Interest rate modelling via SPDE's (STK 4530)
Exercises 4, 7.10.2016

Problem 1 (Estimation of the risk premium) Suppose that the dynamics of overnight rates $r(t)$ is described by the Vasicek model

$$dr(t) = a(b - r(t))dt + \sigma dB_t.$$

Once more let us cast a glance at the data set of Exercises 3, Problem 1, that is consider the Canadian overnight money market financing rates during 9 consecutive trading days given by

Day	r_i %	Day	r_i %
19/09/07	4.4502	27/09/07	4.5421
20/09/07	4.4722	28/09/07	4.5340
21/09/07	4.4555	01/10/07	4.5078
24/09/07	4.4690		
25/09/07	4.4717		
26/09/07	4.4663		

On 05/10/07 the yield of a 1 year Canadian treasury bill is quoted with 4.3800 and the overnight rate with 4.5132. Use the maximum likelihood estimators for a, b and σ in Exercises 3, Problem 1 and the market price of the treasury bill to determine (a rough) estimate for the risk premium.

Hint: Employ the theoretical bond pricing formula.

Problem 2 Assume that the overnight rates $r(t)$ follow the Cox-Ingersoll-Ross model

$$dr(t) = (a - br(t))dt + \sigma\sqrt{r(t)}dB_t.$$

Derive a formula for the present value of a zero-coupon bond and use it to compute the bond price for $a = 0.1$, $b = 0.045$, $\sigma = 0.015$, maturity $T = 1$ year, $r(0) = 0.046$ and the risk premium $q(t) = -\sqrt{r(t)}$.