$.

(b) Plots of fibre stress and interfacial shear stress for $l = \frac{1}{2}l_t$, $l = l_t$ and $l = 4l_t$:




Average fibre stress $\bar{\sigma}_f = \frac{1}{l}\int_0^l \sigma_f \, dz$, where $\int_0^l \sigma_f \, dz$ is the area under the respective graph of $\sigma_f$ against z.

| | | | $\sigma_c = 70$ MPa | $\sigma_c = 210$ MPa |
|---|---|---|---|---|
| For $l = l_t/2$, | $\bar{\sigma}_f = \frac{2}{l_t}\cdot\left(\frac{1}{2}\cdot\frac{l_t}{2}\cdot\frac{\sigma_c}{2}\frac{E_f}{E_c}\right) = \frac{1}{4}\sigma_c\frac{E_f}{E_c}$ | = | 40.7 MPa | 122.1 MPa |
| For $l = l_t$, | $\bar{\sigma}_f = \frac{1}{l_t}\cdot\left(\frac{1}{2}\cdot l_t \cdot \sigma_c \frac{E_f}{E_c}\right) = \frac{1}{2}\sigma_c\frac{E_f}{E_c}$ | = | 81.4 MPa | 244.2 MPa |
| For $l = 4l_t$, | $\bar{\sigma}_f = \frac{1}{4l_t}\cdot\left(\frac{7}{2}l_t \, \sigma_c \frac{E_f}{E_c}\right) = \frac{7}{8}\sigma_c\frac{E_f}{E_c}$ | = | 142.4 MPa | 427.3 MPa |

(c) Plots of fibre strain for $l = l_t$ and $l = \frac{1}{2}l_t$:

These have exactly the same shape as the corresponding plots of $\sigma_f$, but $\epsilon_f = \sigma_f / E_f$ replaces $\sigma_f$.

The peaks are thus $\frac{\sigma_c}{E_c}$ for $l = l_t$ and $\frac{\sigma_c}{2E_c}$ for $l = l_t/2$.

For $\sigma_c = 70$ MPa, $l = l_t$, $\epsilon_{f\,max} = \frac{\sigma_c}{E_c} = \underline{\underline{0.00233}}$

$l = \frac{1}{2}l_t$, $\epsilon_{f\,max} = \frac{\sigma_c}{2E_c} = \underline{\underline{0.00116}}$

For $\sigma_c = 210$ MPa, $l = l_t$, $\epsilon_{f\,max} = \frac{\sigma_c}{E_c} = \underline{\underline{0.00698}}$

$l = \frac{1}{2}l_t$, $\epsilon_{f\,max} = \frac{\sigma_c}{2E_c} = \underline{\underline{0.00349}}$

4.2 Composite as Q4.1. $\sigma_{fu} = 1.4\,GPa = 1400\,MPa$.

We require critical length $l_c$ and plot of composite strength for $l = 0.1\,l_c$ to $100\,l_c$

$l_c$ = smallest fibre length required to develop max. fibre stress = $\sigma_{fu}$.
This is the value of $l_t$ that corresponds to $(\sigma_f)_{max} = \sigma_{fu}$

$$\frac{l_t}{d} = \frac{(\sigma_f)_{max}}{2\tau_y} \quad \text{so} \quad l_c = \frac{\sigma_{fu}\,d}{2\tau_y} = 1.299\text{ mm}$$

Estimates of $\sigma_{cu}$ are given as follows – see lecture notes or AB&C Section 4.3.2:

For $l \le l_c$, matrix fails first, $\sigma_{cu} = \frac{\tau_y\,l}{d} V_f + \sigma_{mu} V_m$

For $l > l_c$, fibres fail first, $\sigma_{cu} = \sigma_{fu}\left(1 - \frac{l_c}{2l}\right) V_f + (\sigma_m)_{\epsilon_f^*} V_m$

Here $\tau_y = 16.17$ MPa, $d = 0.03$ mm, $V_f = 0.4$, $\sigma_{mu} = 28$ MPa, $V_m = 1 - V_f = 0.6$

$\epsilon_f^*$ = fibre strain at failure = $\sigma_{fu}/E_f = 1.4/70 = 0.02$

$(\sigma_m)_{\epsilon_f^*}$ = matrix stress at this strain = $0.02\,E_m = 0.02 \times 3500 = 70$ MPa.

– but this exceeds $\sigma_{mu}$ so we must use $\sigma_{mu} = 28$ MPa instead.

Resulting curve, plotted logarithmically, is as follows: