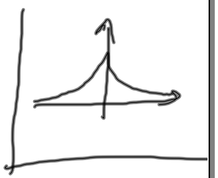
Eksamen 97.

1) $f(x) = c e^{-|x|}, \quad -\infty < x < \infty$

a) Vil bestemme c.



Vi har at

$$\int_{-\infty}^{\infty} c e^{-|x|} dx = 2\int_0^{\infty} c e^{-|x|} dx$$

$$= 2\int_0^{\infty} c e^{-x} dx$$

$$= 2\left[-c e^{-x}\right]_0^{\infty} = 2c$$

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Vi vil at

$$2c = 1 \Rightarrow \underline{c = \tfrac{1}{2}}$$

b) $$P(X < x) = \int_{-\infty}^{x} c e^{-|s|} ds$$

$$= \begin{cases} \int_{-\infty}^{x} c e^{-|s|} ds & \text{hvis } x < 0 \\ \frac{1}{2} + \int_0^x c e^{-|s|} ds & \text{hvis } x \geq 0 \end{cases}$$

$$= \begin{cases} \int_{-\infty}^{x} c e^{s} ds & \text{hvis } x < 0 \\ \frac{1}{2} + \int_0^x c e^{-s} ds \end{cases}$$

$$= \begin{cases} \left[c e^{s}\right]_{-\infty}^{x} & \text{hvis } x < 0 \\ \frac{1}{2} + \left[-c e^{-s}\right]_0^x & \text{hvis } x \geq 0 \end{cases}$$

$$= \begin{cases} \frac{1}{2} e^{x} & \text{hvis } x < 0 \\ 1 - \frac{1}{2} e^{-x} & \text{hvis } x \geq 0. \end{cases}$$

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c) Vil finne

$P(-1 < X \leq 2)$.

Vi har at

$$P(-1 < X \leq 2) = P(X \leq 2) - P(X < -1)$$

$$= 1 - \tfrac{1}{2} e^{-2} - \tfrac{1}{2} e^{-1}$$

$$\underline{\approx 0{,}748}$$

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d) $$E[X] = \int_{-\infty}^{\infty} x f(x) dx$$

$$= \int_{-\infty}^{\infty} x c e^{-|x|} dx$$

$$= \int_{-\infty}^{0} c x e^{x} dx + \int_0^{\infty} c x e^{-x} dx$$

Se på $\int_{-\infty}^{0} c x e^{x} dx$. Gjør substitusjon $u = -x$.

$$\int_{-\infty}^{0} c x e^{x} dx = \int_{\infty}^{0} c(-u) e^{-u} (-du)$$

$$= \int_{\infty}^{0} c u e^{-u} du$$

$$= -\int_{0}^{\infty} c u e^{-u} du$$

$u = -x$

$\frac{du}{dx} = -1$

$du = -dx$

$\int_a^b f(x) dx = -\int_b^a f(x) dx$

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Vi har at

$$E[X] = \int_{-\infty}^{0} c x e^{x} dx + \int_{0}^{\infty} c x e^{-x} dx$$

$$= -\int_{0}^{\infty} x c e^{-x} dx + \int_{0}^{\infty} c x e^{-x} dx$$

$$= 0.$$

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Gammafordeling:

$$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^{\alpha}\Gamma(\alpha)}$$

Vet

$$E[X] = \alpha\beta$$

$$V[X] = \alpha\beta^2$$

72) Vi har at $E[X] = 24$
og $V[X] = 144$

Så $\alpha\beta = 24$ og $\alpha\beta^2 = 144$

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Så $\beta = \frac{24}{\alpha} \Rightarrow \alpha\left(\frac{24}{\alpha}\right)^2 = 144$

$\Rightarrow \alpha = 4$

Det gir
$\beta = 6.$

a) Vil finne $P(12 < X < 24)$.
Husk at (fra side 193)

$$P(X < x) = F(x; \alpha, \beta)$$
$$= F\left(\frac{x}{\beta}; \alpha\right).$$

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Så

$$P(12 < X < 24) = F(24; \alpha, \beta) - F(12; \alpha, \beta)$$
$$= F\left(\frac{24}{6}; \alpha\right) - F\left(\frac{12}{6}; \alpha\right)$$

tabell A.4,

$$\overset{\downarrow}{=} 0{,}567 - 0{,}143$$
$$= 0{,}424.$$

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Eksponensialfordelingen

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0$$

$$F(x) = \int_0^x \lambda e^{-\lambda s}\, ds = 1 - e^{-\lambda x}$$

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La $X \sim \exp(\lambda)$

der $\lambda = 0,01386$

a)
$$P(X<100) = \int_0^{100} \lambda e^{-\lambda s}\, ds$$
$$= 1 - e^{-\lambda s}$$
$$= 1 - e^{-0,01386 \cdot 100} = 0,750.$$

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$$P(X<200) = 1 - e^{-0,01386 \cdot 200}$$
$$= 0,937.$$

og
$$P(100 < X < 200) = P(X<200)$$
$$- P(100) = 937 - 0,750$$
$$= 0,187.$$

Ekstraoppgave: (ikke i boka)

$$P(100 < X < 200 \mid X > 100)$$
$$= P(X<100) = 0,25.$$

↑
glemsomhetsegenskapen

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b)
$$E[X] = \frac{1}{\lambda} = 72,12$$
$$\sigma = \frac{1}{\lambda} = 72,12$$

Så

$$P(X > E[X] + 2\sigma)$$
$$= P(X > 3 \cdot 72,12)$$
$$= 1 - P(X < 216,45)$$
$$= 1 - 1 + e^{-0,01386 \cdot 216,45}$$
$$= 0,0498.$$

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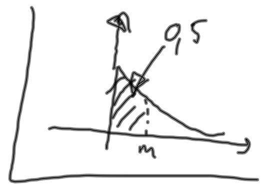
c)



Vi har at
$$0,5 = \int_0^m \lambda e^{-\lambda x}\, dx$$
$$= 1 - e^{-\lambda m}$$

Det gir
$$e^{-\lambda m} = 0,5$$
$$\lambda m = -\ln(0,5)$$
$$m = -\frac{\ln(0,5)}{\lambda}$$
$$= \underline{\underline{50}}$$

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80 Momentgenererende funksjon til gammafordeling:

$$M_x(t) = \frac{1}{(1-\beta t)^\alpha}$$

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a)

$E[X] = M_x'(0)$

$E[X^2] = M_x''(0)$

$\vdots$

Vi har at

$$M_x(t) = \frac{1}{(1-\beta t)^\alpha}$$

$$M_x'(t) = \frac{-\alpha}{(1-\beta t)^{\alpha+1}}(-\beta) = \frac{\alpha\beta}{(1-\beta t)^{\alpha+1}}$$

$$M_x''(t) = \frac{\alpha\beta^2(\alpha-1)}{(1-\beta t)^{\alpha+2}}$$

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Så

$$E[X] = M_x'(0) = \alpha\beta$$

$$E[X^2] = M_x''(0) = \alpha\beta^2(\alpha-1)$$

og

$$V[X] = E[X^2] - E[X]^2$$

$$= \alpha\beta^2(\alpha+1) - \alpha^2\beta^2$$

$$= \alpha\beta^2$$

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La $R_x(t) = \ln(M_x(t))$

Så

$$R_x(t) = -\alpha \ln(1-\beta t)$$

Vet at

$R_x'(0) = E[X]$

$R_x''(0) = V[X]$

$$R_x'(t) = \frac{\alpha\beta}{1-\beta t}$$

$$R_x''(t) = \frac{\alpha\beta^2}{(1-\beta t)^2}$$

Ser at

$$E[X] = R_x'(0) = \alpha\beta$$

$$V[X] = R_x''(0) = \alpha\beta^2$$

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