

ECON 1500 - SEMINAR 6

Oppgave 1

i) a) $f(k_l, l_t) = \alpha k_l + \beta l_t = r(\alpha k + \beta l) = f(k, l)$ konstant skalaavbytte verdi

b) $f(k_l, l_t) = \min\{\alpha k_l, \beta l_t\} = f(k, l)$ konstant variabel

c) ~~$f(k_l, l_t) = A(k_l)^r (l_t)^{\bar{p}}$~~ $= A(k_l)^r (l_t)^{\bar{p}} = r^{\bar{p}} f(k, l)$ konstant om $\alpha + \beta = 1$
økende om $\alpha + \beta > 1$
synkende om $\alpha + \beta < 1$

d) $f(k_l, l_t) = (\alpha k_l^e + \beta l_t^e)^{\frac{r}{e}} = (r^e [\alpha k^e + \beta l^e])^{\frac{r}{e}} = r^{\frac{r}{e}} f(k, l)$ konstant om $r = 1$
økende om $r > 1$
synkende om $r < 1$

i) a) ~~$\min_{k_l, l_t} \{rk_l + wl_t\}$~~ gitt $\alpha k + \beta l = A$ $l = \frac{A - \alpha k}{\beta}$

$$\Rightarrow \min_{k_l, l_t} \left\{ rk_l + w \left(\frac{A - \alpha k}{\beta} \right) \right\} \quad C(r, w, A) = \begin{cases} \frac{w}{\beta} A \text{ om } r \geq \frac{\alpha}{\beta} w \\ \frac{r}{\alpha} A \text{ om } r < \frac{\alpha}{\beta} w \end{cases}$$

b) $\min_{k_l, l_t} \{rk_l + wl_t\}$ gitt $\alpha k + \beta l = A \Rightarrow \alpha k = \beta l = A \Rightarrow C(r, w, A) = \frac{r}{\alpha} A + \frac{w}{\beta} A$

c) ~~$\min_{k_l, l_t} \{rk_l + wl_t\}$~~ gitt $\min_{k_l, l_t} \{rk_l + wl_t\}$ gitt $A k_l^r l^{\bar{p}} = B$

$$L(k_l, l_t, \lambda) = rk_l + wl_t - \lambda (A k_l^r l^{\bar{p}} - B)$$

$$\frac{\partial L}{\partial k_l} = r - \alpha A k_l^{r-1} l^{\bar{p}} = 0$$

$$\frac{\partial L}{\partial l_t} = w - \beta \lambda A k_l^r l^{\bar{p}-1} = 0$$

$$A k_l^r l^{\bar{p}} = B$$

$$\lambda = \frac{r}{\alpha A l^{\bar{p}-1} l^{\bar{p}}} \Rightarrow w = \frac{\beta}{\alpha} \lambda k_l^r \frac{k}{l} \quad l = \frac{\beta}{\alpha} \frac{r}{w} k$$

$$\Rightarrow A k_l^r \left(\frac{\beta}{\alpha} \frac{r}{w} k \right)^{\bar{p}} = B$$

$$k^* = \left[\frac{B}{A} \left(\frac{\beta}{\alpha} \frac{r}{w} \right)^{\bar{p}} \right]^{\frac{1}{r+\bar{p}}}$$

$$l^* = \left[\frac{B}{A} \left(\frac{\beta}{\alpha} \frac{r}{w} \right)^{\bar{p}} \right]^{\frac{1}{r+\bar{p}}}$$

$$\Rightarrow C(r, w, B) = \left(\frac{B}{A} \right)^{\frac{1}{r+\bar{p}}} \left[r^{\frac{r}{e}} \left(\frac{r}{\beta w} \right)^{\bar{p}} + w^{\bar{p}} \left(\frac{r}{\alpha r} \right)^{\bar{p}} \right]^{\frac{1}{r+\bar{p}}}$$

d) $\min_{k_l, l_t} \{rk_l + wl_t\}$ gitt $(\alpha k^e + \beta l^e)^{\frac{r}{e}} = A$

$$L(k_l, l_t, \lambda) = rk_l + wl_t - \lambda ((\alpha k^e + \beta l^e)^{\frac{r}{e}} - A)$$

$$\frac{\partial L}{\partial k_l} = r - \lambda \frac{e}{e} (\alpha k^e + \beta l^e)^{\frac{r}{e}-1} \alpha e k^{e-1} = 0$$

$$\frac{\partial L}{\partial l_t} = w - \lambda \frac{e}{e} (\alpha k^e + \beta l^e)^{\frac{r}{e}-1} \beta e l^{e-1} = 0$$

$$(\alpha k^e + \beta l^e)^{\frac{r}{e}} = A$$

$$\lambda = \frac{r}{A} \frac{1}{e} \frac{1}{e} (\alpha k^e + \beta l^e)^{\frac{r}{e}-1}$$

$$w = r \frac{\beta}{\alpha} \left(\frac{A}{r} \right)^{\frac{1}{e}-1} \quad k = \left(\frac{r}{w} \frac{\beta}{\alpha} \right)^{\frac{1}{e}-1}$$

$$(\alpha \left(\frac{r}{w} \frac{\beta}{\alpha} \right)^{\frac{1}{e}-1} l^e + \beta l^e)^{\frac{r}{e}} = A$$

$$l^* = \frac{A}{\left(\alpha \left(\frac{r}{w} \frac{\beta}{\alpha} \right)^{\frac{1}{e}-1} + \beta \right)^{\frac{1}{e}}}$$

$$k^* = \frac{A}{\left(r + \beta \left(\frac{w}{r} \frac{\beta}{\alpha} \right)^{\frac{1}{e}-1} \right)^{\frac{1}{e}}}$$

$$\Rightarrow C(r, w, A) = r l^* + w k^*$$

PGO Symmetri

Løsning 2

$$a) \max_{l,k} \left\{ \underbrace{\rho \left(l^{\frac{1}{q}} + k_0^{\frac{1}{2}} \right)}_{\pi(l,k)} - wl - vk_0 \right\}$$

$$b) F.O.B: \frac{1}{q} \rho l^{-\frac{3}{q}} - w = 0 \quad l^{-\frac{3}{q}} = 4 \frac{w}{\rho} \quad \underline{l = \left(\frac{1}{4} \frac{\rho}{w}\right)^{\frac{q}{3}}} \quad \textcircled{B} \quad \text{effekspensel arbeidskraft}$$

$$c) \frac{\partial \pi}{\partial l} = \frac{1}{q} \rho l^{-\frac{3}{q}} - w = 0$$

$$\frac{\partial \pi}{\partial k} = \frac{1}{2} \rho k^{-\frac{1}{2}} - v = 0$$

$$d) \boxed{l^* = \left(\frac{1}{4} \frac{\rho}{w}\right)^{\frac{q}{3}}} \quad \textcircled{A} \quad \boxed{k^* = 2 \frac{v}{\rho}} \quad \boxed{k^* = \frac{1}{4} \left(\frac{\rho}{v}\right)^2}$$

$$\pi_{11}^* = -\frac{3}{16} \rho l^{-\frac{3}{q}} \leq 0 \quad \pi_{12}^* = 0 \quad \pi_{22}^* = -\frac{1}{4} \rho l^{-\frac{3}{2}} \leq 0 \quad \pi_{11}^* \pi_{22}^* - (\pi_{12}^*)^2 = \underline{\pi_{11}^* \pi_{22}^* > 0}$$

\Rightarrow Er maksimum

~~Løsning 2~~

9.6

$$q = [k^e + l^e]^{\frac{1}{e}}$$

$$a) \frac{\partial q}{\partial k} = \frac{1}{e} [k^e + l^e]^{\frac{1}{e}-1} e k^{e-1} = [k^e + l^e]^{\frac{1-e}{e}} k^{e-1} = \underline{\left(\frac{k}{l}\right)^{1-e}}$$

$$\frac{\partial q}{\partial l} = \frac{1}{e} [k^e + l^e]^{\frac{1}{e}-1} e l^{e-1} = \underline{\left(\frac{l}{k}\right)^{1-e}}$$

$$b) \text{Rate of technical substitution } RTS = -\frac{\partial k}{\partial l} \Big|_{q=q_0} = \frac{\frac{\partial q}{\partial k}}{\frac{\partial q}{\partial l}} = \frac{\left(\frac{k}{l}\right)^{1-e}}{\left(\frac{l}{k}\right)^{1-e}} = \underline{\left(\frac{k}{l}\right)^{2-e}}$$

$$\text{Elasticity of substitution } \sigma := \frac{df(k)}{dRTS} \frac{RTS}{k} = \frac{1}{\frac{\partial q}{\partial l}} \frac{RTS}{k} = \frac{1}{(1-e)\left(\frac{k}{l}\right)^e} \frac{RTS}{k} = \frac{1}{1-e} \frac{\left(\frac{k}{l}\right)^{1-e}}{\left(\frac{k}{l}\right)^e} = \underline{\frac{1}{1-e}}$$

$$c) \frac{\partial q}{\partial k} \frac{k}{q} = \left(\frac{k}{l}\right)^{1-e} \frac{k}{q} = \left(\frac{k}{l}\right)^e \quad \frac{\partial q}{\partial l} \frac{l}{q} = \left(\frac{l}{k}\right)^{1-e} \frac{l}{q} = \left(\frac{l}{k}\right)^e \quad \left(\frac{k}{l}\right)^e + \left(\frac{l}{k}\right)^e = \underline{\frac{1}{q^e} (k^e + l^e) = \frac{q^e}{q^e} = 1}$$

$$d) \left(\frac{\partial q}{\partial l}\right)^\sigma = \left(\frac{k}{l}\right)^{1-e} = \frac{k}{l} \quad \Rightarrow \underline{\ln\left(\frac{k}{l}\right) = \sigma \ln\left(\frac{q}{l}\right)}$$

9.8

~~$f(kl, lt) = q^{\frac{1}{k+l}}$~~ homogeneous of degree 1

$\Rightarrow f'_k(kl, lt)k + f'_l(kl, lt)l = q$

Evaluated at $l=0$, this gives $\underline{q = f_k k + f_l l}$

$MP_l = \underline{f'_l} \quad AP_l = \frac{q}{l} = \frac{f_k k + f_l l}{l} = \underline{f_k \frac{k}{l} + f_l}$

$MP_l > AP_l \Leftrightarrow f'_l > f_k \frac{k}{l} + f_l \Leftrightarrow \underline{f_k < 0}$

Dette betyr at man vil bruke mindre k , men l \Rightarrow Produktet av marginalproduktet er høyere enn gjennomsnittsproduktet

AP_l øker betyr MP_l > AP_l, så vil ikke altokes geske, så nei.
 (med mindre ressursbegrensninger eller annet står i veien).

avdelinger

10.2

$\bar{q} = s^{\frac{1}{2}} J^{\frac{1}{2}} \quad S: 900 \text{ h}, 3 \text{ per h} \quad J: 12 \text{ per h}$

$a) \bar{q} = \sqrt{900} \sqrt{J} = 30\sqrt{J} \Rightarrow J = \frac{\bar{q}^2}{30^2} \quad J = \frac{1}{900} \bar{q}^2$

$J(150) = 25 \quad J(300) = 100 \quad J(450) = 225$

$b) \cancel{\text{Diagram}} \quad \cancel{\text{dgp}} \quad J'(q) = \frac{1}{450} q \quad \Rightarrow MK = \frac{12}{450} q = \frac{4}{150} q = \frac{2}{75} q$

$MK(150) = \frac{300}{75} = 4 \quad MK(300) = 8 \quad MK(450) = 12$

10,6

$$C = \frac{2}{9} w^{\frac{2}{3}} v^{\frac{1}{3}}$$

a) Shephard's lemma: $\lambda = \frac{\partial C}{\partial w}, k = \frac{\partial C}{\partial v}$

$$\Rightarrow \underline{\lambda = \frac{2}{3} \bar{q} (\bar{w})^{\frac{1}{3}}} \quad \underline{k = \frac{1}{3} \bar{q} (\bar{w})^{\frac{2}{3}}}$$

~~$\lambda = \frac{2}{3} \bar{q} (\bar{w})^{\frac{1}{3}}$~~

~~$\lambda = \frac{2}{3} \bar{q} (\bar{w})^{\frac{1}{3}} + \frac{1}{3} \bar{q} (\bar{v})^{\frac{1}{3}}$~~

~~$\lambda = \frac{2}{3} \bar{q} (\bar{w})^{\frac{1}{3}} + \frac{1}{3} \bar{q} (\bar{v})^{\frac{1}{3}} - \frac{2}{3} \bar{q} (\bar{v})^{\frac{1}{3}}$~~

~~$\lambda = \frac{2}{3} \bar{q} (\bar{w})^{\frac{1}{3}} + \frac{1}{3} \bar{q} (\bar{v})^{\frac{1}{3}} - \frac{2}{3} \bar{q} (\bar{v})^{\frac{1}{3}}$~~

$$b) \lambda = \frac{2}{3} \bar{q} (\bar{w})^{\frac{1}{3}} \quad \text{and} \quad \underline{\lambda = \frac{3}{2} \frac{1}{q}}$$

~~$\Rightarrow k = \frac{1}{3} q + \frac{1}{(\frac{3}{2} \frac{1}{q})^2}$~~

~~$\Rightarrow \frac{1}{3} q + \frac{1}{q^2} \quad q^2 = k l^2 \frac{27}{4}$~~

~~$\Rightarrow q = 3 \left[k \left(\frac{1}{2} l \right)^2 \right]^{\frac{1}{3}}$~~