

26.5.

Problem 5:

- Demand: $q_1 = 22 - 2p_1 + p_2$

$$q_2 = 22 - 2p_2 + p_1$$

- $c(q_1) = 10q_1$

$$c(q_2) = cq_2$$

a) Write the firms payoff functions:

$$\pi_1 = p_1 q_1 - 10q_1 = q_1 (p_1 - 10)$$

$$= (22 - 2p_1 + p_2) (p_1 - 10)$$

$$\pi_2 = p_2 q_2 - cq_2 = q_2 (p_2 - c)$$

$$= (22 - 2p_2 + p_1) (p_2 - c)$$

b) Calculate Best response functions:

Firm 1: $\frac{\partial \pi_1}{\partial p_1} = -2(p_1 - 10) + 22 - 2p_1 + p_2 = 0$

$$4p_1 = 42 + p_2$$

$$BR_1(p_2) \equiv p_1 = \frac{42 + p_2}{4}$$

Firm 2: $\frac{\partial \pi_2}{\partial p_2} = -2(p_2 - c) + 22 - 2p_2 + p_1 = 0$

$$4p_2 = 22 + p_1 + 2c$$

$$BR_2(p_1) \equiv p_2 = \frac{22 + 2c + p_1}{4}$$

c) From 6 we have that

$$1) p_1 = \frac{42 + p_2}{4} = \frac{42}{4} + \frac{1}{4} p_2.$$

$$\text{and } 2) p_2 = \frac{1}{4} (22 + 2c + p_1).$$

Thus enter 2) into 1). thus:

$$p_1 = \frac{42}{4} + \frac{1}{4} \cdot \frac{1}{4} (22 + 2c + p_1) \quad | \cdot 16$$

And enter that $c = 10$ gives:

$$16 p_1 = 42 \times 4 + 22 + 2 \times 10 + p_1$$

$$15 p_1 = 42 \times 5$$

$$p_1 = \frac{42 \cdot 5}{15} = \frac{42}{3} = 14$$

$$\text{and } p_2 = \frac{42 + p_1}{4} = \frac{42 + 14}{4} = 14.$$

d) Assume two types of player 2 one with Low costs ($c=6$) and one with high ($c=14$) each with equal probability.

$$3) BR_2^L(p_1) = \frac{22 + 2 \cdot 6 + p_1}{4} = \frac{34 + p_1}{4}$$

i.e. best response function of firm 2 with low costs:

$$4) BR_2^H(p_1) = \frac{22 + 2 \cdot 14 + p_1}{4} = \frac{50 + p_1}{4}$$

Now firm 1 has a new profit function:

$$1) \pi_1 = \left(22 - 2p_1 + \frac{1}{2} p_2^H + \frac{1}{2} p_2^L \right) (p_1 - 10)$$

i.e. with $\frac{1}{2}$ probability he meets firm 2 which have high costs and $\frac{1}{2}$ probability he meets low costs.

Thus

$$5) BR_1(p_2) = \frac{42}{4} + \frac{1}{4} \left(\frac{1}{2} p_2^H + \frac{1}{2} p_2^L \right)$$

Enter 3) and 4) into 5). gives:

$$p_1 = \frac{42}{4} + \frac{1}{8} \left(\frac{34 + p_1}{4} + \frac{50 + p_1}{4} \right) \quad | \cdot 32$$

$$32p_1 = 8 \cdot 42 + 34 + p_1 + 50 + p_1$$

$$30p_1 = 8 \cdot 42 + 84 = 10 \cdot 42$$

$$p_1 = \frac{10 \cdot 42}{30} = 14$$

$$p_2^L = \frac{34 + 14}{4} = 12 \quad p_2^H = \frac{50 + 14}{4} = 16.$$