Teorem side 217.

La X ha pdf $f_X(x)$ og anta $Y = g(X)$ og g er monoton, La videre $h = g^{-1}$, da er

$$f_Y(y) = f_X(h(y))\,|h'(y)|.$$

Oppgave 110

La $f_X(x) = \frac{2}{x^3}$, $x > 1$

Vil finne pdf til $Y = \sqrt{X}$

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Bruker teoremet på side 217. hvor $g(x) = \sqrt{x}$ og $h(y) = y^2$

Vi får da

$$f_Y(y) = f_X(h(y))\,|h'(y)|$$
$$= \frac{2}{(h(y))^3}\,|2y|$$
$$= \frac{2}{y^6}\,2y = \underline{\underline{\frac{4}{y^5}}}.$$

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Kap 5.

Oppg 6

Pmf til X er gitt ved

x	0	1	2	3	4
$p_X(x)$	0,1	0,2	0,3	0,25	0,15

a) $P(X=4,\ Y=2)$

$$= P(Y=2 \mid X=4)\,P(X=4)$$
$$= \binom{4}{2} 0{,}6^2\, 0{,}4^2 \cdot 0{,}15 = \underline{\underline{0{,}05184}}$$

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Oppg 10

La X og Y være uniformt fordelt på 5 til 6

Vi anta at X og Y er uavhengige. Det betyr at

$$f(x,y) = f_X(x)\,f_Y(y)$$

Dermed er

$$f_{X,Y}(x,y) = \begin{cases} 1 & \text{hvis } 5<x,y<6 \\ 0 & \text{ellers} \end{cases}$$

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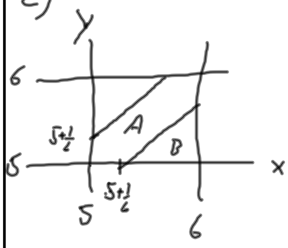
b) $P(5{,}25 < X < 5{,}75,\ 5{,}25 < Y < 5{,}75)$

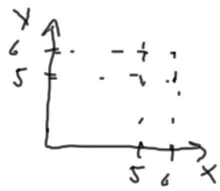
X og Y er uavh.

$\overset{\downarrow}{=} P(5{,}25 < X < 5{,}75)\, P(5{,}25 < Y < 5{,}75)$

$= \frac{1}{2} \cdot \frac{1}{2} = 1/4$

c)





Areal B $= \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{2}$

P(spise sammen) $= 1 - 2 \cdot$ (Areal B)

$= 1 - 2 \cdot \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{2}$

$= \frac{11}{36}$

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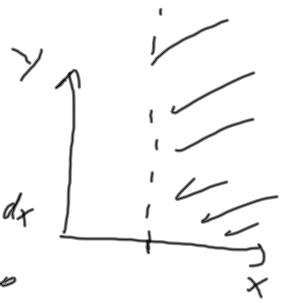
Opg 12

$$f(x,y) = \begin{cases} x e^{-x(1+y)} & x,y > 0 \\ 0 & \text{ellers} \end{cases}$$

a)



$P(X > 3)$

$$= \int_3^\infty \int_0^\infty x e^{-x(1+y)}\, dy\, dx$$

$$= \int_3^\infty \left[-e^{-x(1+y)} \right]_{y=0}^{y=\infty} dx$$

$$= \int_3^\infty e^{-x}\, dx = \left[-e^{-x} \right]_3^\infty = 0 + e^{-3} = e^{-3} \approx 0{,}050$$

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b) $$f_X(x) = \int_0^\infty x e^{-x(1+y)}\, dy$$

$$= \left[-e^{-x(1+y)} \right]_0^\infty$$

$$= e^{-x}$$

$$f_Y(y) = \int_0^\infty x e^{-x(1+y)}\, dx$$

$$= \left[\frac{x(-e^{-x(1+y)})}{1+y} \right]_{x=0}^{x=\infty} \quad \boxed{\int uv' = uv - \int u'v}$$

$$+ \int_0^\infty \frac{e^{-x(1+y)}}{1+y}\, dx$$

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$$= 0 + \left[-\frac{e^{-x(1+y)}}{(1+y)^2} \right]_{x=0}^{x=\infty} \quad \boxed{\lim_{x\to\infty} x e^{-x} = \lim_{x\to\infty} \frac{x}{e^x} = 0}$$

$$= \frac{1}{(1+y)^2}$$

Er X og Y uavh.?

Vi har

$f(x,y) \neq f_X(x) f_Y(y)$

Så X og Y er ikke uavhengige.

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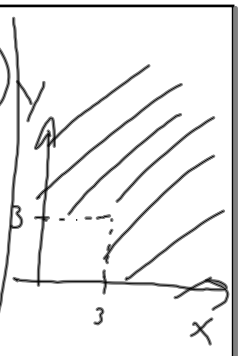
c) $P(X \text{ eller } Y > 3)$



$$= 1 - P(X<3 \text{ og } Y<3)$$

$$= 1 - \int_0^3 \int_0^3 x e^{-x(1+y)} \, dy \, dx$$

$$= 1 - \int_0^3 \Big[-e^{-x(1+y)} \Big]_{y=0}^{y=3} dx$$

$$= 1 - \int_0^3 \left(-e^{-4x} + e^{-x} \right) dx$$

$$= 1 - \left[\frac{1}{4} e^{-4x} - e^{-x} \right]_0^3$$

$$= 1 - \frac{1}{4} e^{-12} + e^{-3} + \frac{1}{4} - 1$$

$$= \frac{1}{4} + e^{-3} - \frac{1}{4} e^{-12} \approx \underline{0,300}$$

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