

MEK4540 ASSIGNMENT 1 AUTUMN 2011 – NUMERICAL SOLUTIONS

All values shown here were obtained using Matlab. Some small deviations must be expected if the elastic ply properties are first calculated by hand and then input to Matlab.

The units are consistently newtons and millimetres (and MPa), except for the density in part a).

PART a)

Wf = 0.7467
Wm = 0.2533
rhoc = 1.9074 (g/cm³)
Vf = 0.5607
Vm = 0.4393
t = 0.1053

PART b)

EL = 4.2089e+004
ETsimple = 7.3021e+003
Halpin-Tsai: ET = 1.3141e+004
vLT = 0.2659

Gf = 3.0167e+004
Gm = 1.2593e+003
GLTsimple = 2.7216e+003
Halpin-Tsai: GLT = 3.9419e+003

PART c)

Q =
1.0e+004 *
4.3039 0.3573 0
0.3573 1.3438 0
0 0 0.3942

angle = 0

Qk =
1.0e+004 *
4.3039 0.3573 0
0.3573 1.3438 0
0 0 0.3942

angle = 90

Qk =
1.0e+004 *
1.3438 0.3573 -0.0000
0.3573 4.3039 0.0000
-0.0000 0.0000 0.3942

angle = 45

Qk =
1.0e+004 *
1.9848 1.1964 0.7400
1.1964 1.9848 0.7400
0.7400 0.7400 1.2333

angle = -45

Qk =

1.0e+004 *

1.9848	1.1964	-0.7400
1.1964	1.9848	-0.7400
-0.7400	-0.7400	1.2333

PART d)

Lay-up (i)

A1 =

1.0e+004 *

2.6493	0.6546	0.0000
0.6546	1.4023	0.0000
0.0000	0.0000	0.6856

B1 =

1.0e-012 *

-0.2274	-0.0284	0
-0.0284	-0.2274	0
0	0	0.0284

D1 =

1.0e+003 *

1.6757	0.3480	0.0692
0.3480	0.7996	0.0692
0.0692	0.0692	0.3664

Qtot1 =

1.0e+004 *

2.6493	0.6546	0.0000	-0.0000	-0.0000	0
0.6546	1.4023	0.0000	-0.0000	-0.0000	0
0.0000	0.0000	0.6856	0	0	0.0000
-0.0000	-0.0000	0	0.1676	0.0348	0.0069
-0.0000	-0.0000	0	0.0348	0.0800	0.0069
0	0	0.0000	0.0069	0.0069	0.0366

Lay-up (ii)

A2 =

1.0e+004 *

2.0258	0.6546	-0.0000
0.6546	2.0258	0.0000
-0.0000	0.0000	0.6856

B2 =

1.0e-012 *

-0.2274	-0.0284	0
-0.0284	-0.2274	0
0	0	0.0284

D2 =

1.0e+003 *

1.6527	0.3480	0.0692
0.3480	0.8227	0.0692
0.0692	0.0692	0.3664

Qtot2 =
 1.0e+004 *

2.0258	0.6546	-0.0000	-0.0000	-0.0000	0
0.6546	2.0258	0.0000	-0.0000	-0.0000	0
-0.0000	0.0000	0.6856	0	0	0.0000
-0.0000	-0.0000	0	0.1653	0.0348	0.0069
-0.0000	-0.0000	0	0.0348	0.0823	0.0069
0	0	0.0000	0.0069	0.0069	0.0366

Lay-up (iii)

A3 =
 1.0e+004 *

1.3247	0.3273	0.0000
0.3273	0.7011	0.0000
0.0000	0.0000	0.3428

B3 =

0	0.0000	-82.0889
0.0000	0	-82.0886
-82.0889	-82.0886	0.0000

D3 =

250.1089	28.7986	0
28.7986	88.7226	0
0	0	31.0967

Qtot3 =
 1.0e+004 *

1.3247	0.3273	0	0	0.0000	-0.0082
0.3273	0.7011	0	0.0000	0	-0.0082
0	0	0.3428	-0.0082	-0.0082	0.0000
0	0.0000	-0.0082	0.0250	0.0029	0
0.0000	0	-0.0082	0.0029	0.0089	0
-0.0082	-0.0082	0.0000	0	0	0.0031

PART e)

Here we have used just the A matrices since B = 0 and there is no bend-stretch coupling. We could instead have used the full 6x6 stiffness matrices with the bending moments in the N-vector set to zero.

Lay-up (i)

A1inv =
 1.0e-003 *

0.0427	-0.0199	-0.0000
-0.0199	0.0806	-0.0000
-0.0000	-0.0000	0.1459

Strain vector for applied $N_x = 1$ (MPa) x 8t (mm)

EpsilonFx1 =
 1.0e-004 *

0.3595
-0.1678
-0.0000

Strain vector for applied $N_y = 1$ (MPa) x 8t (mm)

EpsilonFy1 =

1.0e-004 *
-0.1678
0.6792
-0.0000

Strain vector for applied $N_{xy} = 1$ (MPa) x 8t (mm)

EpsilonFxy1 =

1.0e-003 *
-0.0000
-0.0000
0.1229

Resulting moduli:

$E_{x1} = 2.7817e+004$ (MPa)

$\nu_{xy1} = 0.4668$

$E_{y1} = 1.4724e+004$ (MPa)

$\nu_{yx1} = 0.2471$

$G_{xy1} = 8.1373e+003$ (MPa)

Lay-up (ii)

A2inv =

1.0e-003 *
0.0551 -0.0178 0.0000
-0.0178 0.0551 -0.0000
0.0000 -0.0000 0.1459

Mid-plane strain and curvature vector for applied $N_x = 1$ (MPa) x 8t (mm)

EpsilonFx2 =

0.0464
-0.0150
0.0000

Mid-plane strain and curvature vector for applied $N_y = 1$ (MPa) x 8t (mm)

EpsilonFy2 =

-0.0150
0.0464
-0.0000

Mid-plane strain and curvature vector for applied $N_{xy} = 1$ (MPa) x 8t (mm)

EpsilonFxy2 =

0.0000
-0.0000
0.1229

Resulting elastic properties:

$E_{x2} = 21\ 533$ (MPa)

$\nu_{xy2} = 0.3231$

$E_{y2} = 21\ 533$ (MPa)

$\nu_{yx2} = 0.3231$

$G_{xy2} = 8\ 137$ (MPa)

NB: This lay-up is quasi-isotropic!

PART f)

Ply strength properties (given)

$$s_{1t} = 490 \quad (\text{MPa})$$

$$s_{1c} = 463 \quad (\text{MPa})$$

$$s_{2t} = 170 \quad (\text{MPa})$$

$$s_{2c} = 175 \quad (\text{MPa})$$

$$s_{12} = 189 \quad (\text{MPa})$$

$$F_{12\text{star}} = -0.5000$$

$$F_{12} = -6.0861\text{e-}006$$

Applied stresses (given)

$$\sigma_{\text{max}} = 300 \quad (\text{MPa})$$

$$\sigma_{\text{may}} = 140 \quad (\text{MPa})$$

$$\tau_{\text{axy}} = 65 \quad (\text{MPa})$$

$$\text{Total thickness } T = 0.8426$$

Convert stresses to N-vector:

$$N =$$

$$252.7712 \quad (\text{N/mm})$$

$$117.9599 \quad (\text{N/mm})$$

$$54.7671 \quad (\text{N/mm})$$

Resulting mid-plane strains

$$\epsilon =$$

$$0.0084$$

$$0.0045$$

$$0.0080$$

Stresses in first 2 plies in x,y co-ordinates. Ply 1 in 1st column, ply 2 in 2nd column

$$\sigma =$$

$$379.0424 \quad 280.0705 \quad (\text{MPa})$$

$$90.2693 \quad 248.8434 \quad (\text{MPa})$$

$$31.4873 \quad 194.0506 \quad (\text{MPa})$$

Stresses in first 2 plies transformed to L,T co-ordinates

$$\sigma_{\text{ply}} =$$

$$379.0424 \quad 458.5076 \quad (\text{MPa})$$

$$90.2693 \quad 70.4063 \quad (\text{MPa})$$

$$31.4873 \quad -15.6133 \quad (\text{MPa})$$

Application of Tsai-Wu criterion to first 2 plies:

$$F_{\text{TW}} =$$

$$0.4885 \quad 0.6644$$

Positive roots only of equation for R:

$$R_{\text{factor}} =$$

$$1.4180 \quad 1.2198$$

$$\text{inv}R_{\text{factor}} =$$

$$0.7052 \quad 0.8198$$

Both plies are OK!

BH 19.10.2011