# MEK 4540 Autumn 2011: Compulsory Assignment no. 1

**To be submitted by 11:15 on 4 October 2011 as described below.**

A glass fibre laminate consists of the following materials:

| | | |
|---|---|---|
| Glass fibres: | E-modulus: | $E$ = 72.4 GPa |
| | Poisson's ratio: | $\nu$ = 0.2 |
| | Density: | $\rho$ = 2.54 g/cm$^3$ |
| Vinylester: | E-modulus: | $E$ = 3.4 GPa |
| | Poisson's ratio: | $\nu$ = 0.35 |
| | Density: | $\rho$ = 1.1 g/cm$^3$ |

The fibre fabric in each unidirectional ply weighs 150 g/m$^2$. After a burn-off test on 7.5 g of laminate, 5.6 g of glass is left.

a) Calculate the density of the composite, the fibre volume fraction and the thickness of a single ply containing 150 g/m$^2$ of reinforcement.

b) Estimate $E_L$, $E_T$, $G_{LT}$ and $\nu_{LT}$. Use the simple combination laws and, where appropriate, also the Halpin-Tsai formulae. Explain the differences.

c) With the elastic properties derived in (b) above (taking the Halpin-Tsai values for $E_T$ and $G_{LT}$), set up the stiffness matrix ($Q$) in the $L$-$T$ axes. Use the transformation matrix **T** as defined in the lectures (or the matrices $\mathbf{T_1}$ and $\mathbf{T_2}$ as defined in the text-book) to calculate the stiffness matrices ($\overline{Q}$) in directions 0º, 90º, 45º and -45º. Explain briefly how these matrices express the relationship between stress and strain for a single ply, and indicate clearly whether you are using the tensor shear strain or the engineering shear strain. NB: This part should be solved first *without* using MATLAB, but the results can be checked with MATLAB in connection with Part (d).

d) Study the MATLAB script provided and modify it to calculate the **A**-, **B**- and **D**-matrices for laminates consisting of the above plies in the three lay-ups:
   (i) $(0/45/\text{-}45/0)_s$
   (ii) $(0/45/\text{-}45/90)_s$ and
   (iii) (0/45/-45/0).
   Comment on the differences, identifying any special properties displayed by the lay-ups. Note: The MATLAB script may also be used to check the individual ply stiffness matrices ($\overline{Q}$) calculated in part (c).

e) Calculate the total thickness of each of the laminates (i) and (ii). Modify the same MATLAB script to calculate the strains for each of these two laminates when they are exposed to a tensile stress in the 0º direction of 1 MPa (which must first be converted to a force per unit length). Hence find the effective E-modulus $E_x$ and Poisson's ratio $\nu_{xy}$ for each of these two laminates, where the $x$-axis lies parallel to the 0º direction. By applying a tensile stress in the 90º direction, find the effective $E_y$ and $\nu_{yx}$ and verify that $\nu_{yx}/\nu_{yx} = E_y/E_x$. Similarly, by applying a shear stress find $G_{xy}$.

   Comment on the differences between the results for the two laminates.

f) Laminate (i) is subjected simultaneously to stresses $\sigma_x$ = 300 MPa, $\sigma_y$ = 140 MPa and $\tau_{xy}$ = 65 MPa. By extending the MATLAB script, calculate the direct and shear stress components parallel and perpendicular to the fibre direction in each ply. Hence calculate also the Tsai-Wu factor $F_{TW}$ and the inverse of the Tsai-Wu strength ratio index (1/*R*) in each ply, and deduce whether the Tsai-Wu failure criterion is violated, given the following ply strength properties:

| | |
|---|---|
| Tensile strength in fibre direction, $\hat{\sigma}_{1t}$ : | 490 MPa |
| Compressive strength in fibre direction, $\hat{\sigma}_{1c}$ : | 463 MPa, |
| Tensile strength in transverse direction, $\hat{\sigma}_{2t}$ : | 170MPa |
| Compressive strength in transverse direction, $\hat{\sigma}_{2c}$ : | 175 MPa |
| In-plane shear strength, $\hat{\tau}_{12}$ : | 189 MPa |
| Normalised interaction coefficient, $F_{12}^*$ : | – 0.5 |

Note: Since there are no bending moments and deformations, in part (f) it is only necessary to consider one ply in each of the directions 0º and 45º. Thus it is sufficient to consider only the first two plies.

**Important: Submission of solutions**

Solutions are to be submitted by 11:15 on 04.10.2011 by one of the following means:

- directly by email to brianha at math.uio.no
- placed in Brian's mail pigeon-hole on the 7th Floor
- delivered by hand at lecture on 4 October

Solutions may be either handwritten or typed using suitable word/text processing software. Solutions sent by email must be in either MS Word or (preferably) PDF format.

The MATLAB script that is developed to solve parts (d), (e) and (f) is to be handed in together with the solution. Even if the main solution is handed in on paper, the script must be sent in digital format (MATLAB input file) by email to brianha at math.uio.no.

Brian Hayman, 30.08.2011