Oppgave 14 (kap 6)

Anta $X_1, \dots, X_{40}$ er uavh. og $\mu_x = 10$ og $\sigma_x = 6$.

La

$T_0 = X_1 + \cdots + X_{40}$

Vi har at

$\mu_{T_0} = 6 \cdot 40 = 240$

$\sigma_{T_0} = \sqrt{40}\, \sigma_x = 37,9$

$$Var\left(\sum_{i=1}^{40} X_i\right) = \sum_{i=1}^{40} Var(X_i) = 40\sigma_x^2$$

mai 3-11:15

Fra sentralgrenseteoremet er

$$\frac{T_0 - \mu_{T_0}}{\sigma_{T_0}} \overset{\text{til.}}{\sim} N(0,1)$$

Så

$$P(T_0 < 250) = P\left(\frac{T_0 - \mu_{T_0}}{\sigma_{T_0}} < \frac{250 - \mu_{T_0}}{\sigma_{T_0}}\right)$$

$$\approx \Phi\left(\frac{250-240}{37,9}\right) = \Phi(0,264)$$

$$\approx 0,6026$$

mai 3-11:15

b) $$P(T > 260) = P\left(\frac{T_0 - 240}{\sigma_{T_0}} > \frac{260 - 240}{\sigma_{T_0}}\right)$$

$$\approx 1 - \Phi\left(\frac{20}{37,9}\right)$$

$$\approx 0,2981$$

mai 3-11:24

16) La $X_i \sim N(10, 4)$

Dag 1 La $\bar{X}_1$ være gjennomsnittlig brukt tid på dag 1.

$$\bar{X}_1 \sim N\left(10, \frac{4}{5}\right)$$

tilsvarende:

Dag 2

$$\bar{X}_2 \sim N\left(10, \frac{4}{6}\right)$$

$$Var\left(\frac{1}{n}\sum X_i\right) = \frac{1}{n^2}\sum Var(X_i) = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

mai 3-11:26

Vi har at

$$P(\bar{X}_1 < 11) = \Phi\left(\frac{11-10}{\sqrt{4/5}}\right)$$

$$= 0{,}8686$$

$$P(\bar{X}_2 < 11) = \Phi\left(\frac{11-10}{\sqrt{4/6}}\right)$$

$$= 0{,}8888$$

Siden er

$$P(\bar{X}_1 < 11 \text{ og } \bar{X}_2 < 11)$$

$$= P(\bar{X}_1 < 11) \cdot P(\bar{X}_2 < 11)$$

$$= 0{,}772.$$

mai 3-11:30

E93
Oppg. 2

La $f_{X,Y}(x,y) = \begin{cases} 2e^{-x-y} & \text{for } 0<x<y \\ 0 & \text{ellers} \end{cases}$

a)

$$f_X(x) = \int_x^\infty 2e^{-x-y}\,dy$$

$$= \left[-2e^{-x-y}\right]_x^\infty = 0 + 2e^{-x-x}$$

$$= 2e^{-2x} \quad \text{for } x>0$$

mai 3-11:33

Så

$$f_X(x) = \begin{cases} 2e^{-2x} & \text{for } x>0 \\ 0 & \text{ellers} \end{cases}$$

b)

$$f_Y(y) = \int_0^y 2e^{-y-x}\,dx$$

$$= \left[-2e^{-y-x}\right]_0^y = -2e^{-2y} + 2e^{-y}$$

Så

$$f_Y(y) = \begin{cases} -2e^{-2y} + 2e^{-y} & \text{for } y>0 \\ 0 & \text{ellers.} \end{cases}$$

mai 3-11:37

Siden $f_Y(y) f_X(x) \neq f_{X,Y}(x,y)$ så er X og Y ikke uavhengige.

c)

$$E[X] = \int_0^\infty x f_X(x) = \int_0^\infty 2xe^{-2x}\,dx$$

= bruk delvis integrasjon

$$E[Y] = \int_0^\infty y f_Y(y) = \int_0^\infty -2ye^{-2y} + 2ye^{-y}\,dy$$

= bruk delvis integrasjon

mai 3-11:41

d) La $T = \frac{Y}{X} - 1$.

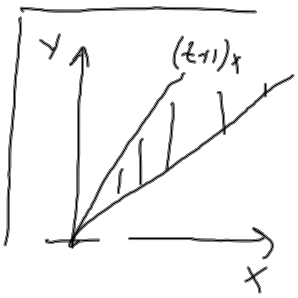
Vi har at

$$P(T < t) = P\left(\frac{Y}{X} - 1 < t\right)$$

$$= P\left(Y < (t+1)X\right)$$

$$= \int_0^\infty \int_x^{(t+1)x} 2e^{-x-y} \, dy \, dx$$

$$= \int_0^\infty \left[-2e^{-x-y}\right]_x^{(t+1)x} dx$$



mai 3-11:45

$$= \int_0^\infty -2e^{-x-(t+1)x} + 2e^{-2x} \, dx$$

$$= \int_0^\infty -2e^{-(t+2)x} + 2e^{-2x} \, dx$$

$$= \left[\frac{2}{t+2} e^{-(t+2)x} - e^{-2x}\right]_0^\infty$$

$$= 0 - \frac{2}{t+2} + 1$$

$$= 1 - \frac{2}{t+2}.$$

mai 3-11:54

så

$$f_t(t) = \frac{2}{(t+2)^2}. \quad \text{for } t > 0$$

derivate du kumulative.

Vi har

$$f_t(t) = \begin{cases} \frac{2}{(t+2)^2} & \text{for } t > 0 \\ 0 & \text{ellers} \end{cases}$$

mai 3-12:00